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Lorentzian quintessential inflation*

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From the assumption that the slow-roll parameter ϵ has a Lorentzian form as a function of the e-folds number N, a successful model of a quintessential inflation is obtained. The form corresponds to the vacuum energy both in the inflationary and in the dark energy epochs. The form satisfies the condition to climb from small values of ϵ to 1 at the end of the inflationary epoch. At the late universe, ϵ becomes small again and this leads to the dark energy epoch. The observables that the models predict fits with the latest Planck data: $r \sim 10^{-3}, n_s \approx 0.965$. Naturally, a large dimensionless factor that exponentially amplifies the inflationary scale and exponentially suppresses the dark energy scale appearance, producing a sort of *cosmological seesaw mechanism*. We find the corresponding scalar Quintessential Inflationary potential with two flat regions — one inflationary and one as a dark energy with slow-roll behavior.

Keywords: Inflation; dark energy; quintessence; cosmology; cosmological seesaw mechanism.

1. Introduction

The inflationary paradigm is considered as a necessary part of the Standard Model of cosmology, since it provides the solution to the fundamental puzzles of the old Big Bang theory, such as the horizon, the flatness, and the monopole problems.[1–9] It can be achieved through various mechanisms, for instance through the introduction of a scalar inflaton field.[10–25] Almost 20 years after the observational evidence of cosmic acceleration, the cause of this phenomenon, labeled as a “dark energy”, remains an

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open question which challenges the foundations of theoretical physics: why there is a large disagreement between the vacuum expectation value of the energy–momentum tensor which comes from quantum field theory and the observable value of dark energy density.[26–28] One way to parametrize dynamical dark energy, uses a scalar field, the so-called quintessence model for canonical scalar fields.[29–31] In such a way that the cosmological constant gets replaced by a dark energy fluid with a nearly constant density today.[32–36] For the slow-roll approximation, the scalar field behaves as an effective dark energy. The form of the potential is clearly unknown and many different potentials have been studied and confronted to observations.

These two regimes of accelerated expansion are treated independently. However, it is both tempting and economical to think that there is a unique cause responsible for a quintessential inflation[37–50] which refers to the unification of both concepts using a single scalar field. Consistency of the scenario demands that the new degree of freedom, namely, the scalar field, should not interfere with the thermal history of the universe, and thereby it should be "invisible" for the entire evolution and reappears only around the present epoch giving rise to late-time cosmic acceleration.

2. Lorentzian Ansatz

In order to formulate an ansatz for the Hubble function that treats symmetrically both the early and late times, we use the Lorentzian function for the slow-roll parameter:

$$\epsilon(N) = \frac{\xi}{\pi} \frac{\Gamma/2}{N^2 + (\Gamma/2)^2} \tag{1}$$

as a function of the number of e-folds $N = \log(a/a_i)$, where a_i is the scale parameter at some time (which we may choose as the initial state of the inflationary phase). ξ is the amplitude of the Lorentzian, Γ is the width of the Lorentzian. In that way, the ϵ parameter increases from the initial value to 1 at the end of inflation, then continues to increase, peak and then decreases until it gets down to the value 1 and this represents the beginning of a the new dark energy phase that will eventually dominate the late evolution of the universe. The upper panel of Fig. 1 presents the qualitative shape of this behavior.

The dominant energy condition yields another bound on the coefficients. The equation of states w is in the range $|w| \leq 1$. From the relation $\epsilon = \frac{3}{2}(w+1)$, we obtain the bound $0 \leq \epsilon \leq 3$. The ansatz for the vacuum energy evolution (1) is positive always, hence the lower bound is preserved. The largest value of the ansatz (1) is $2\xi/\pi\Gamma$. From the upper bound of ϵ, we obtain the condition:

$$\Gamma < 2\xi/3\pi. \tag{2}$$

In general, the calculation of the above observables demands a detailed perturbation analysis. Nevertheless, one can obtain approximate expressions by imposing the

