



# Proceeding Paper Distribution of Forces in RC Interior Beam-Column Connections <sup>+</sup>

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**Abstract:** The beam-column connection is a fundamental element in frame structures and requires special attention in the calculation of the forces passing through it and the corresponding modeling. For the study of the moment-resisting frame in the leading countries in seismic research, uniform procedures have introduced. However, in their seismic codes, there is still a discrepancy in how the shear force is determined in the beam-column connection. In the present paper, a new mathematical model is proposed for the analytical determination of the forces passing through the joint in the beam-beam and column-column direction. The occurring large deformations in the beam and the column, which be realized during an earthquake, are take into account. The material is elastic. The obtained values are compared with results determined by mathematical procedures proposed in other literature sources. The results show values of the shear force determined by the new model about 20% greater than those available in the literature.

**Keywords:** beam-column connection; shear force; reinforced concrete; Elastic material; Larges deformations

# 1. Introduction

The beam-to-column connection is an important element in frame structures for transferring loads between connected elements. Modern codes give uniform recommendations regarding the static load calculation of moment resisting reinforced concrete frames. However, sudden failures occur in many frame structures under cyclic loading (such as earthquakes) due to joint shear.

In [1], the authors give the first quantitative definition of the shear force, as the horizontal force transferred at the mid-height of a horizontal section of a beam-column connection. They propose limiting the shear stress to the level at which shear failure occurs at the joint. This definition has been adopted worldwide and subsequent studies [2–7] lead to the creation of design provisions providing a limit value of joint shear stress.

The study in [8] shows an irrationality in the joint shear model adopted in the most current design codes. The conclusion is based on the test data of twenty reinforced concrete internal beam-to-column-joint failed in joint shear. The analysis showed that the joint shear stress increased in most specimens, even after apparent joint shear failure starts.

#### 2. Materials

Hanson and Connor [1] defined the joint shear  $V_{jh}$  in an interior beam-column connection from Figure 1 as given in (1).

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**Figure 1.** Definition of joint shear  $V_i$  in interior RC beam-column connection.

$$V_{ih} = T + C'_{s} + C'_{c} - V_{c} = T + T' - V_{c}, \qquad (1)$$

where  $C_s$  and  $C_c$  are compressive force in longitudinal reinforcing bars in beam passing through the connection; and in concrete; T —tensile forces in longitudinal reinforcing bars in beam;  $V_c$  —column shear force.

The difficulty encountered in determining the forces from (1) leads to the adoption:

$$V_{jh} = \frac{M_{b}}{j_{b}} + \frac{M'_{b}}{j'_{b}} - V_{c} , \qquad (2)$$

where  $M_{b}$  and  $M'_{b}$  are moment at column face;  $j_{b}$  and  $j'_{b}$ —the length of bending moment arm at the column face. It is assumed to be constant and unchanging in the process of deformation.

With assumption of uniform distribution of the stresses along each face of the joint [7], the shear stresses in horizontal and vertical directions of the joint should equal each other и с достатьчна точност можем да запишем:

$$V_{j\nu} = \frac{h_{\rm b}}{h_{\rm c}} V_{jh} \,, \tag{3}$$

where:  $h_{b}$  and  $h_{c}$  – височината на гредата и колоната, съответно.

In this article, the following tasks are set: 1. to compare the force values from Figure 1, at the face of the column and those from (2). 2. to compare the force values from Figure 1, at bottom of the column and those from (3).

The solution is made by taking into account the actual dimensions of the beams and columns in the mathematical model.

