Maxwell’s Demon and Comoving Observers in General Relativity: What Do They Have in Common?

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Abstract: We elaborate on the similarities between the explanation of the well known Maxwell’s demon paradox, based on the theory of information, and the rationale behind the fact that real dissipative (entropy producing) processes may be detected by non–comoving observers (tilted), in systems that appear to be isentropic for comoving observers, in general relativity.

1. Introduction

More than a century ago, J.C. Maxwell [1] put forward a paradox, usually referred to as Maxwell’s demon. In one of its many (but equivalent) versions, the Maxwell’s demon is a small “being” living in a cylinder filled with a gas, and divided in two equal portions, by a partition with a small door. Then the demon may open the door when the molecules come from the right, while closing it when the molecules approach from the left. Doing so, the demon is able to concentrate all the molecules on the left, reducing the entropy by $Nk \ln 2$ (where $N$ is the number of molecules, and $k$ is the Boltzman constant), thereby violating the second law of thermodynamics.

In order to solve this paradox, some researchers (e.g., Brillouin [2]) argued that in the process of selection of molecules, the demon increases the entropy by an amount equal or larger than the decreasing of entropy achieved by concentrating all molecules on one side of the cylinder.

However, soon after, different researchers were able to propose different ways by means of which the demon could select the molecules in a reversible way (i.e., without entropy production). It was necessary to wait for more than a century, until Bennet [3], gave a satisfactory resolution of this paradox.

It is our purpose in this work to stress the fact that an argument similar to the one put forward by Bennet to solve the Maxwell’s demon paradox, may be used to explain the very different pictures of a given system, presented by different congruence of observers in general relativity.

Indeed, there is an ambiguity in the description of the source of the gravitational field (whenever it is represented by a fluid distribution), which is related to the arbitrariness in the choice of the four–velocity in terms of which the energy–momentum tensor is split.

The above mentioned arbitrariness, in its turn, is related to the well known fact, that different congruences of observers would assign different four–velocities to a given fluid distribution. We have in mind here, the situation when one of the congruences corresponds to comoving observers, whereas the other is obtained by applying a Lorentz boost to the comoving observer’s frame (this Lorentz boosted congruence is usually referred to as the tilted congruence).

The strange fact then appears, that systems that are isentropic for comoving observers, may become dissipative for tilted observers (see [4–13] and references therein).

When we go from comoving observers (which assign zero value to the three-velocity of any fluid element) to tilted observers, for whom the three-velocity represents another degree of freedom, the erasure of the information stored by comoving observers (vanishing three velocity), explains the presence of dissipative processes (included gravitational radiation) detected by tilted observers.
In this work we illustrate this situation by analyzing, from the point of view of the tilted congruence, an axially symmetric fluid distribution which, for the comoving congruence of observers, is geodesic, shear–free, irrotational and non–dissipative.

As expected from previous works, the fluid distribution appears to be dissipative for the tilted observer, however, and unlike previously analyzed examples, the tilted observer, in this case, will detect a flux of gravitational radiation, associated to the magnetic part of the Weyl tensor, which for the tilted observers is non vanishing.

An explanation for these results is given in terms of the information theory, in analogy with the explanation of the Maxwell’s demon paradox.

2. Comoving and Tilted Observers

Let us consider a congruence of observers which are comoving with an arbitrary fluid distribution, then the four–velocity for that congruence, in some globally defined coordinate system, reads

\[ V^\mu = (V^0, 0, 0, 0). \]  

(1)

In order to obtain the four–velocity corresponding to the tilted congruence (in the same globally defined coordinate system), one proceeds as follows.

We have first to perform a (locally defined) coordinate transformation to the Locally Minkowskian Frame (LMF). Denoting by \( L^\nu_\mu \) the local coordinate transformation matrix, and by \( \bar{V}^\alpha \) the components of the four velocity in such LMF, we have:

\[ \bar{V}^\mu = L^\mu_\nu V^\nu. \]  

(2)

Next, let us perform a Lorentz boost from the LMF associated to \( \bar{V}^\alpha \), to the (tilted) LMF with respect to which a fluid element is moving with some, non–vanishing, three–velocity.