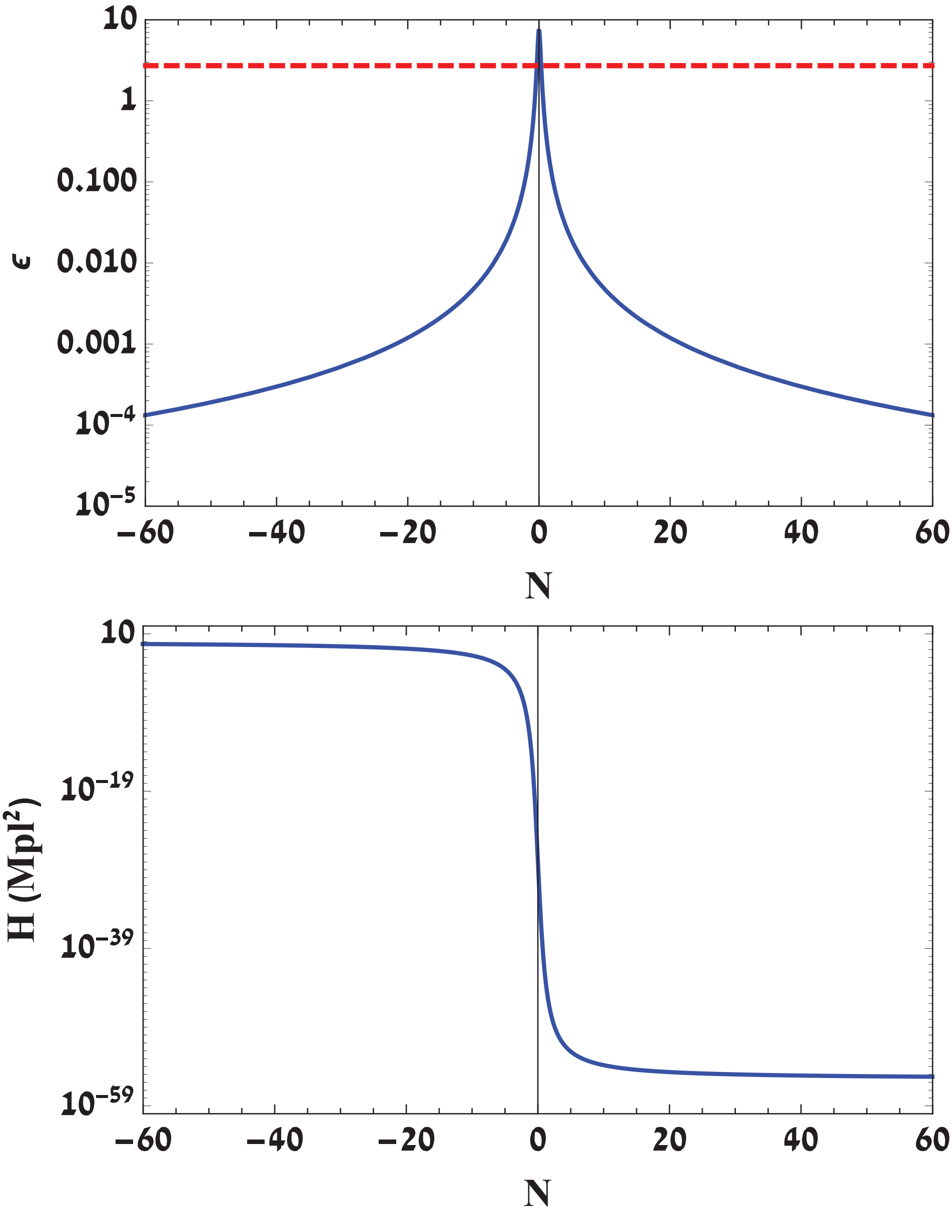

Fig. 1. The upper panel shows the slow-roll parameter ϵ versus the number of e-folds for the ansatz (1), in a logarithmic scale. The lower panel shows the corresponding Hubble function of the vacuum versus the number of e-folds.

slow-roll assumptions, under which all inflationary information is encoded in the slow-roll parameters. In particular, one first introduces[51]

$$\epsilon_{n+1} = \frac{d}{dN} \log |\epsilon_n|, \tag{3}$$

where $\epsilon_0 \equiv H_i/H$ and n a positive integer. The slow-roll parameters read:

$$\epsilon \equiv \epsilon_1 = -\frac{H'}{H}, \quad \epsilon_2 = \frac{H''}{H'} - \frac{H'}{H},$$

and so on. From the first slow-roll parameter definition with the ansatz (1), we obtain the solution:

$$H = \sqrt{\frac{\Lambda_0}{3}} \exp\left[-\frac{\xi}{\pi}\tan^{-1}\left(\frac{2N}{\Gamma}\right)\right]. \tag{4}$$

