Meeting the Electrical Energy Needs of a Residential Building with a Wind-Photovoltaic Hybrid System

Mehdi Ali Ehyaei¹, Marc .A. Rosen², Mohammad Hossein Ahmadi³*

¹ Department of Mechanical Engineering, Islamic Azad University, Pardis Branch, Pardis New City, Tehran, Iran
² Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, 2000 Simcoe Street North, Oshawa, Ontario, L1H 7K4, Canada.
³ Department of Renewable Energies, Faculty of New Science and Technologies, University of Tehran, Tehran, Iran
E-Mails: Aliehyaei@yahoo.com; Marc.Rosen@uoit.ca; mohammadhosein.ahmadi@gmail.com

* Department of Renewable Energies, Faculty of New Science and Technologies, University of Tehran, Tehran, Iran; Tel.: +989122866205

Received: 7 July 2014 / Accepted: 31 October 2014 / Published: 3 November 2014

Abstract: A complete hybrid system including a photocell and a wind turbine with battery storage is modeled, and the best approach for sizing the system to meet the electrical energy needs of a residential building is investigated. In evaluating system performance, the city of Tehran is used as a case study. Matlab software is used for analyzing the data and optimizing the system for the given application. Further, the price of the system design is investigated, and shows that the electrical cost of the hybrid system in Tehran is 0.62 US$/kWh, which is 78% less expensive than a wind turbine system and 34% less expensive than a photovoltaic system.

Keywords: Residential Building; Electrical Energy Need; Photovoltaic; Wind Turbine; Hybrid System; Energy Modeling.

1. Introduction

Since the oil crises of the 1970s, solar and wind power have become increasingly significant, attractive and cost-effective. In recent years, the hybrid PV/wind system has become a viable alternative to
satisfy environmental protection requirements and electricity demands. With the complementary characteristics of solar and wind energy resources for certain locations, hybrid PV/wind systems with storage units sometimes are advantageous options for supplying small electrical loads at remote locations that lack utility grid power.

Nowadays, hybrid PV/wind systems are able to offer a high reliability of power supply, but some concerns remain regarding their applications. Due to the stochastic behavior of both solar and wind energy, some major challenges in the design of a HPWS are the reliability of the supply of power to consumers under varying atmospheric conditions and at reasonable costs. To use solar and wind energy resources more efficiently and economically, the optimum size of a hybrid PV/wind system with batteries is an important consideration.

Various optimization techniques for hybrid PV/wind system sizing have been reported. Developing a novel description of the production/consumption phenomenon and used this information to propose a new sizing procedure (2008)[2]. Using this procedure, they obtain the optimum battery capacity, with the optimum number of PV modules and wind turbines, subject to minimum costs. Determining energy balances and conversion efficiencies for the components of a system, and proposed potential improvements to increase efficiency and the surplus energy produced by the wind/solar generator. They performed energy and exergy analyses to allow extrapolation of the results to real stand-alone applications providing an uninterrupted power supply to receptors isolated from the grid (2010)[3], presenting an optimal sizing method, and 1) optimize configurations of a hybrid solar-wind system employing battery banks based on a genetic algorithm (GA) to attain the global optimum with relative computational simplicity and 2) calculate the optimum system configuration that can achieve the required loss of power supply probability (LPSP) for customers with a minimum annualized cost of system (ACS) (2007) [4]. Presenting a simulation for analyzing the probability of power supply failure in hybrid photovoltaic-wind power generation systems incorporating a storage battery bank and analyzed the reliability of the systems. They also presented a case study of hybrid solar-wind power supply for telecommunication systems (2002)[5]. Presenting and discussed results of an exergy analysis conducted during the operation of a test-bed hybrid wind/solar generator with hydrogen support (2010) [6]. With a complete PV modeling, The effect of using a multi-axis sun–tracking system on electrical generation and evaluated its performance for the city of Monastir, Tunisia investigated and considered the effect of azimuth and tilt angels on the output of the photovoltaic panels (2010) [7]. Determining the component design and cost of a PV system required to satisfy given energy requirements and calculated the payback period for the suggested stand-alone PV system (2009) [8]. Determining the possibility of improving wind energy capture under low wind speed conditions in built-up areas, and described the design of a small wind generator for domestic use and the methodology of applying physical tests conducted in boundary layer wind tunnel and computer modeling using a CFD code (2007) [9]. An exergy analysis of atypical wind turbine model (Bargey Excel-S) in two cities of Iran showed that, with regard to the annual average wind data, by varying the cut in, the annual average production is increased by about 20% and the entropy generation is decreased about 77% (2009) [10]. Calculating the component design and cost of a PV system required to supply the desired energy and the playback period for the suggested stand-alone PV system (2010) [11]. Developing sizing algorithms for PV modules, wind generators and batteries (2007)
Determining the accumulation of energy obtained from renewable sources, stored as hydrogen, for satisfying the electrical supply of several islands (2007) [13]. Developing an hourly management method for energy generated in grid-connected wind farms using hydrogen storage and an hourly management method for energy generated in grid-connected wind farms by storing electrical energy in batteries proposed (2009) [14].

