An Inexact Fuzzy Optimization Programming with Hurwicz Criterion (IFOPH) for Sustainable Irrigation Planning in Arid Region

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Abstract: In the past decades, sustainability in irrigation planning has been of concern to many researchers and managers. However, uncertainties existed in an irrigation planning system can bring about enormous difficulties and challenges in generating desired decision alternatives with aim of sustainability. In this study, an inexact fuzzy optimization programming with Hurwicz criterion (IFOPH) is developed for sustainable irrigation planning under uncertainty, which incorporates two-stage stochastic programming (TSP), interval-parameter programming (IPP), fuzzy credibility-constraint programming (FCP) and Hurwicz criterion (TCP-CH) within an framework. The developed method is applied to a real case of planning sustainable irrigation
in Tarim Basin, which is one of the aridest regions of China. The results based on confidence degrees are obtained, which can permit in-depth analyses of various policy scenarios of that are associated with different levels of economic penalties. Meanwhile, the results reveal that an appropriate irrigation planning can improve the efficiency of water allocations, which has brought positive effects on remedying water deficit and promoting the sustainable development of agricultural production activities. Moreover, tradeoffs between economic benefit and system-failure risk based on Hurwicz criterion can support generating an increased robustness in risk control, which can facilitate the local decision makers in adjusting water-allocation pattern.

**Keywords:** sustainable irrigation planning; water resources management; two-stage stochastic programming; fuzzy credibility-constraint programming; Hurwicz criterion; uncertainty; arid region.

1. **Introduction**

Water resources are lifeline of oasis agriculture development in arid region [1]. Practically, around 70% of global freshwater diverted to agriculture, at the same time, water demand of irrigation is still increasing because the farmland being irrigated continues to be expanded. Particular in decades, controversial and conflict-laden water resources allocation issue has challenged decision makers due to rising demand pressure for freshwater associated with a variety of factors such as population growth, economic development, food security, environmental concern, and climate change [2]. Water shortage is subject to increasing pressure particularly for arid regions that are mainly characterized by low rainfall and high evaporation. On the contrary, increased population shifts and shrinking water supplies have exacerbated competition among different users. When the demand for water has reached the limits of what the natural system can provide with, sustainable irrigation planning has been of concern to many researchers and managers, which not only contributes to remit pressure of water shortage characterized, but also improve deteriorated water quality and endangered ecosystems [3-4]. Moreover, irrigation planning systems are complicated with a variety of uncertainties (e.g., imprecise economic data, random stream flows, uncertain economic benefits and varied water allocations) and their interactions which may intensify the conflict laden issues of water allocation [5]. Therefore, comprehensive, complex and ambitious plans for sustainable irrigation planning under uncertainties is required, with the aim of developing and implementing appropriate water resources infrastructure and management strategies [6].

Previously, various mathematical programming models were developed for supporting water resources planning including irrigation planning under uncertainties [7-13]. For example, Maqsood et al. [14] developed an interval-fuzzy two-stage stochastic programming method for planning water resources management systems associated with multiple uncertainties, in which techniques of interval-parameter programming (IPP) and fuzzy programming were integrated into a TSP framework. Li and Huang [15] proposed a fuzzy-stochastic-based violation analysis (FSVA) for the planning for agriculture water resources management, in which can deal with uncertainties expressed as probability distributions and
fuzzy sets. Vidoli [16] developed a two stage method for evaluating the water resources service, through integrating the conditional robust nonparametric frontier and multivariate adaptive regression splines into a TSP framework. In general, TSP can provide an effective linkage between policies and the economic penalties, which has advantages in reflecting complexities of system uncertainties as well as analyzing policy scenarios when the pre-regulated targets are violated.

However uncertainties may be related to errors in acquired data, variations in spatial and temporal units, and incompleteness or impreciseness of observed information in water resources planning [17]. Fuzzy programming (FP) is effective in dealing with decision problems under fuzzy goal or constraints and in handling ambiguous coefficients of objective function and constraints caused by imprecision and vagueness, when the quality and quantity of uncertain information is often not satisfactory enough to be presented as probabilistic distribution [15]. Fuzzy credibility constrained programming (FCP) can measure the confidence levels in fuzzy water system to tackle uncertainties expressed as fuzzy sets, when detailed information is not able to be presented by interval or stochastic numbers [18-20]. However, FCP can not tackle uncertainties expressed fuzzy sets which existed in constraint’s left and right-hand sides contemporarily, particularly in function [21]. Therefore, Hurwicz criterion is introduced into FCP, which can tackle uncertainties in function when different type uncertainties expressed fuzzy sets in function and constraints contemporarily. A compromise stroked by optimistic criterion (maximum payoff and minimin loss) and pessimistic criterion (minimin payoff and maximum loss) of Hurwicz criterion makes that decision maker is neither adventurous nor conservative in decision process with uncertain importations [22]. In addition, the major problem of TSP and FCP methods is that the increased data requirement for specifying the probability distributions of coefficients may affect their practical applicability [23]. Interval-parameter programming (IPP) is introduced to handle uncertainties in the model’s left- and/or right-hand sides as well as those that cannot be quantified as membership or distribution functions, since interval numbers are acceptable as its uncertain inputs [23]. Previously, few studies were reported in the presentation and interpretation of multiple uncertainties in hybrid formats previously, although multi-types of uncertainties may exist within the practical irrigation planning.