where Λ_0 is an integration constant. The Hubble function interpolates from the inflationary values $H_{-\infty}$ to the dark energy value $H_{+\infty}$ that corresponds to:

$$H_\pm = \sqrt{\frac{\Lambda_0}{3}} \exp^{\mp\xi/2}. \tag{5}$$

The magnitude of the vacuum energy at the inflationary phase reads $10^{-8}\mathrm{Mpl}^4$, while the magnitude of the vacuum energy at the present slowly accelerated phase of the universe is $10^{-120}\mathrm{Mpl}^4$. From the Friedmann equations, the values of the energy density is $3H^2$ in the Planck scale. Therefore, the coefficients of the model are:

$$\xi \approx 129, \quad \Lambda_0 = 1.7 \cdot 10^{-32}\mathrm{Mpl}^4. \tag{6}$$

We calculate the other slow-roll parameters using (3):

$$\epsilon_2 = -\frac{8N}{\Gamma^2 + 4N^2}, \quad \epsilon_3 = \frac{1}{N} - \frac{8N}{\Gamma^2 + 4N^2}. \tag{7}$$

For $\Gamma \to 0$, all of the slow-roll parameters with $n \geq 3$ yields the value $-1/N$. However, in the general case, all of the slow parameters have small values if the ϵ_2 is small.

As usual inflation ends at a scale factor a_f where $\epsilon_1(a_f) = 1$ and the slow-roll approximation breaks down. Therefore, the end of inflation takes place when the number of e-folds read:

$$N_f = \pm\sqrt{\frac{\Gamma}{4\pi}(2\xi - \pi\Gamma)}. \tag{8}$$

Note that with the condition (2), we get a definite value. In order to have an inflationary phase, the condition $2\xi > \pi\Gamma$ must be satisfied. The negative value of N_f is the final state of the inflationary phase, while the positive value of N_f is the initial value of the slow-rolling dark energy at the late universe. Therefore, in order to calculate the inflationary observables, we must take the minus sign of N_f. Consequently, the initial N_i satisfies the condition: $N_f - N_i = \mathcal{N} \approx 50 - 60$, where we impose 60 e-folds for the inflationary phase. Hence, the initial state of the inflationary phase reads:

$$N_i = -\sqrt{\frac{\Gamma}{4\pi}(2\xi - \pi\Gamma)} - \mathcal{N}. \tag{9}$$

The inflationary observables are expressed as:[51]

$$r \approx 16\epsilon_1, \quad n_{\rm s} \approx 1 - 2\epsilon_1 - \epsilon_2, \quad \alpha_{\rm s} \approx -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3, \quad n_{\rm T} \approx -2\epsilon_1, \tag{10}$$

where all quantities are calculated at N_i. Therefore, the tensor to scalar ratio and the primordial tilt give:

$$r = \frac{32\Gamma\xi}{\pi\Gamma^2 + 4\pi N_i^2}, \quad n_s = \frac{\pi\left(\Gamma^2 + 4N_i(N_i+2)\right) - 4\Gamma\xi}{\pi\left(\Gamma^2 + 4N_i^2\right)}. \tag{11}$$

For 60 e-folds and $\Gamma = 0.1$, the observables read:

$$r = 0.0076, \quad n_s = 0.961754. \tag{12}$$

These values are in agreement with the latest 2018 Planck data:[52,53]

$$0.95 < n_s < 0.97, \quad r < 0.064. \tag{13}$$

Figure 2 shows the predicted distribution of the observables.[54] Figure 3 shows the predicted distribution of the observables.[54] We assume a uniform prior: $N \in [50; 70]$, $\xi \in [100; 200]$, $\Gamma \in [0; 1]$, with 10^7 Markov Chain Monte Carlo samples. We find the posterior yields:

$$r = 0.045^{+0.065}_{-0.053}, \tag{14}$$

$$n_s = 0.9624^{+0.0087}_{-0.011}, \tag{15}$$

$$\alpha_s = -\left(33^{+27}_{-30}\right) \cdot 10^{-5}, \tag{16}$$

in a good agreement with the recent Planck values.