The objective of this research is to determine an optimum hybrid system for meeting the electrical energy needs of a residential building. The hybrid system includes a wind turbine and PV power generation system as well as a storage battery. With the method, which is proposed and explained in this paper, the optimum capacity of the wind turbine, the photocell and the battery is determined based on minimum electricity cost.

2. Description of building

The residential building considered in this study is located in Tehran (Province of Iran) and has an average occupancy (4 adults). The building has a total floor area of about 200 m$^2$, a height of 3 m, a length of 20 m (aligned in an east-west direction), and a width of 10 m (in a north-south direction). The areas of the windows account for 30% of the area of the south and north walls and 20% of the area of the east and west walls of the building. The external and internal walls are 22 and 12 cm thick, respectively, and are all made of brick with gypsum plaster on the interior walls. The roof is 22 cm thick, and made of brick and roofing materials. No thermal insulation or other energy saving measures is employed in the walls or the roof.

To calculate electrical load of the building, it is assumed that the 15th day of each month is representative of all the days of the month. To estimate the electrical energy needs of the residential building, we measure the electrical utilization and period of use for each electrical appliance and light. These data are shown in Figure 1 for in January 15, while Figure 2 displays the total electrical consumption on that day. The electrical loads of the residential building on the 15th of every month are given in Table 1.

**Figure 1.** Electrical usage and period of use for electrical appliances and lights on January 15

![Figure 1](image)

**Figure 2.** Total electrical consumption of the residential building on January 15

![Figure 2](image)
Table 1. Electrical loads (in kW) of the residential building on the 15th of every month