Therefore, an inexact fuzzy optimization programming with Hurwicz criterion (IFOPH) is developed to better account for optimizing irrigation planning under uncertainty with aim of sustainability, which incorporate two-stage stochastic programming (TSP), interval-parameter programming (IPP), fuzzy credibility-constraint programming (FCP) and Hurwicz criterion (HC) within an framework. The developed IFOPH method will be applied to a real case study of sustainable irrigation in Tarim Basin, which is one of the aridest regions in Northwest China. The proposed IFOPH can provide an effective linkage between conflicting economic benefits and the associated penalties attributed to the violation of the pre-regulated policies. The modeling results can be used for supporting the adjustment of the existing irrigation patterns to raise the water demand, as well as the capacity planning of water resources to satisfy the basin’s increasing water demands. Satisfaction degrees for constraints and Hurwicz criterions can be represented using interval credibility levels and Hurwicz parameters (i.e., optimistic and pessimistic criterion), which can provide a scientific support for large-scale regional irrigation under uncertainties at the watershed level.
2. Model development

In a sustainable irrigation planning problem, a water manager is responsible for allocating water to multiple crops, with the aim of maximizing system benefits (e.g., economic, and social benefits) based on limited water. An appropriate planning concluding many factors such as food security, population growth, ecosystem deterioration and economic-social development should be considered, which can not only improve system benefits, but also optimal water availabilities. Based on local irrigation policies, a prescribed quantity of water is promised to each crop. If the promised water is delivered, it will result in net benefits to the local economy; otherwise, crops will have to either obtain water from more expensive sources or curtail their development plans, resulting in economic penalties. In such a problem, the water flow levels are uncertain (expressed as random variables), while a decision of water-allocation target (first stage decision) must be made before the realization of random variables, and then a recourse action can be taken after the disclosure of random variables (second-stage decision) [23]. Therefore, this problem under consideration can be formulated as a two-stage stochastic programming (TSP) model as follows:

Max \( f = \sum_{i=1}^{I} \sum_{j=1}^{J} c_i e_i x_j - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_h d_i e_i y_{ih} \)  

subject to

\( \sum_{i=1}^{I} \sum_{j=1}^{J} e_j (x_{ij} - y_{ij}) \leq Q_{ij} \)  

\( y_{ij} \leq x_{ij} \leq x_{ij}^{\max} \)  

\( x_{ij} \geq 0 \)  

\( y_{ij} \geq 0 \)

where \( i \) denotes types of crop (\( i = 1, 2, \cdots, I \)); \( j \) denotes types of district (\( j = 1, 2, \cdots, J \)); \( h \) denotes probability level of random water availability (\( h = 1, 2, \cdots, H \)); \( f \) presents net benefit of the entire system ($); \( c_{ij} \) is net benefit for crop \( i \) in district \( j \) per area ($ / ha); \( e_{ij} \) is irrigative coefficient of water consumption per area for crop \( i \) in district \( j \) (\( 10^3 \) m$^3$/ ha); \( x_{ij} \) is irrigated area target of crop \( i \) in district \( j \) (ha); \( Q_{ij} \) is total water availability of the entire system under probability \( P_h \) (\( 10^3 \) m$^3$); \( P_h \) denotes probability of random water availability \( Q_{ij} \) under level \( h \) (%); \( d_{ij} \) is economic loss for crop \( i \) in district \( j \) per area ($ / ha) when \( x_{ij} \) is not delivered ($); \( y_{ij} \) is water deficiency are for crop \( i \) in district \( j \) when demand is not met (\( 10^3 \) m$^3$). Where \( x_i \) is vector of first-stage decision variables, which have to be decided before the actual realizations of the random variables; \( c_{ij} e_{ij} x_{ij} \) is first-stage benefits; \( p_h \) is probability of random event; \( y_{ih} \) is recourse at the second-stage under the occurrence of event; \( \sum_{h=1}^{H} p_h d_i e_i y_{ih} \) is expected value
of the second-stage penalties [5]. Let $Q_{ijh}$ be a fuzzy set of imprecise right-hand sides with possibility distributions. Fuzzy credibility constrained programming (FCP) is effective for problems where system analysis is desired and the related stochastic distribution data are unavailable [24]. In FCP, credibility constraints can be addressed through credibility measures.

\[
\text{Max } f = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} d_{ij} e_{ij} y_{ijh} \tag{2a}
\]

subject to

\[
Cr\left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij} (x_{ij} - y_{ijh}) \leq Q_{ijh} \right\} \geq \alpha \tag{2b}
\]

\[
y_{ijh} \leq x_{ij} \leq x_{ij\text{max}} \tag{2c}
\]

\[
x_{ij} \geq 0 \tag{2d}
\]

\[
y_{ijh} \geq 0 \tag{2e}
\]