3. Scalar Field Dynamics

The above ansatz is of general applicability in any inflation realization, whether this is driven by a scalar field, or it arises effectively from modified gravity, or from any other mechanism. In order to provide a more transparent picture, let us consider a realization of these ideas in the context of a canonical scalar field theory ϕ moving in a potential $V(\phi)$. The Friedmann equations are:

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \quad \dot{H} = -4\pi G\dot{\phi}^2, \tag{17}$$

while the variation for the scalar field is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{18}$$

Let us apply the ansatz in order to reconstruct a physical scalar-field potential that can generate the desirable inflationary observables. From the Friedmann equation (17) that holds in every scalar-field inflation, we extract the following solutions:

$$\phi = \int_0^N \sqrt{-2\frac{H'}{H}}\, dN, \quad V(\phi) = HH' + 3H^2. \tag{19}$$

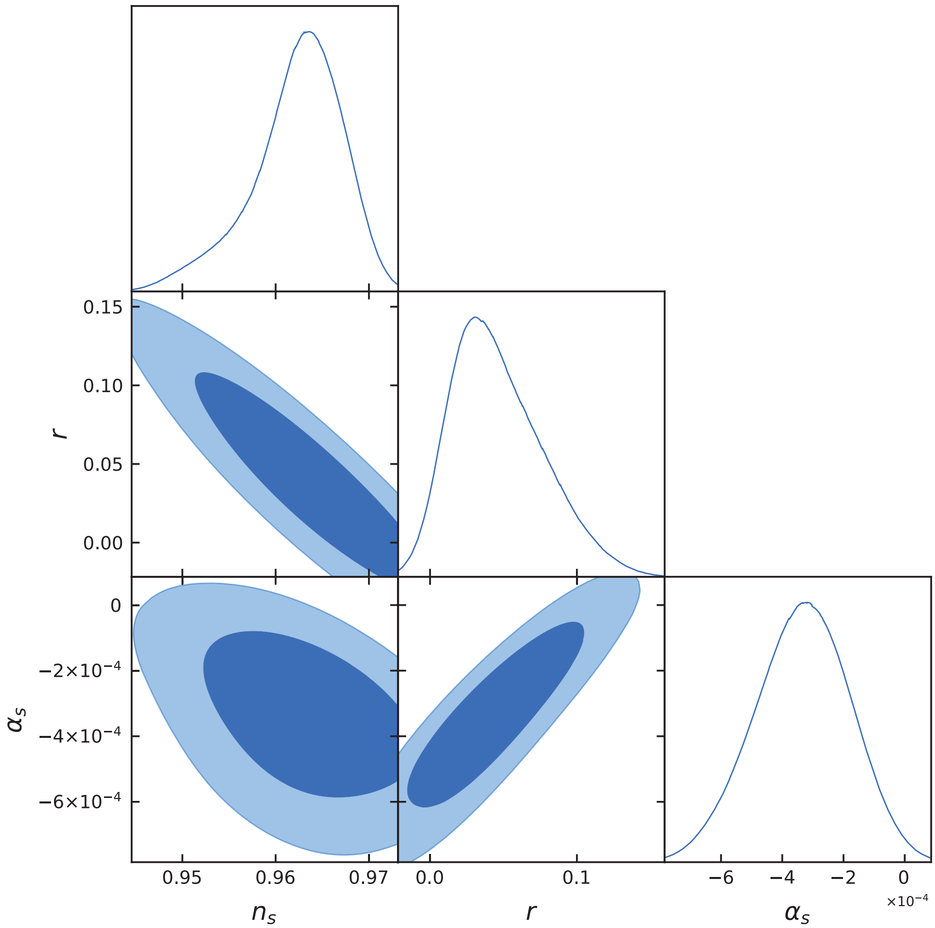

Fig. 2. The predicted scalar to tensor ratio versus the primordial tilt of the model.

with $8\pi G = 1$. From the integration of the Hubble parameter, we get:

$$N = \frac{\Gamma}{2}\sinh\left(\sqrt{\frac{\pi}{\xi\Gamma}}\phi\right), \quad V(N) = \Lambda_0 e^{-\frac{2\xi}{\pi}\tan^{-1}\left(\frac{2N}{\Gamma}\right)}\left(1 - \frac{2\Gamma\xi}{3\pi\Gamma^2 + 12\pi N^2}\right). \tag{20}$$

