GUP Modified Phase-Space and Thermodynamics Self-Consistency

Asmaa G. Shalaby

Department of Physics, Faculty of Science, *13518* Benha University, Egypt.

Asmaa.shalaby@fsc.bu.edu.eg

ECEA 2017

4th International Electronic Conference on Entropy and Its Applications

Abstract

Based on the generalized uncertainty principle (GUP) with the modification of the phase space, the partition function has recently been modified . In the present work, we analyze the self-consistency of the axiomatic thermodynamic derived from the modified partition function. This work studies the self-consistency for the thermodynamic quantities pressure, energy density, entropy, number density and some correlated quantities. The thermal parameters at the moment of freeze-out are extracted, as well as the occupation number of the fermionic states. We found that, the deformed phase space distribution does not obey the thermodynamic consistency conditions. On the other hand, the thermodynamic quantities and freeze-out parameters are well reproduced.

Introduction

The generalized uncertainty Principle (GUP), GUP is supposed to combine quantum mechanics and the general relativity. GUP is supposed to solve what so called trans-Planckinan problem in which some physical quantities appeared in a range beyond the Planck scale. **The Planck length**

$$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \simeq 1.616 \times 10^{-35} m \text{ or } \ell_p = \frac{\hbar}{m_p}$$

Based on the approach of modified commutation relation .

$$[x_i, p_j] = i\hbar\left[\delta_{ij} - \alpha\left(p\delta_{ij} + \frac{p_i p_j}{p}\right) + \alpha^2\left(p^2\delta_{ij} + 3p_i p_j\right)\right]$$

where $p^2 = \sum_{j=1}^{3} p_j p_j$, $\alpha = \frac{\alpha_0}{(M_{p\ell} c)}$, $M_{p\ell}$ is Planck mass.

$$\Delta x \Delta p \geq \frac{\hbar}{2}[1 - 2\alpha < p > + 4\alpha^2 < p^2 >]$$
$$\geq \frac{\hbar}{2}\left[1 + \left(\frac{\alpha}{\sqrt{< p^2 >}} + 4\alpha^2\right)\Delta p^2 + 4\alpha^2 < p >^2 - 2\alpha\sqrt{< p^2 >}\right]$$

The modified partition function is written as;

$$\ln Z = \pm \sum_i \frac{V g_i}{(2\pi)^2 \hbar^3} \int \ln[1 \pm \exp(-(\varepsilon_i - \mu_i)/k_B T)] \frac{p^2 dp}{(1 - \alpha p)^4},$$

The derived thermodynamics quantities such as pressure, number density, energy density and entropy

- Pressure, P

$$P = \frac{T}{V}\left(\frac{\partial \ln Z}{\partial V}\right),$$
$$= \pm \sum_i \frac{g_i}{(2\pi)^2} \int \ln[1 \pm \exp(-(\varepsilon_i - \mu_i)/T)] \frac{p^2 dp}{(1 - \alpha p)^4},$$

- Number density, n

$$n = \frac{T}{V}\left(\frac{\partial \ln Z}{\partial \mu_i}\right),$$
$$= \pm \sum_i \frac{g_i}{(2\pi)^2} \int \frac{\exp(-(\varepsilon_i - \mu_i)/T)}{\ln[1 \pm \exp(-(\varepsilon_i - \mu_i)/T)]} \frac{p^2 dp}{(1 - \alpha p)^4},$$

- Energy density, ϵ

$$\epsilon = \frac{-1}{V}\left(\frac{\partial \ln Z}{\partial (1/T)}\right),$$
$$= \mp \sum_i \frac{g_i}{(2\pi)^2} \int \frac{\exp(-(\varepsilon_i - \mu_i)/T)(\mu_i - \varepsilon_i)}{[1 \pm \exp(-(\varepsilon_i - \mu_i)/T)]} \frac{p^2 dp}{(1 - \alpha p)^4},$$

- entropy density, s

$$s = \frac{1}{V}\left(\frac{\partial T \ln Z}{\partial T}\right),$$
$$= \pm \sum_i \frac{g_i}{(2\pi)^2} \int p^2 dp \left[\frac{\exp(-(\varepsilon_i - \mu_i)/T)(-\frac{\mu_i}{T^2} + \frac{\varepsilon_i}{T^2})}{[1 \pm \exp(-(\varepsilon_i - \mu_i)/T)](1 - \alpha p)^4} + \frac{\ln(1 + \exp(-(\varepsilon_i - \mu_i)/T))}{(1 - \alpha p)^4}\right],$$