Where $\alpha$ is credibility level, which indicated relationships between satisfaction and risk of system [25]. Formula (2b) shows that credibility of satisfying $\sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij} (x_{ij} - y_{ijh}) \leq Q_{ijh}$ should be greater than or equal to level $\alpha$. However, FCP has difficulties in tackling uncertainties expressed fuzzy sets existing in left- and right-hand sides of constraints even in both sides of objective function synchronously. When $\tilde{c}_{ij}$, $d_{ij}$, $\tilde{e}_{ij}$ are fuzzy sets, Hurwicz criterion analysis is effective to tackle such a problem by introducing optimistic and pessimistic criterion, which can prove fuzzy determination by neutralizing alternative under uncertainties [26]. Therefore, introducing Hurwicz criterion into the FCP framework, a credibility-constrained programming with Hurwicz criterion (FCPH) model can be formulated as follows:

\[
\text{max } \bar{f} = \left\{ \lambda f_{opt} + (1-\lambda) f_{pec} \right\} \tag{3a}
\]

subject to

\[
Cr\left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} \tilde{c}_{ij} e_{ij} x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} d_{ij} e_{ij} y_{ijh} \leq f_{opt} \right\} \geq \beta \tag{3b}
\]

\[
Cr\left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} \tilde{c}_{ij} \tilde{e}_{ij} x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} d_{ij} \tilde{e}_{ij} y_{ijh} \geq f_{pec} \right\} \geq \beta \tag{3c}
\]

\[
Cr\left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij} (x_{ij} - y_{ijh}) \leq Q_{ijh} \right\} \geq \alpha \tag{3d}
\]

\[
y_{ijh} \leq x_{ij} \leq x_{ij\text{max}} \tag{3e}
\]
\[ x_{ij} \geq 0 \]  \hspace{1cm} (3f)

\[ y_{ijh} \geq 0 \]  \hspace{1cm} (3e)

where the decision payoffs are weighted by a coefficient of optimism \( \lambda \) (realism), where \( 0 \leq \lambda \leq 1 \). Conversely, \( 1 - \lambda \) represent a measure of the decision maker’s pessimism. The Hurwicz criterion requires that, for each decision alternative, the maximum payoff (minimum cost) be multiplied by the coefficient of optimism, and the minimum payoff (maximum cost) be multiplied by the coefficient of pessimism [21]. Therefore, by varying the coefficient \( \lambda \), the Hurwicz criterion becomes various criteria, e.g., when \( \lambda = 1 \), the criterion is the optimistic criterion; when \( \lambda = 0 \), it degenerate to a pessimistic criterion. \( \beta \) is credibility levels to (3b) and (3c), which indicated relationships between satisfaction and risk of system under optimistic and pessimistic situations.

The possibility of a fuzzy event, characterized by \( \xi \leq r \), is defined by \( Pos\{\xi \leq r\} = \sup_{u \leq r} \mu(u) \), while the necessity of a fuzzy event, characterized by \( \xi \leq r \), is defined by \( Nec\{\xi \leq r\} = 1 - \sup_{u > r} \mu(u) \) [27]. The credibility measure (Cr) is an average of the possibility measure and the necessity measure [19]:

\[
Cr\{\xi \leq r\} = \frac{1}{2} \left( Pos\{\xi \leq r\} + Nec\{\xi \leq r\} \right)
\]  \hspace{1cm} (4)

Also, the expected value of \( \xi \) can be determined based on the credibility measure as follows [19]:

\[
E[\xi] = \int_0^\infty Cr\{\xi \leq r\}dr - \int_0^{-\infty} Cr\{\xi \leq r\}dr
\]  \hspace{1cm} (5)

Triangular fuzzy number and trapezoidal fuzzy number are two kinds of special fuzzy variables in fuzzy set theory. Also, they are always employed in dealing with fuzziness. Let \( \xi = (\xi_1, \xi_2, \xi_3, \xi_4) \) be a trapezoidal fuzzy number. If \( \xi_2 = \xi_3 \), then trapezoidal fuzzy number \( \xi \) degenerates to a triangular fuzzy number. According to the Eq. (5), the expected value of \( \xi \) is \( (\xi_1, \xi_2, \xi_3, \xi_4)/4 \). Let \( \tilde{s} \) be a trapezoidal fuzzy variable \( (s_1, s_2, s_3) \), \( \tilde{s} \) is the estimation value by hierarchical agglomerative clustering method, which contains some estimation errors \( \varepsilon \), so we can set \( s_1 = s_2 (1-\varepsilon) \) and \( s_3 = s_2 (1+\varepsilon) \). Let \( \tilde{t} \) be a trapezoidal fuzzy variable \( (t_1, t_2, t_3, t_4) \), and \( \zeta \) the multiplication of \( \tilde{s} \) and \( \tilde{t} \). (i.e., \( \zeta = \tilde{s} \cdot \tilde{t} \)), then we have [22]:

\[
\zeta = s \cdot t \Rightarrow \zeta_\beta = s_\beta \cdot t_\beta = [s_{\beta L} L, s_{\beta U} U] = [\xi_{\beta L}, \xi_{\beta U}]
\]  \hspace{1cm} (6a)
\[ \xi^L = s_1 t_1 + \beta(s_2 t_2 - s_1 t_1) + (s_2 t_1 - s_1 t_1) + \beta^2[(s_2 - s_1)(t_2 - t_1)] \]  
(6b)

\[ \xi^U = s_4 t_4 + \beta(s_3 t_3 - s_4 t_4) + (s_3 t_4 - s_2 t_4) + \beta^2[(s_2 - s_3)(t_3 - t_4)] \]  
(6c)