Then the four–velocity in the tilted LMF is defined by:

\[ \tilde{\bar{V}}_\beta = \Lambda^\alpha_\beta \bar{V}^\alpha, \]  

(3)

where \( \Lambda^\alpha_\beta \) denotes the Lorentz matrix.

Finally, we have to perform a transformation from the tilted LMF, back to the (global) frame associated to the line element under consideration. Such a transformation, which obviously only exists locally, is defined by the inverse of \( L^\nu_\mu \), and produces the four–velocity of the tilted congruence, in our globally defined coordinate system, say \( \bar{V}^\alpha \).

Let us now consider a given spacetime, which according to comoving observers, is sourced by a dissipationless anisotropic fluid distribution, so that the energy momentum–tensor in the “canonical” form reads:

\[ T_{\alpha\beta} = (\mu + P)V_\alpha V_\beta + P g_{\alpha\beta} + \Pi_{\alpha\beta}, \]  

(4)

where as usual, \( \mu, P, \Pi_{\alpha\beta}, V_\beta \) denote the energy density, the isotropic pressure, the anisotropic stress tensor and the four velocity, respectively.

If we now impose the shear–free and the geodesic conditions, and assume that the fluid is non–dissipative, the line element becomes [14]

\[ ds^2 = -dt^2 + B^2(t) \left[ dr^2 + r^2 d\theta^2 + R^2(r, \theta) d\phi^2 \right]. \]  

(5)

where \( B(t) \) and \( R(r, \theta) \) are functions of their arguments satisfying the Einstein equations, and from regularity conditions at the origin we must require \( R(0, \theta) = 0 \). Also it can be shown that all, geodesic and shear–free fluids, are necessarily irrotational (see [14] for details).

For our comoving observer the four–velocity vector reads
\[ V^\alpha = (1, 0, 0, 0) ; \quad V_\alpha = (-1, 0, 0, 0). \] (6)

We shall next define a canonical orthonormal tetrad (say \( e^{(a)}_\alpha \)), by adding to the four–velocity vector \( e^{(0)}_\alpha = V_\alpha \), three spacelike unitary vectors (these correspond to the vectors \( K, L, S \) in [15])

\[ e^{(1)}_\alpha = K_\alpha = (0, B, 0, 0); \quad e^{(2)}_\alpha = L_\alpha = (0, 0, Br, 0), \] (7)

\[ e^{(3)}_\alpha = S_\alpha = (0, 0, 0, BR), \] (8)

with \( a = 0, 1, 2, 3 \) (latin indices labeling different vectors of the tetrad)

For the energy density and the isotropic pressure, we have

\[ \mu = T_{\alpha\beta} e^{(0)}_\alpha e^{(0)}_\beta, \quad P = \frac{1}{3} h^{\alpha\beta} T_{\alpha\beta}, \] (9)

with

\[ h^{\alpha\beta} = \delta^{\alpha\beta} + V^{\alpha} V^{\beta}, \] (10)

whereas the anisotropic tensor may be expressed through three scalar functions defined as (see [15], but notice the change of notation):

\[ \Pi_{(2)(1)}(\alpha) = e^{(2)}_\alpha e^{(1)}_\beta T_{\alpha\beta}, \] (11)

\[ \Pi_I = \left( 2e^{(1)}_\alpha e^{(1)}_\beta - e^{(2)}_\alpha e^{(2)}_\beta - e^{(3)}_\alpha e^{(3)}_\beta \right) T_{\alpha\beta}, \] (12)

\[ \Pi_{II} = \left( 2e^{(2)}_\beta e^{(2)}_\beta - e^{(3)}_\beta e^{(3)}_\beta - e^{(1)}_\beta e^{(1)}_\beta \right) T_{\alpha\beta}. \] (13)