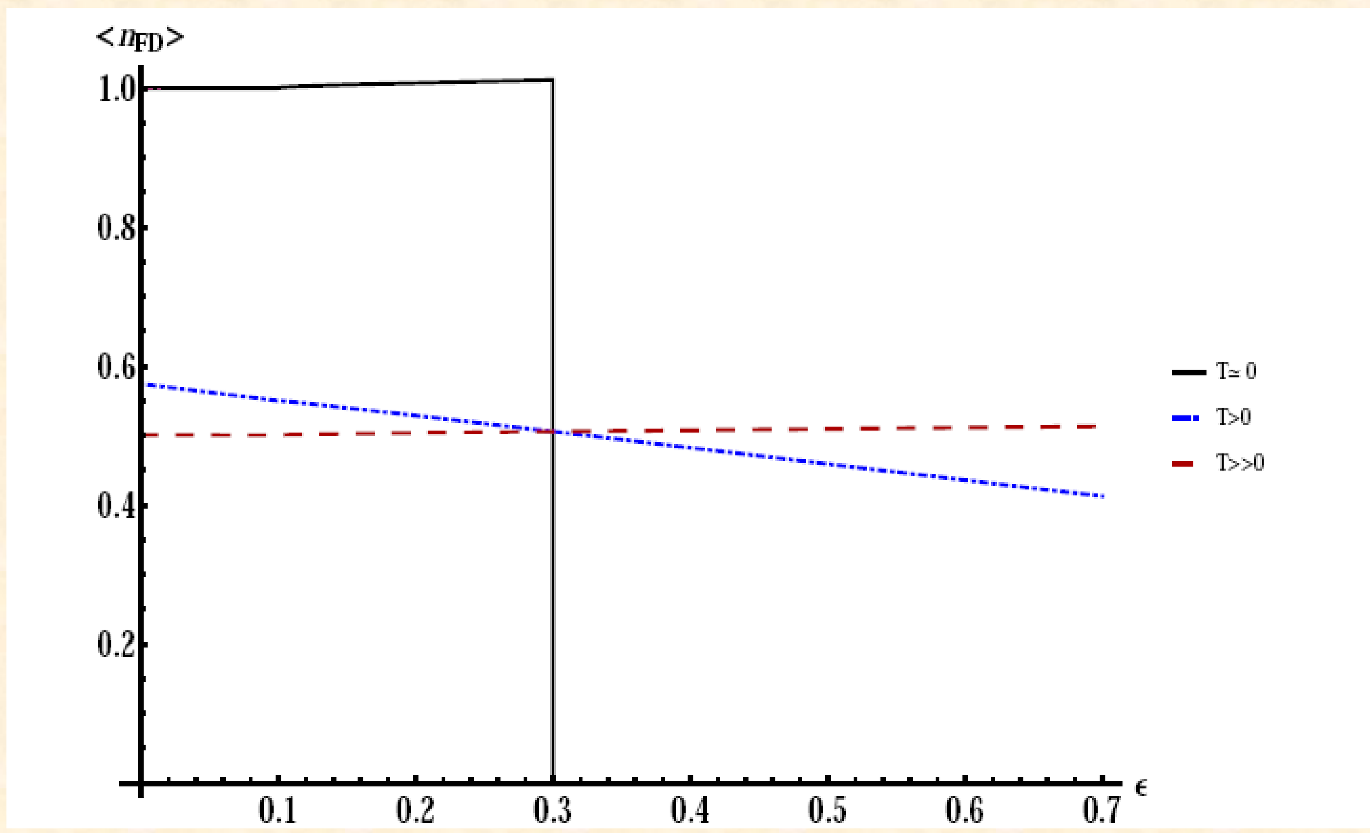
The thermodynamic consistency can be proven if the following conditions are satisfied . The conditions are:-

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T,$$
$$T = \left.\frac{\partial \epsilon}{\partial s}\right|_n,$$
$$\mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s,$$
$$s = \left.\frac{\partial P}{\partial T}\right|_\mu.$$

