Toward a Coupled Oscillator Model of the Mechanisms of Universal Evolution and Development

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Outline

1. The big questions: Sagan, Chaisson, Kurzweil
2. The search for universality across different systems
3. The Principle of Least Action as a driver for self-organization
4. Positive feedback model: exponential acceleration
5. Negative feedback model: sinusoidal oscillations
6. Combine the two: exponential sinusoidal model
7. External noise – stochastic
8. Examples: Cities, Economy, techno, metabolic cycle, photosynthesis
9. Conclusions: A, f, H all increase exponentially
Cosmic Calendar

**Known from telescopes looking back in time, physical models**

- **January**
  - The Big Bang

- **February**
  - Milky Way disk forms

- **March**
  - Solar System and life

- **April**
  - Photosynthesis

- **May**
  - Eukaryotic cells

**Geologic record, fossils, genetic drift**

- **June**
  - 10:15 AM: Ape / gibbon divergence
  - 8:10 PM: Human / chimpanzee divergence

- **July**
  - 10:48 PM: Homo erectus evolves
  - 11:54 PM: Anatomically modern humans evolve

- **August**
  - 11:58 PM: Modern humans migrate out of Africa
  - 11:59 PM: Neanderthals die out, megaflora stressed

**Written record**

- **September**
  - Columbus arrives in America (one second to midnight)

- **October**
  - First cities in Mesopotamia
  - Roman republic, Old Testament, Buddha

- **November**
  - Agriculture, permanent settlements
  - Dynastic China

- **December**
  - Peak of last glacial period, humans migrate to the Americas

*By Carl Sagan*
Accelerating rate of self-organization

By Ray Kurzweil
Cosmic Evolution

The arrow of time, from origin of the Universe to the present and beyond, spans several major epochs throughout all of history. Cosmic evolution is the study of the many varied changes in the assembly and composition of energy, matter and life in the thinning and cooling Universe.
FERD as measure for complexity

By Eric Chaisson
Exponential Growth of Computing
Twentieth through twenty first century

Logarithmic Plot

Calculations per Second per $1,000

Year

By Ray Kurzweil
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A first principle

- The **Least Action Principle** for a system states: all processes in nature occur with the least expenditure of action, which is the product of time and energy for them.

\[ \delta \sum_{ij} I_{ij} = \delta \sum_{ij} \int_{t_1}^{t_2} L_{ij} \, dt = 0 \]
Quantity of organization

\[ \alpha = \frac{hnm}{\sum_{ij} I_{ij}} \]

- Organization, \( \alpha \), is inversely proportional to the average number of quanta of action per one element and one edge crossing of a network.

- \( n \) is the number of elements in the system and \( m \) is the number of edge crossings per unit time.
Total flow and number of quanta

• Recognize that \( nm \), the total number of edge crossings, is the flow, \( \phi \), of elements per unit time in the network: \( \phi = nm \).

\[
\sum_{i,j}^{nm} I_{ij}
\]

• Recognize that \( Q = \frac{ij}{h} \) is the total number of quanta of action in the system in certain interval of time.

• Therefore:

\[
\alpha = \frac{\phi}{Q}
\]
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Positive feedback model – exponential solutions

If $\alpha$ or $\phi$ stops increasing, or decreases, the other changes in the same direction.

\[ \dot{\alpha} = \frac{a_{12} Q + a_{13} \phi}{a_{31} \alpha + a_{32} Q} \phi \]

\[ \alpha = \eta \phi^\gamma \]

\[ \phi = \chi Q^\mu \]

\[ \alpha = \alpha_0 e^{\alpha_0} \]

\[ Q = Q_0 e^{\beta_0} \]

\[ \phi = \phi_0 e^{\delta_0} \]
Data for CPUs since 1971 (closed circles) and an exponential fit (solid line). The transition from single to multicore processors around time $10^9$[sec], does not affect the trend. $\alpha$ and Q do not increase smoothly but in steps.

An edge in this system is defined to be one computation.
To calculate $\alpha$, the potential energy of the electrons was taken to be constant.
The Lagrangian was then calculated using the kinetic energy.
The data for Million Instructions Per Second (MIPS) for each processor was divided by the thermal design power and multiplied by the table value of the Planck’s constant, to solve for $\alpha$. 

Exponential growth of $\alpha$ and Q in time
Data for $\alpha$ and $Q$ (closed circles) and an power law fit (solid line) with variations around the average.
Confirming Chaisson’s data for CPUs

$\Phi$ (FERD) as a function of $t$ (time). The data are from 1982 starting with Intel 286, to 2012.
Power law relations between $\alpha$, $Q$ and $\Phi$.