#### 3. Method

#### 3.1. Simple Beam-Support Reactions

In [9] beam from a frame structure is considered. The beam is statically indeterminate, prismatic and symmetric. The beam is under the conditions of special bending with tension/compression and Bernoulli-Euler hypothesis is considered (Figure 2). The solutions give the formulas of the horizontal support reactions below:



Figure 2. Supports of simple beam to columns.

$$H_{1} = \frac{-qL^{3}k_{1}\{2EAh_{1} + Lk_{2}n_{1} - Lk_{3}n_{2}\}}{6\{2EA[8EI + LD_{1}] + 8EILK + LD_{2}\}}; H_{2} = \frac{qL^{3}k_{2}\{4EAa + Lk_{1}n_{1} + Lk_{3}4a\}}{6\{2EA[8EI + LD_{1}] + 8EILK + LD_{2}\}};$$

$$H_{3} = \frac{qL^{3}k_{3}\{4EAa + Lk_{1}n_{2} + Lk_{2}4a\}}{6\{2EA[8EI + LD_{1}] + 8EILK + LD_{2}\}}$$

$$(4)$$

where  $k_1 = \frac{E_1 A_1}{L}$ ;  $k_2 = \frac{E_2 A_2}{L}$ ;  $k_3 = \frac{E_3 A_3}{L}$  — The coefficients of the linear springs;

 $E_1, E_2, E_3 [kN / cm^2]$  — the modulus of elasticity of the concrete, of the reinforcement  $A_1, A_2, A_3 [cm^2]$  — the area of the cross-section of the concrete; and of reinforcement;

$$h_{1} = 2b + h; \qquad n_{1} = 2a + h_{1}; \qquad n_{2} = 2a - h_{1}; K = k_{1} + k_{2} + k_{3}; \qquad K_{12} = k_{1}k_{2}; \qquad K_{13} = k_{1}k_{3}; K_{23} = k_{2}k_{3}; D_{1} = (k_{2} + k_{3})4a^{2} + k_{1}h_{1}^{2}; \qquad D_{2} = L \Big[ k_{1}k_{2}(2a + h_{1})^{2} + k_{1}k_{3}(2a - h_{1})^{2} + k_{2}k_{3}16a^{2} \Big],$$
(5)

The solution was performed in the symbolic environment of the MATLAB R2017b program [11].

#### 3.2. Cantilever Beam-Support Reactions

We will consider the columns coming out of the joint as a cantilever beam (Figure 3). All geometric and material characteristics introduced up to this point are preserved [10].



Figure 3. Supports of cantilever beam (column) to a joint.

$$H_{4} = \frac{-FLk_{1} \left\{ EAh_{1}N_{1} + 2EIL\left[k_{2}n_{1} - k_{3}n_{2}\right] + 2L^{2}a^{2}K_{23}n_{2} \right\}}{EI \left\{ EAD_{1} + D_{2} \right\}};$$

$$H_{5} = \frac{FLk_{2} \left\{ EAaN_{2} + 4EIL\left[k_{1}n_{1} + k_{3}4a\right] + L^{2}aK_{13}n_{2}^{2} \right\}}{2EI \left\{ EAD_{1} + D_{2} \right\}};$$

$$H_{6} = \frac{FLk_{3} \left\{ 4EAaN_{1} + 4EIL\left[k_{1}n_{2} + k_{2}4a\right] - L^{2}aK_{12}n_{1}n_{2} \right\}}{2EI \left\{ EAD_{1} + D_{2} \right\}}$$
(6)

where the notations from (5) and also are used:

$$N_{1} = 2EI - k_{2}La^{2}; \qquad N_{2} = 8EI + L(k_{1}h_{1}^{2} + k_{3}4a^{2}),$$
(7)

## 4. Results and Discussion

For the numerical results, a beam and column with a cross-section of  $25/25 \ cm$  was introduced. The transverse load is q = 0,05kN/cm' and F = 25kN. And more e = 3[cm] and a = 9,5[cm],  $A_2 = A_3 = 12,5[cm^2]$  and  $E_2 = E_3 = 21000[kN/cm^2]$ . The distance b[cm] varies in the interval [12,5;0) and is monitored by the ratio h/b. The length of the simple beam is L = 1000[cm] and of the column is  $L = 300 \ cm$ . Two examples with a difference in the modulus of elasticity of concrete are considered. The modules used are  $E_1 = 1700[kN/cm^2]$  for normal concrete and  $E_1 = 3700[kN/cm^2]$  for High-strength concrete.