In [14] it was shown, that for the geodesic, shear–free non–dissipative fluid, we have: \( \Pi_{(2)(1)} = \Pi_I = \Pi_{II} = \Pi \), accordingly, the anisotropic tensor may be written in the form:

\[ \Pi_{\alpha\beta} = \Pi \left( e^{(1)}_\beta e^{(1)}_\beta + e^{(2)}_\beta e^{(2)}_\beta + e^{(3)}_\beta e^{(3)}_\beta - \frac{2h_{\alpha\beta}}{3} \right). \] (14)

As mentioned before, for the comoving observer, and the line element Equation (5), the four–acceleration, the shear and the vorticity vanish, whereas for the expansion we get:

\[ \Theta = \frac{3\dot{B}}{B}, \] (15)

where overdot denotes derivatives with respect to \( t \).

Also, as shown in [14], the magnetic part of the Weyl tensor calculated by means of the four–velocity vector Equation (6) vanishes, and the electric part is defined through a unique scalar function.

We shall now analyze the line element Equation (5) from the point of view of the tilted observer (see [16] for details).

For doing so, we have to obtain first the tilted congruence and all the associated kinematical variables, applying the procedure sketched above.

In our case we have for \( L_\mu^\beta \), where \( x^\mu \) denotes the Locally Minkowskian coordinates:

\[ \nabla^\mu = L_\mu^\beta V^\beta, \] (16)
where

\[ L_3^0 = 1; \quad L_1^1 = B; \quad L_2^2 = Br; \quad L_3^3 = BR. \]  

Next, the boost is applied along the two independent directions \((\tilde{x}^1, \tilde{x}^2)\), thus we have:

\[ \Lambda^0_0 = \Gamma; \quad \Lambda^0_i = -\Gamma \tilde{v}_i; \quad \Lambda^i_j = \delta^i_j + \frac{(\Gamma - 1)v_i v_j}{v^2}, \]  

where latin indices \(i, j\) run from 1 to 3, \(\Gamma \equiv \frac{1}{\sqrt{1 - v^2}}, \quad v^2 = v_1^2 + v_2^2\), and \(v_1, v_2\) are the two non-vanishing components of the three-velocity of a fluid element as measured by the tilted observer.

This produces:

\[ \tilde{\varepsilon}^{(0)}_a = \tilde{\varphi}_a = \left( -\Gamma, B\Gamma v_1, Br\Gamma v_2, 0 \right); \quad \tilde{\varepsilon}^a = \left( \Gamma, \frac{\Gamma v_1}{B}, \Gamma v_2, 0 \right). \]  

\[ \tilde{\varepsilon}^{(1)}_a = \left( -\Gamma v_1, B \left[ 1 + \frac{(\Gamma - 1)v_1^2}{v^2} \right], \frac{Br(\Gamma - 1)v_1 v_2}{v^2}, 0 \right), \]  

\[ \tilde{\varepsilon}^{(2)}_a = \left( -\Gamma v_2, \frac{B(\Gamma - 1)v_1 v_2}{v^2}, Br \left[ 1 + \frac{(\Gamma - 1)v_2^2}{v^2} \right], 0 \right), \]  

and

\[ \tilde{\varepsilon}^{(3)}_a = \tilde{\varepsilon}^{(3)} = (0, 0, 0, Br), \]

where for simplicity we have omitted the bar over the components of the three velocity.

We can now calculate all the kinematical variables for the tilted congruence.

The four acceleration

\[ \tilde{a}_a = \tilde{\varphi}_a \tilde{V}_{a,\beta}, \]

may be expressed through two scalar functions as:

\[ \tilde{a}_\mu = \tilde{a}^{(1)}_a \tilde{\varepsilon}^{(1)}_a + \tilde{a}^{(2)}_a \tilde{\varepsilon}^{(2)}_a. \]

From the coordinate components of \(a_\mu\), we can easily find the explicit expressions for the two scalars \(a^{(1)}_a\) and \(a^{(2)}_a\).