<table>
<thead>
<tr>
<th>Hour</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>2</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>3</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>4</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>5</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>6</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>7</td>
<td>0.41</td>
<td>0.41</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>8</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.217</td>
<td>0.221</td>
<td>0.217</td>
<td>0.223</td>
<td>0.22</td>
<td>0.215</td>
<td>0.21</td>
<td>0.204</td>
</tr>
<tr>
<td>9</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.217</td>
<td>0.221</td>
<td>0.217</td>
<td>0.223</td>
<td>0.22</td>
<td>0.215</td>
<td>0.21</td>
<td>0.204</td>
</tr>
<tr>
<td>10</td>
<td>0.18</td>
<td>0.18</td>
<td>0.189</td>
<td>0.189</td>
<td>0.193</td>
<td>0.197</td>
<td>0.2</td>
<td>0.199</td>
<td>0.196</td>
<td>0.191</td>
<td>0.185</td>
<td>0.181</td>
</tr>
<tr>
<td>11</td>
<td>0.18</td>
<td>0.18</td>
<td>0.189</td>
<td>0.189</td>
<td>0.193</td>
<td>0.197</td>
<td>0.2</td>
<td>0.199</td>
<td>0.196</td>
<td>0.191</td>
<td>0.185</td>
<td>0.181</td>
</tr>
<tr>
<td>12</td>
<td>0.18</td>
<td>0.18</td>
<td>0.189</td>
<td>0.189</td>
<td>0.193</td>
<td>0.197</td>
<td>0.2</td>
<td>0.199</td>
<td>0.196</td>
<td>0.191</td>
<td>0.185</td>
<td>0.181</td>
</tr>
<tr>
<td>13</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>14</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>15</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.064</td>
<td>0.068</td>
<td>0.07</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>16</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.055</td>
<td>0.064</td>
<td>0.068</td>
<td>0.071</td>
<td>0.07</td>
<td>0.067</td>
<td>0.061</td>
<td>0.056</td>
<td>0.067</td>
</tr>
<tr>
<td>17</td>
<td>0.714</td>
<td>0.242</td>
<td>0.246</td>
<td>0.25</td>
<td>0.254</td>
<td>0.258</td>
<td>0.261</td>
<td>0.26</td>
<td>0.257</td>
<td>0.252</td>
<td>0.718</td>
<td>0.714</td>
</tr>
<tr>
<td>18</td>
<td>0.714</td>
<td>0.714</td>
<td>0.718</td>
<td>0.723</td>
<td>0.254</td>
<td>0.258</td>
<td>0.261</td>
<td>0.732</td>
<td>0.729</td>
<td>0.724</td>
<td>0.718</td>
<td>0.714</td>
</tr>
<tr>
<td>19</td>
<td>0.805</td>
<td>0.806</td>
<td>0.807</td>
<td>0.814</td>
<td>0.817</td>
<td>0.822</td>
<td>0.817</td>
<td>0.823</td>
<td>0.82</td>
<td>0.815</td>
<td>0.809</td>
<td>0.805</td>
</tr>
<tr>
<td>20</td>
<td>0.805</td>
<td>0.806</td>
<td>0.807</td>
<td>0.814</td>
<td>0.817</td>
<td>0.822</td>
<td>0.817</td>
<td>0.823</td>
<td>0.82</td>
<td>0.815</td>
<td>0.809</td>
<td>0.805</td>
</tr>
<tr>
<td>21</td>
<td>0.71</td>
<td>0.71</td>
<td>0.714</td>
<td>0.719</td>
<td>0.723</td>
<td>0.727</td>
<td>0.73</td>
<td>0.729</td>
<td>0.726</td>
<td>0.721</td>
<td>0.715</td>
<td>0.711</td>
</tr>
<tr>
<td>22</td>
<td>0.52</td>
<td>0.52</td>
<td>0.524</td>
<td>0.53</td>
<td>0.533</td>
<td>0.538</td>
<td>0.54</td>
<td>0.539</td>
<td>0.536</td>
<td>0.53</td>
<td>0.524</td>
<td>0.52</td>
</tr>
<tr>
<td>23</td>
<td>0.405</td>
<td>0.405</td>
<td>0.409</td>
<td>0.414</td>
<td>0.814</td>
<td>0.422</td>
<td>0.424</td>
<td>0.417</td>
<td>0.421</td>
<td>0.416</td>
<td>0.41</td>
<td>0.406</td>
</tr>
<tr>
<td>24</td>
<td>0.051</td>
<td>0.051</td>
<td>0.055</td>
<td>0.06</td>
<td>0.064</td>
<td>0.058</td>
<td>0.071</td>
<td>0.07</td>
<td>0.067</td>
<td>0.062</td>
<td>0.056</td>
<td>0.052</td>
</tr>
</tbody>
</table>
3. Mathematical models of the systems

Modeling the energy generated from wind turbines and PV modules together with the storage battery is a critical step for optimization. Data on the operating performances of the system components, which are important in building energy use modeling, are given in Fig.3. There are a number of mathematical models in the literature for both PV modules and wind generators, and many of these models have considered a variety of physical factors for improving accuracy. But as the aim of this study is to illustrate a novel insight into the WPHS modeling and to introduce an efficient sizing strategy, complicated component models are avoided and basic mathematical models are for clarity to characterize the system.

Fig. 3. General diagram of a hybrid (wind-PV) system [2]

3.1. Basic mathematical model of PV modules

The electrical power generated from a PV module can be evaluated as follows:

\[ P_{pv} = A \eta G_t \]  

(1)

Where \( A \) is the solar cell array area (m\(^2\)), \( G_t \) is the solar radiation incident on the solar cell in its current location (W/m\(^2\)) and \( \eta \) is the solar cell efficiency. Basic energy models are selected in this study to illustrate the new methodology for WPHS sizing clearly, but the ideas proposed here remain valid and applicable if more precise energy models are used for characterizing the system.