Results

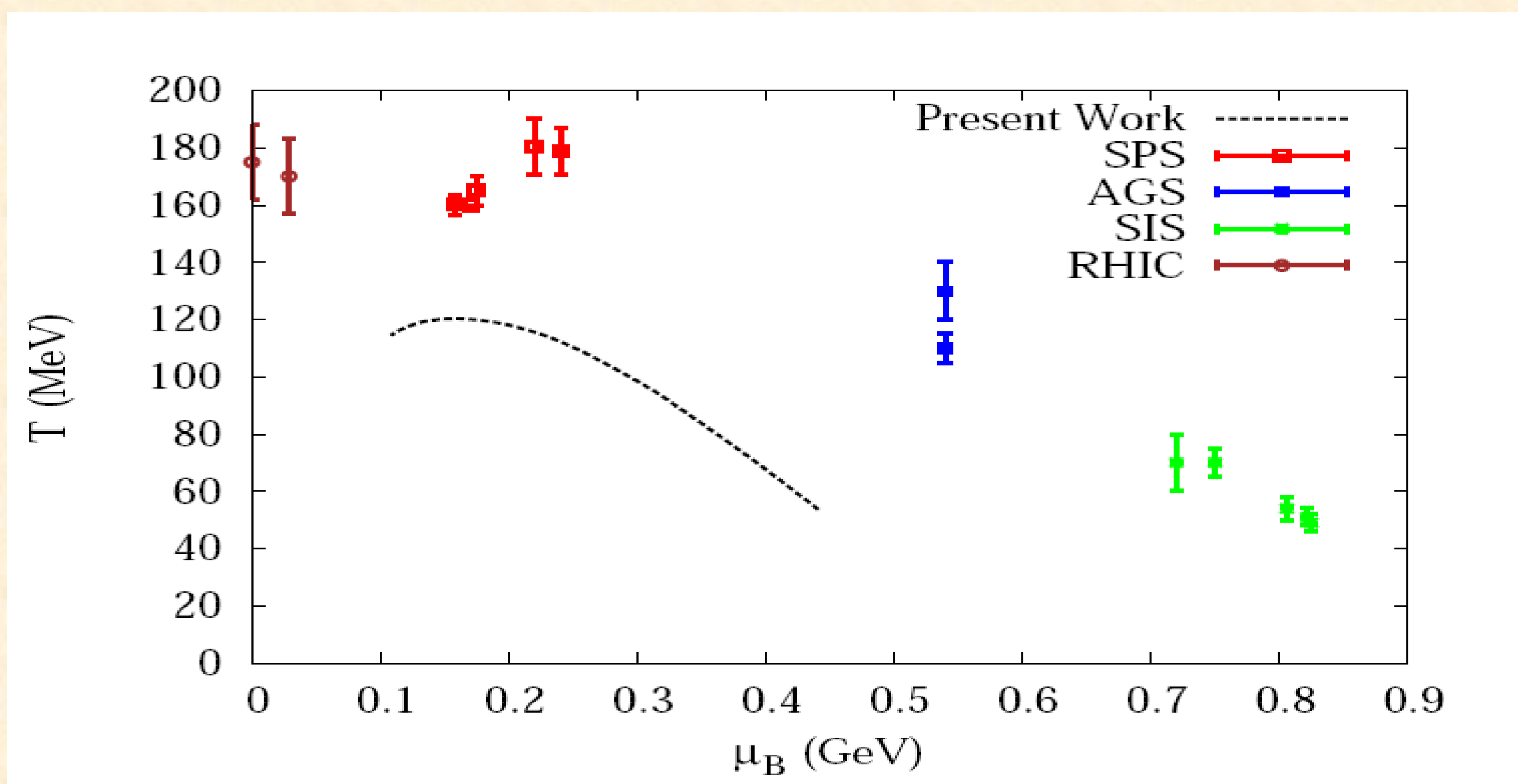
1- GUP effect on the occupation number

$$< n_{FD} >_{GUP} = \frac{1}{\lambda^{-1} e^{\beta \varepsilon_i} + 1}\left[\frac{1}{(1 - \alpha k)^4}\right]$$



Modified occupation number due to deformed phase-space α **= 0:01GeV** $^{-1}$

2- FREEZE-OUT PARAMETERS CONFRONTING TO EXPERIMENTAL DATA



$T - \mu_B$ **extracted from freeze-out condition** $\frac{\epsilon}{n} \approx 1$ **the online colored symbols are the different experimental data.**

Conclusion

We derived the different thermodynamic quantities from the modfied partition function. This modification is due to GUP which affects the phase-space by an extra term. We concluded that, this kind of thermodynamics is reproduced from the corrected phase-space partition function at least qualitatively. Concretely, we found that the thermodynamic consistency could not be satisfied. So, thus the occupation number of the accessible states is modified too, which proved that, Pauli's principle is still preserved at $T \simeq 0$. And the extracted thermal parameters appeared lower than the experimental one by applying the freeze-out condition $\frac{\epsilon}{n} \approx 1$.