It is a simple matter to check that if we put \(v = 0, \Gamma = 1\), we obtain \(\tilde{a}_\mu = 0\), as expected.

Next, the shear tensor

\[ \tilde{\sigma}_{a\beta} = \tilde{\varepsilon}^{(a)}_{(a)(b)} \tilde{\varepsilon}^{(b)}_{a\beta} = \tilde{\varphi}_{(a)\beta} + \tilde{a}_{(a)\beta} - \frac{1}{3} \tilde{\varphi}_{\alpha\beta}, \]

may be defined through two independent tetrad components (scalars) \(\tilde{\sigma}^{(1)}_{(1)}\) and \(\tilde{\sigma}^{(2)}_{(2)}\), defined by:

\[ \tilde{\sigma}^{(1)}_{(1)} = 3\tilde{\varepsilon}^{(1)}_{(1)} \tilde{\varepsilon}^{(1)}_{a\beta}, \quad \tilde{\sigma}^{(2)}_{(2)} = 3\tilde{\varepsilon}^{(2)}_{(2)} \tilde{\varepsilon}^{(2)}_{a\beta}. \]

These two scalars may be easily obtained from Equation (26) and the expressions for the non-vanishing coordinate components of the shear tensor.

Again, if we go back to the comoving congruence by assuming \(v = 0\) (\(\Gamma = 1\)), we get \(\tilde{\sigma}_{a\beta} = 0\). For the vorticity vector defined as:

\[ \tilde{\omega}_a = \frac{1}{2} \eta_{a\beta\mu} \tilde{\varphi}^{\beta\mu} \tilde{V}^\nu = \frac{1}{2} \eta_{a\beta\mu} \tilde{\Omega}^{\beta\mu} \tilde{V}^\nu, \]

where \(\tilde{\Omega}_{a\beta} = \tilde{\varphi}_{(a)\beta} + \tilde{a}_{(a)\beta}\) denotes the vorticity tensor; we find a single component different from zero, producing:
with the scalar function \( \tilde{\Omega} \) given by
\[
\tilde{\Omega} = -\tilde{\Omega}^{(3)}_a.
\]
(29)

Finally, we may write for the heat flux vector:
\[
\tilde{q}_\mu = \tilde{q}^{(1)}_\mu + \tilde{q}^{(2)}_\mu.
\]
(37)

Since, both congruences of observers are embedded within the same space–time Equation (5), then it is obvious that the Einstein tensor is the same for both congruences, and therefore the energy–momentum tensors, although split differently, also must be the same.

Then equating Equations (4) and (32), and projecting on all possible combinations of tetrad vectors (tilded and non–tilted), we find expressions of physical variables measured by comoving observers, in terms of the tilted ones, and viceversa. These are exhibited in [16].

For the tilted congruence, the electric part of the Weyl tensor has three independent non-vanishing components, and the magnetic part of the Weyl tensor is non–vanishing, and defined through two
components. Thus we may write these two tensors, in terms of five tetrad components \( (\tilde{\xi}_1^{(1)}, \tilde{\xi}_2^{(1)}, \tilde{\xi}_1^{(2)}, \tilde{\xi}_2^{(2)}, \tilde{\xi}_3^{(1)}, \tilde{\xi}_3^{(2)}) \), respectively as:

\[
\tilde{E}_{\alpha\beta} = \left[ (2\tilde{\xi}_1^{(1)} + \tilde{\xi}_2^{(2)})(\tilde{\xi}_\alpha^{(1)}\tilde{\xi}_\beta^{(1)} - \frac{1}{2}h_{\alpha\beta}) + (2\tilde{\xi}_2^{(2)} + \tilde{\xi}_1^{(1)})(\tilde{\xi}_\alpha^{(2)}\tilde{\xi}_\beta^{(2)} - \frac{1}{2}h_{\alpha\beta}) \right] + \left[ (2\tilde{\xi}_2^{(2)} + \tilde{\xi}_1^{(1)})(\tilde{\xi}_\alpha^{(1)}\tilde{\xi}_\beta^{(1)} - \frac{1}{2}h_{\alpha\beta}) + \left( 2\tilde{\xi}_1^{(1)} + \tilde{\xi}_2^{(2)} \right) \right] \]

and

\[
\tilde{H}_{\alpha\beta} = \tilde{H}_{1(3)}^{(1)} \left( \tilde{\xi}_\alpha^{(1)}\tilde{\xi}_\beta^{(1)} + \tilde{\xi}_\alpha^{(2)}\tilde{\xi}_\beta^{(2)} \right) + \tilde{H}_{2(3)}^{(3)} \left( \tilde{\xi}_\alpha^{(2)}\tilde{\xi}_\beta^{(2)} + \tilde{\xi}_\alpha^{(3)}\tilde{\xi}_\beta^{(3)} \right).
\]

From the above expressions we can calculate the super–Poynting vector, in terms of only two scalar functions as:

\[
P_\alpha = P_1^{\alpha(1)} + P_2^{\alpha(2)}.
\]

Where

\[
P_1^{(1)} = 2\tilde{H}_{2(3)}^{(3)} \left( 2\tilde{\xi}_2^{(2)} + \tilde{\xi}_1^{(1)} \right) + 2\tilde{H}_{1(3)}^{(1)}\tilde{\xi}_2^{(2)} + 32\pi^2\tilde{q}_{(1)} \left[ (\tilde{\mu} + \tilde{\rho}) + \frac{\tilde{\Pi}}{\tilde{T}} \right]
\]

and

\[
P_2^{(2)} = -2\tilde{H}_{1(3)}^{(1)} \left( 2\tilde{\xi}_1^{(1)} + \tilde{\xi}_2^{(2)} \right) - 2\tilde{H}_{2(3)}^{(3)}\tilde{\xi}_1^{(1)} + 32\pi^2\tilde{q}_{(2)} \left[ (\tilde{\mu} + \tilde{\rho}) + \frac{\tilde{\Pi}}{\tilde{T}} \right]
\]

We can identify two different contributions in Equations (41) and (42). On the one hand we have contributions from the heat transport process. These are in principle independent of the magnetic part of the Weyl tensor, which explains why they may be present in the spherically symmetric limit.

Next we have contributions related to the gravitational radiation. These require, both, the electric and the magnetic part of the Weyl tensor to be different from zero.

We recall that in the theory of the super–Poynting vector, a state of gravitational radiation is associated to a non–vanishing component of the latter. This is in agreement with the established link between the super–Poynting vector and the news functions, in the context of the Bondi–Sachs approach [17].

For the comoving observer and the line element Equation (5), the magnetic part of the Weyl tensor vanishes identically, and the fluid is non–dissipative, implying at once that \( P_1 = P_2 = 0 \). In other words, no gravitational radiation, or dissipative processes of any kind, are detected by the comoving observer.

However, as it should be already clear, important differences appear in the tilted congruence, with respect to the comoving one. Among them, there is one which rises the most intriguing question, namely: how it is possible that tilted observers may detect irreversible processes, whereas comoving observers describe an isentropic situation?

As we shall see, the above quandary becomes intelligible, if we appeal to the discussion on the Maxwell’s demon presented by Bennet.

3. The Maxwell’s Demon and Tilted and Comoving Observers

We have seen that tilted observers detect dissipation in a system that appears non–dissipative for comoving observers. As mentioned before this result is not new, and appears systematically for a wide class of spacetimes.

It is worth mentioning that in the present case, the difference between the pictures described by both congruences of observers, is still sharper, since the tilted observer not only detect a dissipative process, but also gravitational radiation.