The parameter \( \beta \), which is used subsequently in equations (5) and (8), is given as [15]:

\[ \beta = \frac{360(n-1)}{365} \]  

(2)

Where \( n \) is the ordinal date (\( n = 1 \) for January 1 and \( n = 365 \) for December 31). In term of ordinal date (n), the declination is expressed as follows [14-16]:

\[ \delta = 23.45 \sin \left( 360 \frac{284+n}{365} \right) \]  

(3)
Also the solar time $t_s$ is expressed as follows:

$$t_s = t_c + \frac{1}{15}Z_c + E$$ (4)

Here, $Z_c$ is the time zone east of GMT, $t_c$ is local time, and

$$E = (3.82 \times 10^{-6})(75 + 1868 \cos \beta - (32,077) \sin \beta - (14,619) \cos(2\beta) - (40,890) \sin(2\beta))$$ (5)

The hour angle can be written as follows:

$$w = (t_s - 12) \times 15^0$$ (6)

The zenith angle is expressed in terms of latitude $\phi$, declination $\delta$ and hour angle was follows [17]:

$$\cos \theta_z = \cos \phi \cos \delta \cos w + \sin \phi \sin \delta$$ (7)

The angle of incidence can be expressed as [17]:

$$\cos \theta = \sin \phi \sin \delta \cos \beta - \cos \phi \sin \delta \sin \beta \cos \gamma + \cos \phi \cos \delta \cos \beta \cos w + \sin \phi \cos \delta \sin \beta \cos \gamma \cos w + \cos \delta \sin \beta \sin \gamma \sin w$$ (8)

where $\phi$ is the latitude, $\beta$ the mean anomaly, $\gamma$ the longitude and $w$ the hour angle.

The normal extraterrestrial radiation $G_{on}(W/m^2)$ can be expressed with an accuracy adequate for most engineering calculations as follows[14-16]:

$$G_{on} = G_{sc}(1 + 0.033 \cos(\frac{360n}{365}))$$ (9)

Where $G_{sc}$ is solar constant (1367 W/m$^2$) and $n$ is ordinal date. The horizontal extraterrestrial radiation can be calculated in terms of the zenith angle $\theta_z$ (degree) as follows:

$$G_{oh} = G_{on} \cos \theta_z$$ (10)

The average extraterrestrial horizontal radiation over a time step can be calculated as [17]:

$$G_{oh} = \frac{12}{\pi}G_{on}[\cos \phi \cos \delta \sin w_2 - \sin w_1 + \pi \frac{w_2 - w_1}{180} \sin \phi \sin \delta]$$ (11)

Here, $w_1$ and $w_2$ are the hour angles at the beginning and end of the time step, respectively.

The mean monthly global radiation $\overline{G}(W/m^2)$ is expressed as follows:

$$\overline{G} = \overline{G}_h + \overline{G}_d$$ (12)
where $\overline{G}_b$ (W/m$^2$) is the mean direct radiation and $\overline{G}_d$ (W/m$^2$) is the mean diffuse radiation.

The clearness index factor $K$ is defined as the ratio of a particular global radiation to the horizontal extraterrestrial radiation:

$$K = \frac{\overline{G}}{G_{oh}}$$  \hspace{1cm} (13)

Where $G_{oh}$ (W/m$^2$) is the horizontal extraterrestrial radiation.

From [14,16] we have:

$$\frac{\overline{G}_d}{\overline{G}} = \begin{cases} 
0.98K \text{ (for } K < 0.2) \\
0.61092 + 3.6259K - 10.17K_2 + 6.388K_3 \text{ (for } 0.22 < K < 0.8) \\
0.672 - 0.474K \text{ (for } K > 0.8) 
\end{cases}$$  \hspace{1cm} (14)

The beam radiation on a tilted surface $G_{bt}$ (W/m$^2$) is expressible as:

$$G_{bt} = G_b R_b$$  \hspace{1cm} (15)

Where the geometric factor $R_b$ is given by:

$$R_b = \frac{\cos \theta}{\cos \theta_2}$$  \hspace{1cm} (16)

The Hay-Davies-Klucher-Riendl (HDKR) model estimates the absorbed beam, diffuse and ground reflected solar radiation. According to the HDKR model the diffuse component of radiation incident on a tilted surface can be expressed as follows [17]:

$$G_{dt} = G_d (R_b A_i) + (1-A_i) \left(1 + f \sin^2 \left(\frac{\beta}{2}\right)\right)$$  \hspace{1cm} (17)

Where $\beta$ is photovoltaic panel slope (degree).