Expression (20) cannot be inversed, in order to find $N(\phi)$ and then through insertion into (20) to extract $V(\phi)$ analytically:

$$V(\phi) = \Lambda_0 e^{-\frac{2\xi}{\pi}\tan^{-1}(\sinh x)}\left(1 - \frac{2\xi}{3\pi\Gamma}\mathrm{sech}^2 x\right). \tag{21}$$

with $x \equiv \sqrt{\pi/\Gamma\xi}\phi$. Figure 2 shows the scalar potential $V(\phi)$. The universe in this picture begins with $\phi \to \infty$ with a slow-roll behavior and goes to the left-hand side. After approaching the minimum, the universe evolves with another slow-roll behavior that corresponds to the dark energy epoch when $\phi \to -\infty$. The asymptotic values of the potential are:

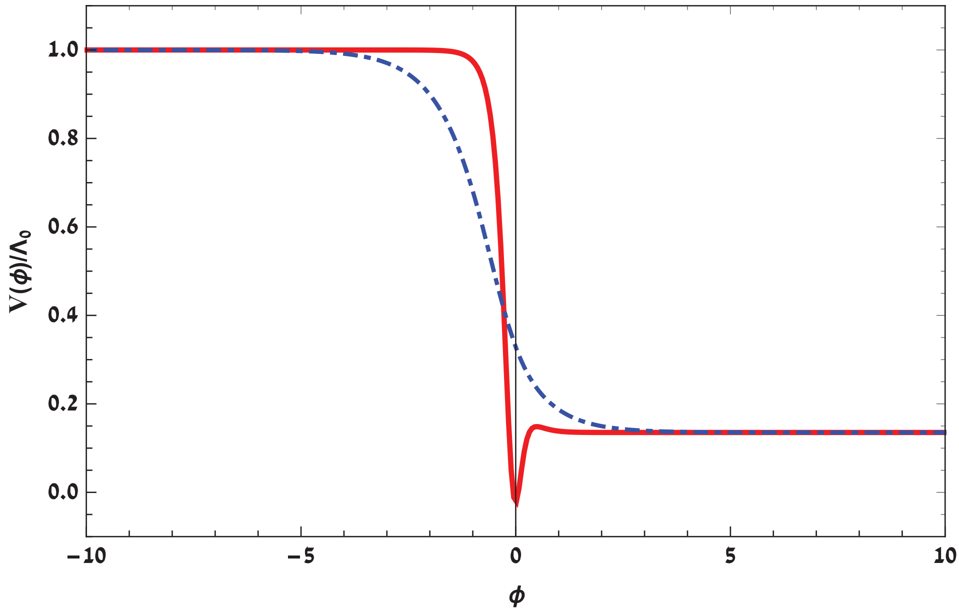

Fig. 3. (Color online) The corresponding scalar field potential for the Lorenzian ansatz, with different values of Γ: 0.1 red smooth line, 1 blue dashed line.

$$V_{+\infty} = \Lambda_0 e^{\xi}, \quad V_{-\infty} = \Lambda_0 e^{-\xi}. \tag{22}$$

Note that this represents a seesaw cosmological effect, that is if Λ_0 represents an intermediate scale, we see that in order to make the inflationary scale big this forces the present vacuum energy to be small. Λ_0 represents the geometric average of the inflationary vacuum energy and the present dark energy vacuum energies.

4. Discussion

This paper introduces a model where we start with an ansatz for the slow-roll parameter ϵ for the whole history of the universe. We choose a Lorentzian form for ϵ, which peaks at some point and goes to zero for the early and late universe, so these two epochs have an accelerated phase. The magnitude of the vacuum energies at the early and late universe obeys a seesaw mechanism, since the asymptotic values of the potential are $\Lambda_0 e^{\pm\xi}$, represents a seesaw cosmological effect, where the requirement is that one scale (the inflationary scale) to be large which pushes the dark energy scale to be very low. Seesaw cosmological effects in modified measure theories with spontaneously broken scale invariance have been studied in Refs. 55–57. For the situation presented in this paper to work, we must choose Λ_0 as an intermediate scale, and indeed then we see that in order to make the inflationary scale big, this forces the present vacuum energy to be small. Λ_0 represents the geometric average of the inflationary vacuum energy and the present dark energy vacuum energies.

The model formulates the vacuum energies both in the inflationary epoch and in the dark energy epoch. However, to compare the basis of the model with the whole history of universe, we have taken into account particle creation models with temperature, as well as radiation production.