Let \( \zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4) \) be a trapezoidal fuzzy number. According to the Eq. (6), the corresponding credibility measures are as follows:

\[
Cr\{\zeta \leq f_{\text{opt}}\} = \begin{cases} 
0 & \text{if } f_{\text{opt}} \leq \zeta_1, \\
\frac{f_{\text{opt}} - \zeta_1}{2(\zeta_2 - \zeta_1)} & \text{if } \zeta_1 \leq f_{\text{opt}} \leq \zeta_2, \\
\frac{f_{\text{opt}} + \zeta_4 - 2\zeta_3}{2(\zeta_4 - \zeta_3)} & \text{if } \zeta_3 \leq f_{\text{opt}} \leq \zeta_4, \\
1 & \text{if } f_{\text{opt}} \leq \zeta_4, 
\end{cases}
\]  
(7a)

\[
Cr\{\zeta \geq f_{\text{pec}}\} = \begin{cases} 
0 & \text{if } f_{\text{pec}} \leq \zeta_1, \\
\frac{2\zeta_2 - \zeta_1 - f_{\text{pec}}}{2(\zeta_2 - \zeta_1)} & \text{if } \zeta_1 \leq f_{\text{pec}} \leq \zeta_2, \\
\frac{\zeta_4 - f_{\text{pec}}}{2(\zeta_4 - \zeta_3)} & \text{if } \zeta_3 \leq f_{\text{pec}} \leq \zeta_4, \\
1 & \text{if } f_{\text{pec}} \leq \zeta_4, 
\end{cases}
\]  
(7b)

Based on (7a) and (7b), it can be proven that if \( \zeta \) is a trapezoidal fuzzy number and \( \beta > 0.5 \) then:

\[
Cr\{\zeta \leq f_{\text{opt}}\} \geq \beta \iff f_{\text{opt}} \geq (2\beta - 1)\zeta_4 - 2(\beta - 1)\zeta_3 \iff (2\beta - 1)s_4 t_4 - 2(\beta - 1)s_3 t_3
\]  
(8a)

\[
Cr\{\zeta \geq f_{\text{pec}}\} \geq \beta \iff f_{\text{pec}} \leq (1 - 2\beta)\zeta_4 + 2\beta\zeta_3 \iff f_{\text{pec}} \leq (1 - 2\beta)s_4 t_4 + 2\beta s_3 t_3
\]  
(8b)

Eq. (8a) and (8b) can be applied directly and more conveniently when compared to a-critical values proposed by, to convert fuzzy chance constraints into their equivalent crisp ones [19]. In real case, some variables are integer. Therefore, mixed integer programming could be employed. TFCHP can be incorporated within the integer programming framework. This leads to an integer fuzzy credibility constrained programming problems as follows:

\[
\max \overline{f} = [\lambda f_{\text{opt}} + (1 - \lambda) f_{\text{pec}}]
\]  
(9a)
subject to

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [(2\beta - 1)c_{ij}^3e_{ij}^4 + 2(\beta - 1)c_{ij}^3e_{ij}^3] x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} [(2\beta - 1)e_{ij}^3g_{ij}^4 - e_{ij}^3g_{ij}^3] y_{ijh}^- \leq f_{opt}^- \quad (9b)
\]

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [2c_{ij}^3e_{ij}^3 + (1 - 2\beta)c_{ij}^4e_{ij}^4] x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} [2e_{ij}^3g_{ij}^3 + (1 - 2\beta)e_{ij}^4g_{ij}^4] y_{ijh}^- \geq f_{pec}^- \quad (9c)
\]

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [(2\beta - 1)c_{ij}^3e_{ij}^4 + 2(\beta - 1)c_{ij}^3e_{ij}^3] x_{ij} - [(2\beta - 1)d_{ij}^3e_{ij}^4 - d_{ij}^3e_{ij}^3] y_{ijh} \leq Q_{ijh}^3 + (1 - 2\alpha)(Q_{ijh}^3 - Q_{ijh}^4) \quad (9d)
\]

\[
y_{ijh} \leq x_{ij} \leq x_{ij}^{max} \quad (9e)
\]

\[
x_{ij} \geq 0 \quad (9f)
\]

\[
y_{ijh} \geq 0 \quad (9g)
\]

However, the parameter of a model may fluctuate within a certain interval, and it is difficult to state a meaningful probability distribution for this variation. Interval-parameter programming (IPP) can deal with uncertainties in objective function and system constraints which can be expressed as interval without distribution information. Therefore, an inexact fuzzy optimization programming with Hurwicz criterion (IFOPH) for sustainable irrigation planning has been developed. In the IFOPH model, when the target of water for each user in each district (\(x_{ij}^\pm\)) is expressed as interval number, decision variable \(z_i\) is introduced to identify the optimal target value. Let \(x_{ij}^+ = x_{ij}^- + \Delta x_{ij}z_i\), where \(\Delta x_{ij} = x_{ij}^+ - x_{ij}^-\) and \(z_i \in [0,1]\). Thus, when \(x_{ij}^+\) reach their upper bounds, a higher net benefit of the water system would be achieved. However, a high risk of excess the water permit for each user in different district would be generated, leading to high loss of water deficiency. When \(x_{ij}^+\) reach their lower bounds, the system may get a lower net benefit with a low risk of water deficiency loss. Thus, model (9) can be transformed into the following two deterministic submodels as follows:

Submodel 1:

\[
\max f^+ = \{ \lambda \max f_{opt}^+ + (1 - \lambda) \min f_{pec}^+ \} \quad (10a)
\]

subject to

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [(2\beta - 1)c_{ij}^3e_{ij}^4 + 2(\beta - 1)c_{ij}^3e_{ij}^3] x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} [(2\beta - 1)d_{ij}^3e_{ij}^4 - d_{ij}^3e_{ij}^3] y_{ijh}^- \leq f_{opt}^- \quad (10b)
\]