A correction factor for the diffuse radiation that includes the influence of cloudiness is expressed by:

$$F = \sqrt{\frac{G_b}{G}}$$  \hspace{1cm} (18)

The anisotropy index is given by:

$$A_i = \sqrt{\frac{G_b}{G_o}}$$  \hspace{1cm} (19)

where $G_0$ (W/m$^2$) is the extraterrestrial radiation.
All models assume that the ground reflected component $G_r(W/m^2)$ is isotropic as follows\cite{2,4}:

$$G_r = G \rho \left( \frac{1 - \cos \beta}{2} \right)$$ \hspace{1cm} (20)

Where $\rho$ is the albedo coefficient of the ground.

The global radiation on a tilted surface is expressible as

$$G_t = G_{bt} + G_{dt} + G_r$$ \hspace{1cm} (21)

Where $G_{bt}(W/m^2)$ is the beam radiation and $G_{dt}(W/m^2)$ is diffuse component radiation, both incident on a tilted surface.

In this study, the HDKR model is applied assuming that the global radiation on tilted surface is as follows:

$$G_t = (G_b + G_d A_i) R_b + G_d (1 - A_i) R_b \left[ 1 + f \sin^2 \left( \frac{\beta}{2} \right) \right] + G_\rho R_b$$ \hspace{1cm} (22)

Here, $f$ is correction factor of diffuse radiation and $R_b$ is a geometric factor.

The energy balance for a unit area of the module, which is cooled by losses to the surroundings, can be written as:

$$\tau \alpha G_t = \eta_c G_t + U_L (T_c - T_a)$$ \hspace{1cm} (23)

where $\tau$ is the solar transmittance of the PV array, $\alpha$ the solar absorptance of the PV array, $G_t$ the global radiation striking the PV array, $\eta_c$ the electrical conversion efficiency of the PV array, $U_L$ the coefficient of heat transfer to the surroundings and $T_a$ the ambient temperature.

Accordingly, the PV cell temperature can be expressed as follows\cite{14-16}:

$$T_c = T_\text{a} + G_T \left( \frac{\alpha \tau}{U_L} \right) \left( 1 - \frac{\eta_c}{\alpha \tau} \right)$$ \hspace{1cm} (24)

To estimate the value of $\frac{\alpha \tau}{U_L}$, we report the nominal operating cell temperature (NOCT), which is defined as the cell temperature that results in an incident radiation of 0.8 kW/m$^2$, an ambient temperature of 20$^\circ$C, no load operation ($\eta = 0$) and an average speed of 1m/s. Substituting these values into the above equation and solving for $\frac{\alpha \tau}{U_L}$:

$$\left( \frac{\alpha \tau}{U_L} \right) = \frac{T_c,\text{NOCT} - T_\text{a,NOCT}}{G_T,\text{NOCT}}$$ \hspace{1cm} (25)

where $T_c,\text{NOCT}$ the is nominal operating cell temperature, $T_\text{a,NOCT}$ the nominal ambient temperature and $G_T,\text{NOCT}$ the nominal global radiation on a tilted surface.
Therefore we have:
\[ T_c = T_a + (T_{c,NOCT} - T_{a,NOCT})(\frac{\theta_T}{G_{T,NOCT}})^{\frac{1-\eta_{mp}}{\tau_\alpha}} \]  \hspace{1cm} (26)

and
\[ \eta_{mp} = \eta_{mp,stk} [1 + \alpha_p (T_c - T_{c,stk})] \]  \hspace{1cm} (27)

The PV array output power is given by:
\[ P_{py} = Y_{pv} f_{pv} \left( \frac{g_T}{G_{T,stk}} \right) [1 + \alpha_p (T_c - T_{c,stk})] \]  \hspace{1cm} (28)

Here, \( Y_{pv} \) is the rated capacity of the PV array, \( f_{pv} \) is its power output under standard test conditions, \( G_{T,stk} \) is the PV derating factor (used to account for such factors as shading, snow cover, aging, etc.), \( \alpha_p \) is the incident radiation at standard test conditions, \( T_c \) is the temperature coefficient of power (which indicates how strongly the PV array power output depends on the cell temperature), and \( T_{c,stk} \) is the PV cell temperature in the current time step.

Operating and design parameters for standard PV technologies are shown in Table 2. In this study we use the Mono-Si model.