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References

1. A. H. Guth, *Phys. Rev. D* **23** (1981) 347; *Adv. Ser. Astrophys. Cosmol.* **3** (1987) 139.
2. A. H. Guth and S. Y. Pi, *Phys. Rev. Lett.* **49** (1982) 1110.
3. A. A. Starobinsky, *JETP Lett.* **30** (1979) 682.
4. D. Kazanas, *Astrophys. J.* **241** (1980) L59.
5. A. A. Starobinsky, *Phys. Lett. B* **91** (1980) 99.
6. A. D. Linde, *Phys. Lett. B* **108** (1982) 389; *Adv. Ser. Astrophys. Cosmol.* **3** (1987) 149.
7. A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48** (1982) 1220; *Adv. Ser. Astrophys. Cosmol.* **3** (1987) 158.
8. J. D. Barrow and A. C. Ottewill, *J. Phys. A* **16** (1983) 2757.
9. S. K. Blau, E. I. Guendelman and A. H. Guth, *Phys. Rev. D* **35** (1987) 1747.
10. J. D. Barrow and A. Paliathanasis, *Phys. Rev. D* **94** (2016) 083518, arXiv:1609.01126 [gr-qc].
11. J. D. Barrow and A. Paliathanasis, *Gen. Rel. Grav.* **50** (2018) 82, arXiv: 1611.06680 [gr-qc].
12. K. A. Olive, *Phys. Rept.* **190** (1990) 307.
13. A. D. Linde, *Phys. Rev. D* **49** (1994) 748, arXiv:astro-ph/9307002 [astro-ph].
14. A. R. Liddle, P. Parsons, and J. D. Barrow, *Phys. Rev.* **1150** (1994) 7222, arXiv:astro-ph/9408015 [astro-ph].
15. C. German and A. Kehagas, *Phys. Rev. Lett.* **105** (2010) 011302, arXiv:1003.2635 [hep-ph]
16. T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Phys. Rev. Lett.* **105** (2010) 231302, arXiv:1008.0603 [hep-th].
17. C.-J. Feng, X.-Z. Li and E. N. Saridakis, *Phys. Rev. D* **82** (2010) 023526, arXiv:1004.1874 [astro-ph.CO]
18. C. Burrage, C. de Rham, D. Seery and A. J. Tolley, *J. Cosmol. Astropart. Phys.* **1101** (2011) 014, arXiv: 1009.2497 [hep-th].
19. T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Prog. Theor. Phys.* **126** (2011) 511, arXiv:l 105.5723 [hep-th].
20. J. Ohashi and S. Tsujikawa, *J. Cosmol. Astropart. Phys.* **1210** (2012) 035, arXiv:1207.4879 [gr-qc].