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [2c_{ij}^3e_{ij}^3 + (1 - 2\beta)c_{ij}^4e_{ij}^4] x_{ij} - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{h=1}^{H} p_{ih} [2d_{ij}^3e_{ij}^3 + (1 - 2\beta)d_{ij}^4e_{ij}^4] y_{ijh}^- \geq f_{pec}^- \quad (10c)
\]
\[
\sum_{i=1}^{l} \sum_{j=1}^{j} \left\{ [(2\beta - 1)c_{ij}e_{ij}^3 + 2(\beta - 1)c_{ij}e_{ij}^3](x_{ij}^- + \Delta x_{ij}z_i) - [(2\beta - 1)e_{ij}^3g_{ij}^4 - e_{ij}^3g_{ij}^3]y_{ij}^- \right\} \\
\leq Q_{ijh} + (1 - 2\alpha)(Q_{ijh}^0 - Q_{ijh}^4) \\
y_{ijh}^- \leq (x_{ij}^- + \Delta x_{ij}z_i) \leq x_{ij}^\text{max} \\
y_{ijh}^- \geq 0, \ \forall i 
\]

Submodel 2:

\[
\max \ f^-_1 = \{ \lambda \max f^\text{ops} + (1 - \lambda) \min f^\text{pec} \} 
\]

subject to

\[
\sum_{i=1}^{l} \sum_{j=1}^{j} [(2\beta - 1)c_{ij}e_{ij}^3 + 2(\beta - 1)c_{ij}e_{ij}^3] x_{ij} - \sum_{i=1}^{l} \sum_{j=1}^{j} \sum_{h=1}^{H} p_{ih} [(2\beta - 1)e_{ij}^3g_{ij}^4 - e_{ij}^3g_{ij}^3] y_{ijh}^- \leq f^\text{ops}^- 
\]

\[
\sum_{i=1}^{l} \sum_{j=1}^{j} [2c_{ij}e_{ij}^3 + (1 - 2\beta)c_{ij}e_{ij}^3] x_{ij} - \sum_{i=1}^{l} \sum_{j=1}^{j} \sum_{h=1}^{H} p_{ih} [2e_{ij}^3g_{ij}^4 + (1 - 2\beta)e_{ij}^3g_{ij}^4] y_{ijh}^+ \geq f^\text{pec} 
\]

\[
\sum_{i=1}^{l} \sum_{j=1}^{j} \left\{ [(2\beta - 1)c_{ij}e_{ij}^3 + 2(\beta - 1)c_{ij}e_{ij}^3](x_{ij}^- + \Delta x_{ij}z_i) - [(2\beta - 1)e_{ij}^3g_{ij}^4 - e_{ij}^3g_{ij}^3]y_{ijh}^+ \right\} \\
\leq Q_{ijh}^1 + (1 - 2\alpha)(Q_{ijh}^0 - Q_{ijh}^4) \\
y_{ijh}^+ \leq (x_{ij}^- + \Delta x_{ij}z_i) \leq x_{ij}^\text{max} \\
y_{ijh}^+ \geq 0, \ \forall i 
\]

3. Case study

The Tarim River is located in northwest of China, which is formed by the unions of Aksu, Hotan, Yarkant and Kaidu-kongque rivers, and flows east along the northern edge of the desert, which is flanked by the Tianshan Mountains to the north and by the Kunlun Mountains to the south [28]. It is with a length of 1300 km, and which is the longest inland river all over the country. The study area (including Kuerle, Yanqi, Hejing, Heshuo, Bohu, Yuli and Luntai counties) is located in the middle reaches of the Tarim River Basin, with an area of approximately \(62 \times 10^3 \text{ km}^2\) and a population over one million [29]. It is a typical arid region due to extremely dry climate, low and uneven distribution rainfall. For example, the climate in the basin is extremely dry with the average rainfall about 273mm/year, which more than 80% of the total annual precipitation falls from May to September, and less than 20% of the total falls from November to the following April [1]. It is one of the most important bases of cotton and grain in the Tarim River Basin and the northwest of China, where irrigation water usage occupies 90% of the water in whole area. It is suitable for the growth of crops such as cereal, cotton, oil bearing crop, vegetable, fruit and
forage, which accelerates agricultural products processing and manufacturing [30]. Water demands of crops in seven districts rely on river’s streamflow, which is mainly from its upstream, snow melting, and rainfall. Due to dry climate, low-rainfall, and high evaporation, the water supply capacity of river is quite low, which has difficulties in satisfying the water demands of crops. Particular in recent years, the demand of irrigation has reached the limits of what the natural system can provide, so that water shortage can become a major obstacle to social and economic development for this region. Therefore, population growth, food security challenge, economy development and the potential threat of climate change elevate the attention given to efficient and sustainable irrigation.

The manager of study region desires to create a sustainable plan to allocate water resources to multiple crops, which should be consider the system benefit and system disruption risk attributable to uncertainties simultaneously. On the one hand, appropriate decisions have been made by water manager based on water demands of various crops planting. If the promised water is delivered, a net benefit to the local economy will be generated for each unit of water allocated; otherwise, either the water must be obtained from higher-priced alternatives or the demand must be curtailed by reduced planting, resulting in a reduced system benefit. On the other hand, uncertainties existed in irrigation planning process (e.g., imprecise economic data, random stream flows, dynamic system variables, uncertain economic benefits, various recourse actions, and varied water allocations) increase complexities of water resources system, which amplifies difficulties of water planning (as shown in Figure 1). These components and their interactions must be systematically investigated using an IFOPH in a sustainable irrigation planning system, as shown in Figure 1.
Figure 1. Framework of IFOPH method application of Tarim River Basin