<table>
<thead>
<tr>
<th>PV module type</th>
<th>( \eta_r ) (%)</th>
<th>NOCT(°C)</th>
<th>( \beta_p ) (1/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mono-Si</td>
<td>13</td>
<td>45</td>
<td>0.4</td>
</tr>
<tr>
<td>Poly-Si</td>
<td>11</td>
<td>45</td>
<td>0.4</td>
</tr>
<tr>
<td>a-Si</td>
<td>5</td>
<td>50</td>
<td>0.11</td>
</tr>
<tr>
<td>CdTe</td>
<td>7</td>
<td>46</td>
<td>0.24</td>
</tr>
<tr>
<td>CIS</td>
<td>7.5</td>
<td>47</td>
<td>0.46</td>
</tr>
</tbody>
</table>

3.2. Basic mathematical model of wind generator

Different types of wind generators have different power output performance curves. Consequently, the model used to describe their performance should also be differing. A typical model for a wind turbine is described below[18-20]:
\[ P_{c,ave} = P_{er} \left\{ \frac{\exp \left( -\left( \frac{u_c}{c} \right)^k \right) - \exp \left( -\left( \frac{u_r}{c} \right)^k \right) \left( \frac{u_c}{c} \right)^k - \left( \frac{u_r}{c} \right)^k}{\left( \frac{u_c}{c} \right)^k - \left( \frac{u_r}{c} \right)^k} \right\} \]  \hspace{1cm} (29)

where \( P_{c,ave} \) is the average output power of the wind generator at wind speed \( V \) (m/s), \( P_{er} \) is the nominal power(kW), \( u_c \) is the startup rotating speed(m/s), \( u_r \) is the nominal speed(m/s), \( u_f \) is the final rotating speed(m/s), and \( c \) and \( k \) are the coefficients of correlation.

A BergeyExcel-Stype of wind turbine is considered, for which characteristics are shown in Table 3.
Table 3. Bergey Excel-S wind turbine characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut-in wind speed (m/s)</td>
<td>3.1</td>
</tr>
<tr>
<td>Rate wind speed (m/s)</td>
<td>13.8</td>
</tr>
<tr>
<td>Rated power (kW)</td>
<td>10</td>
</tr>
<tr>
<td>Furling wind speed (m/s)</td>
<td>15.6</td>
</tr>
<tr>
<td>Type</td>
<td>3 blade up wind</td>
</tr>
<tr>
<td>Swept area (m²)</td>
<td>38.47</td>
</tr>
<tr>
<td>Gear box</td>
<td>Non-direct drive</td>
</tr>
<tr>
<td>Temperature range (°C)</td>
<td>-40 to 60</td>
</tr>
<tr>
<td>Generator</td>
<td>Parameters magnet alternator</td>
</tr>
<tr>
<td>Tower height (m)</td>
<td>24</td>
</tr>
</tbody>
</table>

Wind speed data in Tehran during various months are shown in Table 4.

Table 4. Number of observations ($n_i$) of specific wind speed $u_i$ by month in Tehran

<table>
<thead>
<tr>
<th>$u_i$ (m/s)</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>59</td>
<td>62</td>
<td>82</td>
<td>79</td>
<td>71</td>
<td>76</td>
<td>98</td>
<td>106</td>
<td>119</td>
<td>96</td>
<td>64</td>
<td>60</td>
<td>752</td>
</tr>
<tr>
<td>4-6</td>
<td>25</td>
<td>36</td>
<td>65</td>
<td>61</td>
<td>53</td>
<td>67</td>
<td>73</td>
<td>51</td>
<td>42</td>
<td>37</td>
<td>31</td>
<td>8</td>
<td>437</td>
</tr>
<tr>
<td>7-10</td>
<td>15</td>
<td>22</td>
<td>20</td>
<td>32</td>
<td>27</td>
<td>27</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>2</td>
<td>161</td>
</tr>
<tr>
<td>11-16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>29</td>
</tr>
<tr>
<td>&gt;16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The electricity cost for a hybrid PV/wind system can be calculated as:[21-23]:

$$C = C_p + C_T + C_B$$  \hspace{1cm} (30)

Where $C$ is the unit total cost (US$/kWh), $C_p$ is the unit electricity cost for the photovoltaic system (US$/kWh), $C_T$ is the unit electricity cost for the wind turbine system (US$/kWh) and $C_B$ is the unit battery cost (US$/kWh).