Data are filled circles and solid line is the fit. The data are from 1982 starting with Intel 286, to 2012, ending with Intel Core i7 3770k. There is a good agreement between the data and a power law fit.
Expanding to more mutually dependent functions – 

**interfucntions**

\[
\begin{align*}
\alpha &= a_1 \alpha + a_2 Q + a_3 \phi + a_4 N \\
\dot{Q} &= a_{21} \alpha + a_{22} Q + a_{23} \phi + a_{24} N \\
\dot{\phi} &= a_{31} \alpha + a_{32} Q + a_{33} \phi + a_{34} N \\
\dot{N} &= a_{41} \alpha + a_{42} Q + a_{43} \phi + a_{44} N
\end{align*}
\]

\[
\alpha = \eta N^\gamma \\
\phi = \omega N^\lambda \\
N = \sigma Q^\nu
\]

\[
\begin{align*}
\alpha &= \alpha_0 e^{\eta t} \\
Q &= Q_0 e^{\beta t} \\
\phi &= \phi_0 e^{\delta t} \\
N &= N_0 e^{\epsilon t}
\end{align*}
\]
Positive feedback model solutions

The figure shows the positive feedback loop among the system variables, $\alpha$, $\phi$, $Q$, and $N$ and their corresponding scaling relationships.
Figure 2: The figure shows the exponential scaling relationships between the characteristics, $\alpha$, $\phi$, $Q$, and $N$, with respect to time on a semilogarithmic scale (see (9)) with the goodness of fit (inset).
Figure 3: The figure shows the power-law scaling relationships between the characteristics, $\alpha$, $\phi$, $Q$, and $N$ on a double-logarithmic scale (see (11)) with the goodness of fit (inset).
Figure 4: The figure presents a pictorial representation of the power-law scaling relationships between the system variables, $\alpha$, $\phi$, $Q$, and $N$, and the solid diagonal line signifies slope of one.
The cone of development

• This cone has for levels all of the major stages in levels of organization that we know of.

• From here we get a sense that there is discreteness, in progress and self-organization in nature.
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Negative Feedback

Proposed Mechanism of development
A system of coupled oscillators

\[ F_h = -k(f_j - f_{j,H}) \]

Which has a solution of the form:

\[ y = A \cos(\omega t) \]
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Positive and Negative feedback loops

\[ \alpha \leftrightarrow - \alpha_H + + Q_H \leftrightarrow - Q \]

• The solution of the system of coupled oscillators:

\[ F_h = -k(f_j - f_{j,H}) \]

where

\[ f_{j,H} = f_{j,0} e^{ct} \]

On a first approximation, best fit is with:

\[ f_j = f_{j,0} e^{ct} (A + B e^{c_2 t} \cos(\omega t e^{c_3 t})) \]
Logarized fit: $y = A + B \cdot t + \ln(C \cdot \exp(M \cdot t) + D \cdot \cos(G \cdot t \cdot \exp(H \cdot t)) + K)$

On a first approximation, best fit is with:

$$f_j = f_{j,0} e^{c_1 t} (A + Be^{c_2 t} \cos(\omega te^{c_3 t}))$$

A, f, H all increase exponentially
Amplitude (A) is increasing and frequency (f=1/T) is increasing.

The max deviation from the dynamic equilibrium exponential line (Homeostasis) – is the limit of elasticity of those Interfunctions, i.e. the homeostatic Limits. If the interfunctions deviate more from their Homeostatic values, the system destabilizes and falls apart.
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Self-organized criticality as a fundamental property of complex systems

**Neural System.** Phase plot. Network activity versus connectivity for neurons. A **phase transition** is observed at $z^*$ for the analytical solution with infinite $n$, whereas the transition appears in finite systems at slightly higher values of the control parameter $n$ and is smoothed out over a small interval.

[Image: Neural System phase plot]


Benard Cells: Convective heat Flux as a function of time for formation

Benard Cells, Entropy production Not a simple power law

• The Nusselts number is a **Power Law** function of the Rayleigh’s number,

\[ Nu = (0.19 \pm 0.02) Ra^{0.29 \pm 0.03}. \]

JFM13c_ThermBL(Zhou)

• with oscillations, similar to those observed in other systems.

The Evolution of Nu with Ra at different conditions. Kaddiri-ISRN-Thermodynamis-2012
Biology – punctuated equilibrium

Number of tools vs time: Cultural accumulation when innovations may alter subsistence strategy, increasing biological carrying capacity and leading to an increase in population size. Red: leap innovations; Orange: toolkit innovations. Blue dots indicate the occurrence of big innovations that alter the biological carrying capacity.

The curve shows the number of mutation events for a single species. http://jasss.soc.surrey.ac.uk/4/4/reviews/bak.html
Exponential flow increase with oscillations

Projected Performance Development

https://ideagenius.com/the-s-curve-pattern-of-innovation
http://psyberspace.walterlogeman.com/tag/kevin-kelly/
The S-curve wave of a new paradigm as it explodes into growth and then matures.
Conclusions

• Exponential-sinusoidal solution coming from a positive and negative feedback model is the best fit that we found so far of the available data. A, f, H all increase exponentially.

• From dynamical systems approach and general systems theory, this model is based on well studied system dynamics and agrees with previous research.

• Further modeling is necessary to find the next level approximation for the functional dependence and to find all of the influences on the model.

• The exponential-sinusoidal oscillations of the homeostatic level itself provide modulation. The random fluctuations from the environment make the data stochastic.