Sustainable irrigation system

Imprecise economic data
Random in water availabilities
Different recourse actions
Dynamic water demands
Various water polices

Multiply uncertainties

Radom events
Interval-parameter
Fuzzy sets

Two-stage stochastic programming (TSP)
Interval-parameter programming (IPP)
Fuzzy credibility programming (FCP)
Hurwicz criterion (HC)

An inexact fuzzy optimization programming with Hurwicz criterion (IFOPH)

Application

Irrigation system of Tarim River Basin

Logical irrigation pattern
Water supply security
Sustainable development
Risk preference option
Analysis system benefit

Sensitive analysis
Optimal solutions
Generation of decision alternative
### Table 1 Economic data

<table>
<thead>
<tr>
<th></th>
<th>Cereal</th>
<th>Cotton</th>
<th>Oil bearing crops</th>
<th>Vegetable</th>
<th>Forage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net benefit when water demand is satisfied ($)</strong></td>
<td></td>
<td></td>
<td></td>
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### Table 2 Probability levels and total water availabilities

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<tr>
<th>Flow level</th>
<th>Low (h=1)</th>
<th>Low-medium (h=2)</th>
<th>Medium (h=3)</th>
<th>High-medium (h=4)</th>
<th>High (h=5)</th>
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<tbody>
<tr>
<td>Probability (%)</td>
<td>0.15</td>
<td>0.49</td>
<td>0.24</td>
<td>0.08</td>
<td>0.04</td>
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<tr>
<td>Water flow ($10^6$ m$^3$)</td>
<td>[2291.55, 2326.45]</td>
<td>[2357.64, 2393.55]</td>
<td>[2404.8, 2441.42]</td>
<td>[2451.91, 2489.29]</td>
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Table 1 shows basic economic data, which are estimated indirectly based on the statistical yearbook of Xinjiang Uygur Autonomous Region in Uygur Autonomous Region 2005-2012 and water price. Values of $c_y$ and $d_y$ are estimated according to different users’ gross national product in different counties indirectly, which upper bound of values are estimated the highest one from yearbook (2012) and lower bound are the opposite one. Value of $Q_{ijh}$ should be conducted through statistical analyses with the results of annual stream flow of the Tarim River (2005-2012). Due to rain seasons in Tarim River Basin More than 80% of the total annual precipitation falls from May to September, and less than 20% of the total falls from November to the following April. Therefore, the total water availability can be converted into several levels. Table 2 shows total water availability of Tarim River Basin under several level probabilities.

Since different credibility levels in IFOPH of Tarim River Basin, four cases are considered to compare varied water allocations and system benefits changed by different satisfaction levels. Case 1 is based on the current water-resource allocation policies with α-level of 0.60, while β-level of 0.60. Cases 2 to 4 are considered with the equality of α-level and β-level, which are 0.7 to 0.9 respectively.

4. Results and Discussion

4.1 Water allocation

Figure 2 presents the optimized water targets of competitive crops under different cases ($\lambda = 0.9$) in seven districts of study region. Since the optimized water-allocation targets for water consumers of different districts can be obtained based on $x_{ijopt} = x_{ij} + \Delta x_{ij}z_{ijopt}$, the value of $z_{ijopt}$ would affect the optimized allocation targets directly. For example, optimized allocation targets for vegetable targets in Bohu would be $71.90 \times 10^6$ m$^3$ in periods 1 (i.e., $z_{ijopt} = 0.93$), which would approach their upper bounds; while the optimized cotton targets in Hejing would approach their lower bounds (i.e., $23.86 \times 10^6$ m$^3$) corresponding to $z_{ijopt} = 0.26$. The decision of water-allocation target represents a compromise of policy-guided water shortage and water permit (right) surplus under uncertain water availability. A higher target level would lead to a higher benefit but, at the same time, a higher risk of policy-guided water shortage when the water flow is low; however, a lower target level would result in a higher water permit surplus when the water flow is high. Meanwhile, from overall trends of optimized water target showed in Figure 2, it implies that cotton would be the largest water deficit among competitive crops, in which shortages of cotton would exceed 60% of total water shortages.

Figure 2 Water demands and optimal targets under cases 1 and 4 when λ=0.9
By inputting the interval numbers of stream flow and the economic data, water shortages of crops in seven counties are obtained. Water shortages would occur if the available water resource could not meet the regulated target, which indicates that the shortage is the difference between the target and water availability. Based on different credibility-satisfaction levels, water shortage of 4 crops in 7 districts in study basin under case 1 and case 4 ($\lambda = 0.9$) are shown in Figure 3. Solutions indicate that the water shortages would be influenced by the randomness in the total water availabilities. For example, when water flows in wet season, water target could easily be satisfied, which leads water shortage would be less than that in dry season. Meanwhile, shortages would be influenced by $\alpha$-level, since $\alpha$-level is fuzzy credibility measure in the constraint of water availability. The highest water shortages would be achieved under case 4 (i.e., $\alpha = 0.9$), which indicated that a higher $\alpha$-levels led a higher water shortage; by decreasing of $\alpha$-levels, water shortages dropped under case 1 (i.e., $\alpha = 0.6$). For example, water shortage of cereal in Kuerla county would be $[2.26, 3.99] \times 10^6$ m$^3$ at low level under case 4 (i.e., $\alpha = 0.9$); while it would be $[1.74, 3.31] \times 10^6$ m$^3$ under case 1 (i.e., $\alpha = 0.6$). In addition, $\beta$-levels have little effected on water shortages, since $\beta$ is satisfaction levels of the constraints only influenced unit benefit directly.
Figure 3 Water shortages under cases 1 and 4 when $\lambda=0.9$

