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Optimizing Laplacian Estimation for the Finite Dimensions Model of a Commercial Tripolar Concentric Ring Electrode and Comparing It to the Optimal Electrode Configuration via Finite Element Method Modeling †

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Abstract: Concentric ring electrodes are showing promise in noninvasive electrophysiological measurement but electrode design criteria are rarely detailed and justified. Toward that goal, the use of realistic finite dimensions model of concentric ring electrode in this study was two-fold. First, it was used to optimize the surface Laplacian estimate coefficients for tripolar electrode configuration with dimensions approximating the commercially available t-Lead electrodes manufactured by CRE-medical. Two differential signals representing differences between potentials on the middle ring and on the central disc as well as on the outer ring and on the central disc are combined linearly into the Laplacian estimate with aforementioned coefficients representing the weights of differential signals. Second, it was used to directly compare said tripolar configuration to the optimal tripolar concentric ring electrode configuration of the same size via finite element method modeling based computation of relative and normalized maximum errors of Laplacian estimation. Obtained results suggest the optimal coefficients for Laplacian estimate based on the approximation of the t-Lead dimensions to be (6, −1) as opposed to (16, −1) widely used with this electrode in the past. Moreover, compared to the optimal tripolar concentric ring electrode configuration, commercially available tripolar electrode of the same size leads to a median increase in Laplacian estimation errors of over 4 times. These results are consistent with previously obtained results based on both negligible and finite dimensions models but further investigation on real life phantom and human data via physical concentric ring electrode prototypes is needed for conclusive proof.

Keywords: electrophysiology; measurement; wearable sensors; noninvasive; concentric ring electrodes; Laplacian; estimation; optimization; finite element method; modeling

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1. Introduction

Finite element method (FEM) modeling has been previously used to compare concentric ring electrode (CRE) configurations [1–5]. However, it was based on the simplistic negligible dimensions model (NDM) of a CRE where a single point of negligible radius represents the central disc and circles of negligible width represent concentric rings. In [1,2] it is simply referred to as a nine-point method of surface Laplacian (second spatial derivative of the surface potential) estimation as opposed to tripolar (TCRE; two concentric rings) CRE configuration. In [3–5] comparison included CRE configurations with constant inter-ring distances (distances between the recording surfaces of a CRE) and higher numbers of concentric rings than in TCREs including quadripolar (three rings), pentapolar (four rings), sextopolar (five rings), and septapolar (six rings) CREs [3] as well as TCRE and quadripolar CRE configurations with different types of variable inter-ring distances including linearly increasing [4,5], linearly decreasing [4], and quadratically increasing [5] ones respectively.

Realistic finite dimensions model (FDM) of a TCRE that includes the radius of the central disc and individual widths of concentric rings has been proposed as a proof of concept in [6]. This proof of concept was later developed into a comparison framework validated on human electrocardiogram data [7] before ultimately being used to solve a comprehensive FDM based TCRE optimization problem maximizing the accuracy of Laplacian estimation [8]. Resulting optimal TCRE configuration was confirmed by FEM modeling adapted for the first time from NDM to FDM [8]. Furthermore, FEM results suggested that optimal TCRE configuration may also offer improved sensitivity and spatial resolution [8] compared to constant and linearly increasing inter-ring distances TCRE configurations of the same size from [7].

This study utilized FDM for two purposes. First one was to optimize the surface Laplacian estimate coefficients for TCRE with dimensions approximating the commercially available t-Lead electrodes (CREmedical, Kingston, RI, USA) widely used in studies such as [9,10]. Obtained results suggested the optimal Laplacian estimation coefficients based on the approximation of the t-Lead dimensions to be (6, −1) as opposed to (16, −1) widely used with TCREs of these dimensions from as early as in [1,2] till as recently as in [9,11]. Second, it was used to directly compare said tripolar configuration to the optimal TCRE configuration of the same size via FEM modeling based computation of relative and normalized maximum errors of Laplacian estimation. As a result, compared to the optimal tripolar concentric ring electrode configuration, two different approximations of the t-Lead (based on two versions of its dimensions patented in [12] and published in [11] respectively) of the same size led to a median increase in Laplacian estimation errors of over 4 times. The only similar comparison has been previously performed for the optimal TCRE configuration against commercially available CoDe® electrodes (Spes Medica, Genova, Italy) in [13]. However, CoDe® electrodes are bipolar CREs with a single ring so this study is the first comparison of the optimal TCRE configuration against commercially available TCREs. Another fundamental difference between this study and [13] is showing Laplacian estimation coefficients currently used with t-Lead electrodes to be suboptimal.