The unit electricity cost for the hybrid system can be calculated as:[21-23]:

$$C_p = C_I + C_o$$ \hspace{1cm} (31)

Where $C_I$ is the unit installation cost (US$/kWh) and $C_o$ is the unit maintenance cost (US$/kWh).

The unit installation cost can be determined as:

$$C_I = \frac{C}{C_{I\text{avg}}}$$ \hspace{1cm} (32)

Where $C$ is the primary installation cost (US$), $I$ is the initial cost profit, $C_I$ is the average annual electrical use (kWh/day) and $n_d$ is a number of days in a year.

The initial cost profit can be calculated as follows:

$$I = \frac{i(1+i)^L}{(1+i)^L-1}$$ \hspace{1cm} (33)
Here, $P_{e,ave}$ is the power generated by the PV system in a year (kWh) and $P_{er}$ is equal to 4.

The unit maintenance cost can be calculated as:

$$C_o = 3\% \ C_1$$  \hspace{1cm} (35)

With the same procedure we can calculate $C_T$ (US$/kWh) but with $P_{er}$ for a wind turbine equal to 10.

The cost of usage battery can be calculated as follows:

$$C_B = \frac{C \ I}{C_{fm_d}}$$  \hspace{1cm} (36)

Here, $C$ is the initial installation cost (US$), $I$ is the initial cost profit, and $n_d$ is the number of days in a year.

The initial cost profit can be calculated as:

$$I = \frac{i(1+i)^L}{(1+i)^L+1-1}$$  \hspace{1cm} (37)

Using the above equations, the electrical cost of our system is calculated to be 0.6225 US$/kWh.

4. Optimization procedure

The main aim of this research is the optimization of the photovoltaic-wind turbine hybrid system to meet all electrical power needs over the total hours of a year.

The function that we minimize is electricity cost, which includes initial and maintenance costs; these costs are affected by the system capacity (PV area, wind turbine and battery capacity). The battery is the most expensive part of the system.

On the other side of the optimization procedure we have the energy function. This equation demonstrates that the sum of the photovoltaic and wind turbine power generation and battery storage capacity should be equal to the electrical power need of any hour of the day of the year (equation 38). We use a searching algorithm for optimization, and we modify the wind turbine model with a different rated power, PV area and battery storage type and capacity, by noting that the sum of the power generation should meet the electrical power needs of the residential building (equation 38). Also we recognize from equation 30 that the electricity cost should be at the minimum level.

In this study, data for the 15th of every month is taken as representative of that month for designing the system. Calculations for the hybrid system are done for the peak time of usage and producing energy.
to ensure our system can supply the building needs for the entire day. The hybrid system needs to satisfy the following equation:

$$E = \dot{W}_{\text{pv}h} + \dot{W}_{\text{wind}h} + E_{\text{bat}}$$  \hspace{1cm} (38)$$

Here, $E$ is the electrical energy need of the building, $\dot{W}_{\text{wind}}$ is photovoltaic power supply, $\dot{W}_{\text{wind}}$ is the wind turbine power supply and $E_{\text{bat}}$ is the electrical capacity of the battery.

First the electrical power supply of the wind turbine is calculated. The type of wind turbine considered is a BergeyExcel-S. The electrical power supply of the wind turbine can be calculated using equation (29). Next the capacity is determined for the battery, which supplies the electrical need of the building at night. Finally the size of the photovoltaic system is calculated; to do this the power generation for a 1m$^2$ photovoltaic system is obtained from equation (28) and then with equation (38) the total size of the photovoltaic system is found. All calculations are carried out with the Matlab code.

The results are illustrated in Figs.4-7. The surface are required of the photovoltaic system is found to be 26.58 m$^2$ and the capacity of battery is 8.087 kW. The electrical usage for the 15th of every month is shown in Fig. (4), as is the electrical energy produced by the PV and wind turbine, and the battery capacity.

It can be seen in the figures that the capacity of the battery is constant over the whole year, but the electricity produced by the turbine and PV was changes with the month. In summer days, the photovoltaic system produces more electrical energy than needed but in winter days the production and usage rates are the same.