- **Lower bound**
  - Cereal
  - Cotton
  - Oil bearing crops
  - Vegetable
  - Forage

- **Upper bound**
  - Cereal
  - Cotton
  - Oil bearing crops
  - Vegetable
  - Forage

**Figure 4** shows total water allocations under case 1 and case 2 ($\lambda = 0.9$). Results indicate that the actual water allocation would be the difference between the pre-regulated target and the probabilistic shortage (i.e., actual allocation = optimized target - shortage). Each allocated water flow is the difference between the promised target and the probabilistic shortage under a given stream condition with an associated probability level, which indicates that different violation levels would result in varied water-allocation patterns. For example, optimized targets of oil bearing crops in Heshuo county would be $[13.36, 12.64] \times 10^6$ m$^3$ under case 1 and 6. When inflow is low, shortages and actual allocations would be $[0.10, 0.37] \times 10^6$ m$^3$ and $[12.26, 12.27] \times 10^6$ m$^3$ under case 1; while they would be $[0.23, 0.48] \times 10^6$ m$^3$ and $[12.13, 12.16] \times 10^6$ m$^3$ under case 4. In comparison, it obtained that water allocation with higher $\alpha$-level was smaller than that with higher $\alpha$-level, since higher $\alpha$-level led higher credibility-satisfactions and lower violation risks in irrigation planning system, which generated higher water deficiencies and lower water allocations. In addiction, it implied that the largest water allocation existed in cotton among competitive crops in seven counties. **Figure 5** shows water allocations in Yanqi county under cases 2 and 4 when $\lambda=0.9$. The results indicated that the highest water allocation would be cearl in Yanqi county, which would change with various credibility-satisfaction levels. For example, when inflow is low, actual allocations of cearl would be $[72.71, 76.22] \times 10^6$ m$^3$ under case 2; while they would be $[64.63, 64.97] \times 10^6$ m$^3$ under case 4. The results also indicate that a lower credibility satisfaction levels corresponding to a higher water availability would result in a lower water deficiency, which produced a higher water allocation; otherwise, it generates a opposite result.
Figure 4 Total water allocations under cases 1 and 2 when $\lambda=0.9$

Figure 5 Water allocations of cotton under cases 1 and 3 when $\lambda=0.9$
4.2 System benefit

In the IFOPH model, different credibility satisfaction levels and Hurwicz criterions for objective function and constraints were examined, which could help investigate the risks of violating the constraints and generate a range of decision alternatives. Through solving the IFOPH model, system benefits under various credibility-satisfaction levels and Hurwicz parameters are obtained (as shown in Figure 6). Results present that system benefits would increase with the $\lambda$ value. For example, under case 1 (i.e., $\alpha = 0.6, \beta = 0.6$), system benefits would be from $[667.2, 906.4] \times 10^6$ $\$ to $[655.4, 893] \times 10^6$ $\$ when $\lambda$-levels are from 0.1 to 0.9, and under case 4 (i.e., $\alpha = 0.9, \beta = 0.9$) they would be from $[575.9, 816.7] \times 10^6$ $\$ to $[501, 791.5] \times 10^6$ $\$. Meanwhile, system benefits would change with different $\alpha$-levels. Since $\alpha$-level is conducted as the fuzzy credibility measure in the constraint of water availability, which reflects relationship between confidence degree and violation degree of fuzzy water availability, system benefits would decrease as $\alpha$-level is raised. For example, when $\lambda$ is 0.4, system benefit would be $[660.0, 901.0] \times 10^6$ $\$ under case 1 (i.e., $\alpha = 0.6$), while it would be $[537.0, 813.7] \times 10^6$ $\$ under case 4 (i.e., $\alpha = 0.9$). Thus, it indicates that a higher credibility satisfaction level of water availability led an increased system benefit; however, this increase also corresponds to a raised risk level (i.e. lower system credibility and lower satisfaction degree). Moreover, system benefits would vary with different $\beta$-levels, which are much more complex than that with $\alpha$-levels. The results indicate that system benefits would be affected by interaction of $\beta$-level and $\lambda$. Three tendencies of system benefits are obtained: system benefits would be increasing with $\beta$-levels when $\lambda$ is from 0.1 to 0.4, decreasing when $\lambda$ is from 0.6 to 0.9, and invariant when $\lambda$ is 0.5.
4.3 Sensitive analysis