2. Materials and Methods

2.1. Tripolar Concentric Ring Electrode Configurations

Three TCRE configurations included in this study are presented in Figure 1. FDMs for two sets of dimensions corresponding to commercially available t-Lead electrodes (CREmedical, Kingston, RI, USA) were determined first. For the first set of dimensions patented in [12] (Table 1 for 1 cm external diameter) the radius of the central disc is equal to 1.4 mm, internal and external radii of the middle ring are equal to 2.6 mm and 3.2 mm respectively, and internal and external radii of the outer ring are equal to 4.4 mm and 5 mm respectively. For the second set of dimensions published in [11] all the dimensions are identical except for the inner radius of the middle ring equal to 2.4 mm and the inner radius of the outer ring equal to 4.1 mm. Scaling these dimensions to the size of the optimal TCRE configuration from [8] with the outer radius of the outer ring subdivided into 9 equal intervals (Figure 1C; for 1 cm external diameter these dimensions are equivalent to the radius of the central disc equal to 0.56 mm, internal and external radii of the middle ring equal to 1.11 mm and 1.67 mm respectively, and internal and external radii of the outer ring equal to 2.22 mm and 5 mm respectively) results in FDMs from Figure 1A and Figure 1B respectively. Specifically, for the first set of dimensions from [12] scaling to the size of the optimal TCRE configuration from Figure 1C results in the central disc radius equal to 2.52, the inner radius of the middle ring equal to 4.68, the outer radius of the middle ring equal to 5.76, and the inner radius of the outer ring equal to 7.92. Rounded to the nearest integer those correspond to the TCRE from Figure 1A. For the second set of dimensions from [11] scaling to the size of the optimal TCRE configuration from Figure 1C results in the inner radius of the middle ring equal to 4.32 and the inner radius of the outer ring equal to 7.38. Rounded to the nearest integer those correspond to the TCRE from Figure 1B.

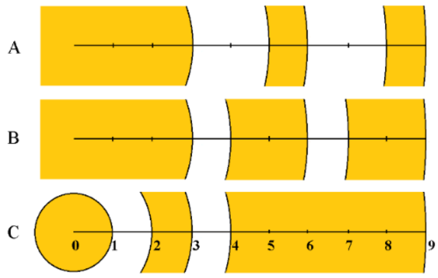

Figure 1. Finite dimensions models of three tripolar concentric ring electrodes including two approximations of t-Lead electrode (patented one in (**A**) and published one in (**B**)) and optimal (**C**) configuration with respect to the accuracy of Laplacian estimation.

To obtain Laplacian estimates for TCRE configurations from Figure 1A,B FDM based analytic approach from [7] was used. First, potentials were calculated for all nine concentric circles as means of potentials at four points on each circle. Next, circle potentials were used to calculate the potentials on the recording surfaces of each TCRE configuration. For example, the potential on the central disc for TCRE configurations from Figure 1A,B is equal to the mean of the potential at the center of the central disc and potentials on the three smallest concentric circles. Finally, two differential signals representing differences between potentials on the middle ring and on the central disc as well as on the outer ring and on the central disc were combined linearly with aforementioned coefficients representing the weights of differential signals and divided by the square of the distance between the concentric circles to produce the Laplacian estimate [7]. Laplacian estimate coefficients (952/1227, −6/409) for the optimal TCRE configuration from Figure 1C were adopted from [8].

2.2. Finite element method modeling

NDM based FEM model from [1–5] was adapted to FDM in [8]. This adaptation was used in this study with the same parameters including an evenly spaced (0.278 mm) square mesh of 700 × 700 points corresponding to roughly 20 × 20 cm located in the first quadrant of the *X*-*Y* plane over a unit charge dipole projected to the center of the mesh and oriented towards the positive direction of the *Z* axis. The medium was assumed to be homogeneous with a conductivity equal to 7.14 mS/cm to emulate biological tissue [14]. Electric potential was generated and analytical Laplacian Δv calculated at each point of the mesh by taking the second spatial derivative of the electric potential for the dipole depth equal to 5 cm [15]. Three TCRE Laplacian estimates were computed at each point of the mesh where appropriate boundary conditions could be applied and compared to Δv using the following error measures adopted from [8]:

$$\text{Relative error}^i = \sqrt{\frac{\sum(\Delta v - \Delta^i v)^2}{\sum(\Delta v)^2}} \quad (1)$$

$$\text{Normalized maximum error}^i = \frac{\max|\Delta v - \Delta^i v|}{\max|\Delta v|} \quad (2)$$

where i represents TCRE configuration, $\Delta^i v$ represents the corresponding Laplacian estimate, and Δv represents the analytical Laplacian at each point of the mesh. Relative error was adopted verbatim from [1–5,8] while normalized maximum error was modified in [8] to make visualization of the improvement in Laplacian estimation accuracy easier by representing the error as a percentage of the maximum absolute value of the analytical Laplacian.

3. Results

3.1. Tripolar Concentric Ring Electrode Configurations

Laplacian estimate coefficients for two TCRE configurations from Figure 1A,B were determined using the FDM based analytic approach from [7]. This approach allows cancelling out the 4th order truncation term of Taylor series expansion which has been shown to be the highest truncation term order (equal to twice the number of concentric rings) that can be cancelled out for a TCRE using both NDM [3] and FDM [6]. The resulting coefficients were equal to (17/63, −1/21) for TCRE from Figure 1A and to (51938/159159, −1202/22737) for TCRE from Figure 1B.