21. Y.-F. Cai, J.-O. Gong, S. Pi, E. N. Saridakis and S.-Y. Wu, *Nucl. Phys. B* **900** (2015) 517, arXiv:1412.7241 [hep-th].
22. V. Kamali, S. Basilakos and A. Mehrabi, *Eur. Phys. J. C* **76** (2016) 525, arXiv: 1604.05434 [gr-qc].
23. D. Benisty and E. I. Guendelman, *Int. J. Mod. Phys. A* **33** (2018) 1850119, arXiv:1710.10588 [gr-qc].
24. I. Dalianis, A. Kehagias and G. Tringas, *J. Cosmol. Astropart. Phys.* **1901** (2019) 037, arXiv:1805.09483 [astro-ph.CO].
25. I. Dalianis and G. Tringas, *Phys. Rev. D* **100** (2019) 083512, arXiv:1905.01741 [astro-ph.CO].
26. S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1.
27. L. Lombriser, *Phys. Lett. B* **797** (2019) 134804, arXiv:1901.08588 [gr-qc].
28. D. Merritt, *Stud. Hist. Phil. Sei. B* **57** (2017) 41, arXiv:1703.02389 [physics.hist-ph].
29. B. Ratra and P. J. E. Peebles, *Phys. Rev. D* **37** (1988) 3406.
30. R. R. Caldwell, R. Dave and P. J. Steinhardt, *Phys. Rev. Lett.* **80** (1998) 1582, arXiv:astro-ph/9708069 [astro-ph].
31. D. Benisty and E. I. Guendelman, *Phys. Rev. D* **98** (2018) 023506, arXiv:1802.07981 [gr-qc].
32. I. Zlatev, L.-M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* **82** (1999) 896, arXiv:astro-ph/9807002 [astro-ph].
33. R. R. Caldwell, *Phys. Lett. B* **545** (2002) 23, arXiv:astro-ph/9908168 [astro-ph].
34. T. Chiba, T. Okabe and M. Yamaguchi, *Phys. Rev. D* **62** (2000) 023511, arXiv:astro-ph/9912463 [astro-ph].
35. M. C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev. D* **66** (2002) 043507, arXiv:gr-qc/0202064 [gr-qc].
36. S. Tsujikawa, *Class. Quantum Grav.* **30** (2013) 214003, arXiv: 1304.1961 [gr-qc].
37. C. Wetterich, *Phys. Rev. D* **89** (2014) 024005, arXiv:1308.1019 [astro-ph.CO].
38. M. W. Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, *Phys. Rev. D* **90** (2014) 023512, arXiv: 1402.6661 [gr-qc].
39. E. Guendelman, E. Nissimov and S. Pacheva, *Bulg. J. Phys.* **44** (2017) 015, arXiv:1609.06915 [gr-qc].
40. E. Guendelman, E. Nissimov and S. Pacheva, Quintessence in Multi-Measure Generalized Gravity Stabilized by Gauss-Bonnet/Inflaton Coupling, *Proc. 2nd Bahamas Advanced Study Institute and Conf. 2017 (BASIC 2017)*, Stella Maris, Long Island, The Bahamas, March 12–18, 2017, *Bulg. J. Phys.* **45** (2018) 152, arXiv:1709.03786 [gr-qc].
41. E. Guendelman, E. Nissimov and S. Pacheva, Metric-Independent Volume-Forms in Gravity and Cosmology, *Proc. Matey Mateev Symp., in commemoration of 75th anniversary of Prof. Matey Mateev*, Sofia, Bulgaria, April 17, 2015, *Bulg. J. Phys.* **42** (2015) 249, arXiv: 1505.07680 [gr-qc].
42. M. W. Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, *Phys. Lett. B* **737** (2014) 191, arXiv:1405.7491 [gr-qc].
43. M. Wali Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, *Int. J. Mod. Phys. D* **24** (2015) 1530014, arXiv:1410.6100 [gr-qc].
44. C.-Q. Geng, M. W. Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, *Phys. Rev. D* **92** (2015) 023522, arXiv: 1502.03597 [gr-qc].
45. C.-Q. Geng, C.-C. Lee, M. Sami, E. N. Saridakis and A. A. Starobinsky, *J. Cosmol. Astropart. Phys.* 1706, (2017) 011, arXiv:1705.01329 [gr-qc].
46. A. B. Kaganovich, *Phys. Rev. D* **63** (2001) 025022 arXiv:hep-th/0007144 [hep-th].

47. M. W. Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, *Phys. Rev. D* **89** (2014) 123513, arXiv: 1404.1445 [gr-qc].
48. P. J. E. Peebles and A. Vilenkin, Quintessential inflation, *Phys. Rev. D* **59** (1999) 063505.
49. Y. Akrami, R. Kallosh, A. Linde *et al.*, *J Cosmol Astropart P*, **1806** (2018) 041, arXiv:1712.09693 [hep-th].
50. D. Benisty, E. I. Guendelman and E. N. Saridakis, *Eur. Phys. J. C* **80**(5) (2020) 480, arXiv:1909.01982 [gr-qc].
51. J. Martin, C. Ringeval and V. Vennin, *Phys. Dark Univ.* **5–6** (2014) 75, arXiv:1303.3787 [astro-ph.CO].
52. Planck Collab., N. Aghanim *et al.* Collab., arXiv: 1807.06209 [astro-ph.CO].
53. Planck Collab., Y. Akrami *et al.* Collab., arXiv:1807.06211 [astro-ph.CO].
54. A. Lewis, arXiv:1910.13970 [astro-ph.IM].
55. E. I. Guendelman, *Mod. Phys. Lett. A* **14** (1999) 1397, arXiv:hep-th/0106084 [hep-th].
56. E. I. Guendelman, *Mod. Phys. Lett. A* **14** (1999) 1043, arXiv:gr-qc/9901017 [gr-qc].
57. E. Guendelman, R. Herrera, P. Labrana, E. Nissimov and S. Pacheva, *Gen. Relativ. Gravit.* **47** (2015) 10, arXiv: 1408.5344 [gr-qc].