A new model to Determination of the Shear Force in RC Interior Beam-Column Connections is proposed (Figure 4a).



**Figure 4.** New definition of joint shear in interior RC beam-column connection. (**a**) distribution of forces; (**b**) moments of the forces in the concrete core of the joint.

$$V_{ih} = H_3 + H_2' + H_1' - V_c,$$
(8)

If the frame is symmetric and other conditions being equal, we will have the equality of  $H_1 = H_1'$ ,  $H_2 = H_2'$  and  $H_3 = H_3'$ . Then (8) becomes

$$V_{jh} = H_3 + H_2 + H_1 - V_c , (9)$$

## 4.1. Comparison of the Results of (2) and (9) (Figure 5)

The difference determined for the extreme values of (9) with these in (2) is respectively:



**Figure 5.** Comparison of the results of (2) and (9)-simple beam. (a) the modulus of elasticity of the concrete is  $E_1 = 1700 \left[ \frac{kN}{cm^2} \right]$ ; (b)  $E_1 = 3700 \left[ \frac{kN}{cm^2} \right]$ .

*Figure* 5a-5,98% at h/b=7,1, the certainty is in the direction of (9) *Figure* 5b-5,98% at h/b=10, the certainty is in the direction of (2)

## 4.2. Comparison of the Results of (3) and Figure 4

From Figure 4 for the vertical force in the joint we write:

$$V_{jv} = H_4 + H_5 + H_6' + N_c, \qquad (10)$$

From (3) and (2) it follows

$$V_{jv} = \frac{h_b}{h_c} \left( \frac{M_b}{j_b} + \frac{M'_b}{j'_b} - V_C \right),$$
 (11)

Then what is proposed by the codes (11) [7] should be equal to what is obtained in (10), namely:

$$H_{4} + H_{5} + H_{6}' + N_{c} = \frac{h_{b}}{h_{c}} \left( \frac{M_{b}}{j_{b}} + \frac{M_{b}'}{j_{b}'} - V_{c} \right),$$
(12)

If  $\frac{h_b}{h_c} = 1$ , for the condition (12) to be fulfilled, the inequality must be fulfilled:

$$H_{4} + H_{5} + H_{6}' < \frac{M_{b}}{j_{b}} + \frac{M_{b}'}{j_{b}'}, \qquad (13)$$

From Figure 6, it is obvious that the direction of the inequality is not satisfied. Therefore, taking into account the real dimensions of the elements leads to forces occurring in the joint exceeding those recommended in the codes.



**Figure 6.** Comparison of the parameters of (13). (a)  $E_1 = 1700 \left[ \frac{kN}{cm^2} \right]$ ; (b)  $E_1 = 3700 \left[ \frac{kN}{cm^2} \right]$ .

*Figure 6a* -275% at h/b = 3.8*Figure 6b* -290% at h/b = 3.3

There is a serious underestimation of the contribution of the beam and of the column forces to the value of the joint shear force, from the expressions known in the literature. The new model shows, that the contribution of the beam and the column forces is greater.

The distribution and direction of forces at the joint suggest that torsion (Figure 4b) is present. It would require additional reinforcement of the joint. This is a recommendation that we also find in [8] where it is proposed to increase the reinforcement in the joint without it passing to the beam. In [12], a model is shown in which a stainless steel wire mesh is wrapped around the concrete core of the joint. The results show an almost 100% increase in deformation with the required amount of energy dissipation and maximum strength capacity.

Therefore, it can be expected that strengthening only the core of the joint with stirrup would lead to an increase in its capacity without impairing the moment response of the frame.

#### 5. Conclusions

Considering the real dimensions and material properties of the elements in a frame structure shows a redistribution of forces in the beams, columns and joints. The derived expressions for the reactions of the horizontal supports give results that clearly show the distribution of forces along the height of the beam in the corresponding support. The obtained results for large deformations of elements show that forces values exceeded those recommended in the design codes.

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