3.2. Figures, Tables and Schemes

Relative and normalized maximum errors computed via the FEM modeling using (1) and (2) are presented in Figures 2 and 3 for CRE diameters ranging from 0.5 cm to 5 cm using linear and semi-log scales respectively. Compared to the optimal TCRE configuration from Figure 1C, TCRE of the same size from Figure 1A corresponds to a median increase in Laplacian estimation error (ratios of respective errors obtained for 10 CRE sizes) of 4.94 (relative error) and 4.9 (normalized maximum error) times while its counterpart from Figure 1B corresponds to an increase of 4.18 (relative error) and 4.16 (normalized maximum error) times.

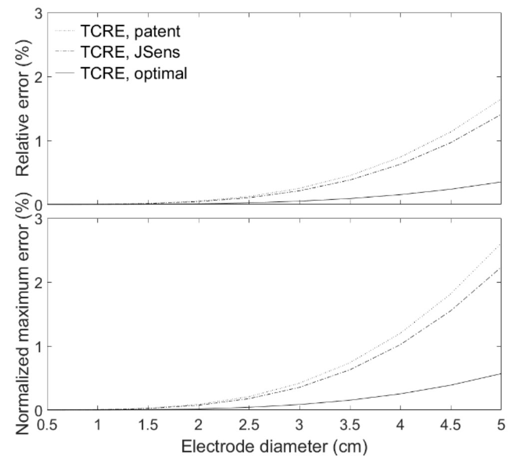

Figure 2. Relative (**top**) and normalized maximum (**bottom**) errors of surface Laplacian estimation via three tripolar concentric ring electrode configurations presented on a linear scale.

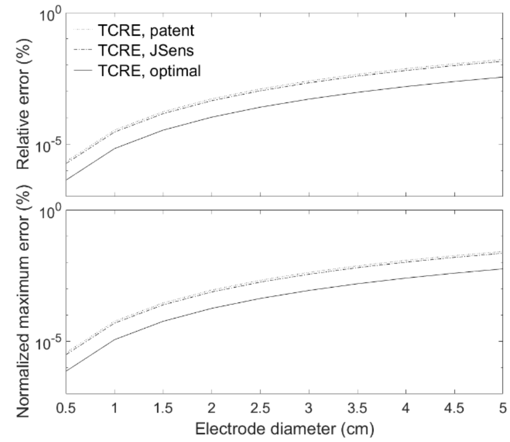

Figure 3. Relative (**top**) and normalized maximum (**bottom**) errors of surface Laplacian estimation via three tripolar concentric ring electrode configurations presented on a semi-log scale.

4. Discussion

This study is the second attempt to directly compare optimal TCRE configuration from Figure 1C to its commercially available counterparts in terms of the accuracy of the surface Laplacian estimation using FDM based FEM modeling and the first one to draw a comparison to a TCRE since in [13] optimal TCRE configuration was compared to bipolar CREs only. Such comparison is important because ability to estimate the surface Laplacian at each electrode constitutes the primary biomedical significance of CREs. Therefore, quantifying the difference between optimal and commercially available configurations could provide an insight to incorporate into the design of future CREs for real-life applications not limited to the ones that already rely on commercially available TCREs such as [9,10].

Obtained optimal Laplacian estimate coefficients for TCREs from Figure 1A,B with the second coefficient scaled to −1 and the first one rounded to the nearest integer are equal to (6, −1) as opposed to (16, −1) widely used with TCREs of these dimensions from as early as in [1,2] till as recently as in [9,11]. Suboptimality of the currently used coefficients stems from NDM approach having been used to determine them in [1,2] based on the assumption of the outer ring radius being twice the middle ring radius that is inconsistent with real life t-Lead dimensions. Potential benefits of replacing the suboptimal coefficients with optimal ones in Laplacian estimation via t-Lead merit further investigation.

Due to the external diameter of the outer ring for TCREs from both [11,12] being equal to 1 cm with the only other two TCRE sizes listed in the same Table 1 in [12] having external diameters of 0.6 cm and 1.6 cm, the three most relevant TCRE sizes out of the 10 sizes total included in this study are TCRE diameters of 0.5 cm, 1 cm and 1.5 cm. As can be seen from Figure 2 for these three sizes the difference in errors is not as substantial for practical real life applications as it is, for example, for TCRE size of 5 cm where TCRE configurations from Figure 1A,B correspond to the Laplacian estimation errors of 1.65% and 1.41% respectively (relative errors) as well as 2.61% and 2.24% respectively (normalized maximum errors) while optimal TCRE from Figure 1C allows decreasing these errors to 0.35% (relative error) and 0.57% (normalized maximum error). However, as can be seen from Figure 3 this difference in Laplacian estimation errors between the TCRE configurations from Figure 1A,B and the optimal TCRE from Figure 1C increases with the decrease in the electrode diameter. This is important since, for example, the difference in Laplacian estimation error between the TCRE from Figure 1A and optimal TCRE configuration from Figure 1C for the three smallest TCRE sizes included in this study increases to over 5 times for both relative and normalized maximum error.

Future work directions include adding measures quantifying sensitivity and spatial resolution as in [8]. Moreover, while these results are consistent with the previously obtained NDM and FDM based ones further investigation on real life phantom and human data via physical TCRE prototypes is needed for conclusive proof.

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