With respect to this last point, it is worth noticing that the tilted observer also detects vorticity, and as has been pointed out in [17], vorticity and gravitational radiation are tightly associated. The reason behind the presence of gravitational radiation in the system analyzed by the tilted observer, is...
basically the same, as the one for any dissipative process (remember that gravitational radiation is a dissipative process too).

To explain such difference in the description of a given system, as provided by different congruences of observers, it has been conjectured in [18], that the origin of this strange situation resides in the fact that passing from one of the congruences to the other, we usually overlook the fact that both congruences of observers store different amounts of information. This is in fact the clue to resolve the quandary, about the presence or not, of dissipative processes, depending on the congruence of observers, that carry out the analysis of the system. Here we shall resort to the resolution of the well known paradox of the Maxwell’s demon [1] to clarify the obtained results.

Roughly speaking, Bennet solved the paradox, by showing that the irreversible act, which prevents the violation of the second law, is not the selection of molecules to put all of them in one side of the cylinder, but the restauration of the measuring appatatus (by means of which the selection is achieved), to the standard state previous to the state where the demon knows from which side comes any molecule. In other words, if we consider the whole system (demon + the gas in the cylinder), the information possesed by the demon before selecting the molecules, is smaller than the information after this process has been achieved. Thus, in order to return to the initial state of the demon, the acquired information has to be erased.

However, according to the Landauer principle, [19], the erasure of one bit of information stored in a system requires the dissipation into the environment of a minimal amount of energy, whose lower bound is given by

\[ \Delta E = kT \ln 2, \]  

where \( k \) is the Boltzmann constant and \( T \) denotes the temperature of the environment.

In other words, to get the demon’s mind back to its initial state, generates dissipation.

A somehow similar picture appears when we apply the operation transforming comoving observers, which assign zero value to the three-velocity of any fluid element, into tilted observers, for whom the three-velocity represents another degree of freedom. The erasure of the information stored by comoving observers (vanishing three velocity), when going to the frame of tilted observers, explain the presence of dissipative processes (included gravitational radiation) observed by the latter.

Thus, the Maxwell’s demon BEFORE knowing where the molecule is coming from, is equivalent to tilted observers: a piece of information is lacking for both of them. Instead, the Maxwell’s demon AFTER knowing where the molecule is coming from, is equivalente to comoving observers: they both have acquired additional information.

Passing from comoving to tilted observers, or returning the demands’s mind to its initial state, requires the erasure of the acquired information, leading to the observed dissipative processes. This explains, on the one hand, why the second law of thermodynamics is not violated by the Maxwell’s demon, and on the other, why tilted observers detect dissipation, there where comoving observers only see an isentropic system.

4. Conclusions

We have considered an axially symmetric fluid distribution, which for comoving observers is geodesic, shear–free, vorticity free, and non–dissipative. However, when viewed by tilted observers, the system appears to be non–geodesic, shearing and endowed with vorticity. Furthermore, tilted observers will detect entropy producing processes (dissipation) and gravitational radiation. Thus, in the example analyzed here, and absent in all previous examples known to the author, the dissipation detected by the tilted observers, expresses itself not only through the heat flow term, but also through the presence of gravitational radiation, as described by the super–Poynting vector. This fact is closely related to the non–vanishing of the vorticity appearing in the fluid flow, as observed by the tilted congruence, and which has been emphasized in the past [20].

To explain the appearance of dissipation, we have stressed that passing from one of the congruences to the other, we usually overlook the fact that both congruences of observers store
different amounts of information. Here resides the clue to resolve the quandary mentioned above, about the presence or not of dissipative processes, depending on the congruence of observers, that carry out the analysis of the system, as well as the challenge posed by the Maxwell’s demon paradox.

At the light of the comments above, the statement by Max Born “Irreversibility is a consequence of the explicit introduction of ignorance into the fundamental laws”, becomes fully intelligible.

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References

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