A number of sensitive analyses are conducted for examining the effects of different credibility-satisfaction levels and Hurwicz parameters for objective function and constraints. Since Hurwicz criterion introduced in models corresponding to various criteria (i.e., optimistic and pessimistic criterion), system benefits with optimistic, pessimistic and normal criterion (i.e., $f_{opt}$, $f_{pec}$, $f$) would be obtained in Figure 7. In Figure 7, various system benefits (i.e., $f_{opt}$, $f_{pec}$, $f$) would change with different $\alpha$- and $\lambda$-levels, which could observe relationships between $\alpha$- and $\lambda$-levels impacted on system benefits. The highest system benefit with optimistic, pessimistic and normal criterion (i.e., $f_{opt}$, $f_{pec}$, $f$) would be achieved under the highest $\lambda$-level (i.e., $\lambda = 0.9$) when $\beta = 0.9$. For example, the highest $f$ would be $[0.53, 0.81] \times 10^9$ (i.e., $\lambda = 0.9$) when $\alpha = 0.6$. By increasing of $\alpha$-level, system benefits with optimistic, pessimistic and normal criterion ($f_{opt}$, $f_{pec}$, $f$) would be decreasing obviously, which indicate that the relationship between $\alpha$-level and system benefit are adverse. For example, $f_{opt}$ would be from $[0.39, 0.53] \times 10^9$ to $[0.32, 0.48] \times 10^9$ when $\alpha$-level are from 0.6 to 0.9 (i.e., $\lambda = 0.6$). Meanwhile, by increasing of $\lambda$-level, system benefits with optimistic (i.e., $f_{opt}$) raised, whereas benefits with pessimistic criterion (i.e., $f_{pec}$) dropped. For example, $f_{opt}$ would be from $[0.06, 0.06] \times 10^9$ to $[0.59, 0.83] \times 10^9$ when $\lambda$ are from 0.1 to 0.9 (i.e., $\alpha = 0.6$), whereas $f_{pec}$ would be from $[0.66, 0.81] \times 10^9$ to $[0.05, 0.08] \times 10^9$. In general, a higher $\lambda$-level can lead to an increased system benefits; however, these increase also are influenced by a dropped $\alpha$-levels (i.e. lower system reliability and lower satisfaction degree).
Figure 7 Sensitive analysis under different α-level and λ-level

![Figure 7: Sensitive analysis under different α-level and λ-level](image)

**Figure 8** shows any changes in β- and λ-levels would lead variation in system benefits. The results indicate that system benefits with three criterions (i.e., $f_{opt}$, $f_{pec}$, $f$) would be vary with interactions between β- and λ-levels. System benefits with three criterions would be decreasing with β-level when λ = 0.1 to 0.4, they would be increasing with β-level when λ= 0.6 to 0.9, and would be invariant with β-level when λ= 0.5. For example, when β are from 0.6 to 0.9, $f_{opt}$ would be from [0.06, 0.09] × 10^9$ to [0.66, 0.91] × 10^9$ (λ = 0.1), $f_{opt}$ would be from [0.33, 0.45] × 10^9$ to [0.33, 0.45] × 10^9$ (λ = 0.5) and $f_{opt}$ would be from [0.59, 0.80] × 10^9$ to [0.56, 0.79] × 10^9$ (λ = 0.9). Meanwhile, by increasing of λ-level, system benefits with optimistic criterion (i.e., $f_{opt}$) raised, whereas benefits with pessimistic criterion (i.e., $f_{pec}$) would drop. For example, $f_{opt}$ would be from [0.06, 0.09] × 10^9$ to [0.59, 0.80] × 10^9$ when λ are from 0.6 to 0.9 (i.e., β= 0.6), whereas $f_{pec}$ would be from [0.6, 0.81] × 10^9$ to [0.06, 0.09] × 10^9$. It implies that system benefits are sensitive to interaction between β- and λ-levels, where a higher λ-level can result in an increased system benefit; while an increasing β-levels can generate various tendencies of system benefits based on different λ-levels.
5. Conclusions

In this study, an inexact fuzzy optimization programming with Hurwicz criterion (IFOPH) is developed for sustainable irrigation planning under uncertainty, which incorporate two-stage stochastic programming (TSP), interval-parameter programming (IPP), fuzzy credibility-constraint programming (FCP) and Hurwicz criterion (TCP-CH) within an framework. IFOPH has three advantages in comparison to other optimization techniques for irrigation planning. Firstly, multiple uncertainties (existed as intervals, random variables, and their combinations) can be directly communicated into the optimization process, leading to enhanced system robustness for uncertainty reflection. Secondly, it can provide an effective linkage between conflicting economic benefits and the associated penalties attributed to the violation of the pre-regulated policies, which can also help generating a sustainable irrigation planning. Thirdly, tradeoffs between economic benefit and system-failure risk are also examined under different risk preferences of decision makers (i.e., optimistic and pessimistic criteria), which support generating an increased robustness in risk control for water resources allocation under uncertainties.

The developed method has been applied to Tarim Basin for sustainable irrigation planning under uncertainties, which a number of various cases based on satisfaction levels, optimistic / pessimistic criterion were listed to compare. Different policies for irrigation planning would lead varied allocation targets, shortages, system benefits, and penalties, which will help generate desired policies for sustainable irrigation planning with maximized economic benefit and minimized system-failure risk. The results discover that severe water deficit in irrigation due to characteristic of aridity has brought negative effects on regional social-economic development in these region. The losses are caused by
several reasons such as unreasonable water plans, inefficient water usage (e.g., behindhand irrigation regime) and unscientific risk option. Secondly, it discover that risk preference of decision makers in decision process with uncertain importations can affect water planning and allocation, which support decision makers making neither adventurous nor conservative decisions in sustainable irrigation planning. Thirdly, the irrigation regime and water saving technology of this region is relative backward, which generate more inefficient water usage. Therefore, the manager of study region should adjust water policy the aim of sustainability in study region, which not only balance the tradeoff between the system benefit and risk of practical water planning, but also support in-depth analysis of different manager preferences toward risk permits. Meanwhile, advanced irrigation regime and water saving technology (e.g., drop irrigation) should be recommended to further improve efficiency of agricultural water usage.

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Author Contributions

Main text paragraph is written by Xueting Zeng, and Yongping Li is responsible for polishing the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References and Notes


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