**Fig.4.** Annual electrical production of the system (PV, wind turbine, battery) and usage

As shown in Figs.5, 6 and 7 the electrical output of the wind turbine and photocell covers 24 hours of the day for three of the months. In all of them our system can supply the electrical needs.
**Fig. 5.** Electrical output of the PV, wind turbine and battery, and electrical usage, on January 15

**Fig. 6.** Electrical output of the PV, wind turbine and battery, and electrical usage, on July 15
Fig. 7. Electrical output of the PV, wind turbine and battery, and electrical usage, on January 15

The electrical costs of the wind turbine and photovoltaic systems are calculated and compared with the hybrid system. If the wind turbine system with battery storage supplies all the needs of the building the unit electrical cost of system will be 1.11 US$/kWh. If the photovoltaic system with battery storage meets all the needs of the building the unit electrical cost will be 0.83 US$/kWh.

A comparison of the results for the hybrid system and the wind turbine system, and also the photovoltaic system, for supplying the electrical need of the building demonstrates that the unit electrical cost of the hybrid system is 0.62 US$/kWh, which is 78% lower than for the wind turbine system and 34% lower than for the photovoltaic system.

Table 5 provides a comparison of different options to supply electrical needs of residential buildings in Tehran.

<table>
<thead>
<tr>
<th>Option</th>
<th>PV model</th>
<th>PV area(㎡)</th>
<th>Wind turbine model</th>
<th>Wind turbine units</th>
<th>Battery capacity (kWh)</th>
<th>Electrical cost (US$/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV+ Battery</td>
<td>Mono-Si</td>
<td>32</td>
<td>-</td>
<td>-</td>
<td>12.9</td>
<td>0.83</td>
</tr>
<tr>
<td>Wind turbine+ Battery</td>
<td>-</td>
<td>-</td>
<td>Bergey Excel-S</td>
<td>12</td>
<td>8.1</td>
<td>1.11</td>
</tr>
<tr>
<td>PV+ Wind turbine + Battery</td>
<td>Mono-Si</td>
<td>26.6</td>
<td>Bergey Excel-S</td>
<td>1</td>
<td>8.9</td>
<td>0.62</td>
</tr>
</tbody>
</table>

5. Conclusions

We investigate successfully the utilization of a photovoltaic-wind turbine hybrid system to supply the electrical needs of residential buildings over the year. For this aim we consider a residential building in Tehran. Using weather data and the location of the building we calculate the power generated by the system. With this data and the electrical need of the building, the optimum size of the system and battery capacity that could supply the electrical need is calculated. The computer code used in this research is able to calculate the size of the photovoltaic-wind turbine and battery capacity. This code is
developed so it could be used for any location to design the most appropriate photovoltaic and wind turbine hybrid system.

**Nomenclature**

- $P_{pv}$: Photovoltaic output power (kW)
- $A$: Surface area of PV panel (m$^2$)
- $\eta$: Electrical conversion efficiency
- $G_t$: Global radiation (W/m$^2$)
- $B$: Mean anomaly (Degree)
- $\delta$: Sun declination (Degree)
- $w$: Hour angle (degree)
- $\theta$: Angle of incidence (degree)
- $\beta$: Slope angle (degree)
- $G_{oh}$: Horizontal extraterrestrial radiation (W/m$^2$)
- $G_{on}$: Normal extraterrestrial radiation (W/m$^2$)
- $\bar{G}$: Global radiation on horizontal panel (W/m$^2$)
- $K$: Clearness index factor
- $G_{bt}$: Beam radiation on tilted surface (W/m$^2$)
- $F$: Correction factor
- $T_a$: Environment temperature ($^\circ$C)
- $T_C$: Cell temperature ($^\circ$C)
- $U_c$: Cut-in wind turbine speed (m/s)
- $U_r$: Rated wind turbine speed (m/s)
- $P_{er}$: Rated power (kW)
- $P_{e,ave}$: Average wind turbine electrical power output (kW)
- $C$: Unit electricity cost (US$/kWh)$
\( C_p \)  Unit electricity cost for photovoltaic system (US$/kWh)

\( C_I \)  Unit installation cost (US$/kWh)

\( C_o \)  Unit maintenance cost (US$/kWh)

\( C_f \)  Average annual electrical use (kWh/day)

**Conflict of Interest**

The authors declare no conflict of interest

**References and Notes**


© 2014 by the authors; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license.