engineering proceedings

MDPI

*Article*

# Designing Unknown Input Observers for Fault Reconstruction in Disturbed Takagi-Sugeno Fuzzy Systems

**Khalida Mimoune [1]*, Mohamed Yacine Hammoudi [1] and Wail Hamdi [1]**

[1] Laboratory of Energy Systems Modeling (LMSE), Department of Electrical Engineering, University of Biskra, BP 145, Biskra 07000, Algeria

* Correspondence: khalida.mimoune@univ-biskra.dz

**Abstract:** Fault occurrence in practical systems, if not addressed, can cause diminished performance or even system breakdown. Therefore, fault detection has emerged as a crucial challenge in ensuring system safety and reliability. This paper presents a novel fuzzy observer aimed at reconstructing actuator and sensor faults in nonlinear systems, even when subjected to external disturbances. The approach we propose utilizes the Takagi-Sugeno fuzzy model and Lyapunov function. Initially, by filtering the system output, we construct a system where actuator faults correspond to the original actuator and sensor faults. Subsequently, the impact of disturbance on state estimations is minimized by employing the H-infinity performance criteria. We demonstrate that, for non-disturbed systems, these estimations gradually converge to their true values. In designing the observer gains, transformation matrices are derived by solving linear matrix inequalities. Our approach boasts some advantages over existing methods. By assuming that the premise variables are immeasurable, we enhance the usability of our approach. As a proof of concept, we evaluate two practical systems. The simulation results underline the benefits of our proposed method in terms of rapid and accurate fault detection performance.

**Keywords:** Takagi-Sugeno fuzzy system; actuator fault; sensor fault; Lyapunov function; linear matrix ine-qualities; H∞ performance.

## 1. Introduction

Takagi-Sugeno Fuzzy (TSF) systems, particularly effective in engineering for observer and fault detection, use observers like the Proportional Integral Observer (PIO) and its enhanced version, the Proportional Multi-Integral Observer (PMIO) [1], for complex nonlinear systems such as the Continuous Stirred Tank Reactor (CSTR) in chemical engineering. The CSTR, a multi-input multi-output (MIMO) system, presents control and estimation challenges due to its nonlinear dynamics [2]. Luenberger's observers, initially for linear systems, have been adapted for nonlinear systems, offering cost-effective state estimation solutions [3].

The TS multi-model approach simplifies state estimation in CSTRs by interpolating between linear models for different behaviors [4]. This is crucial in scenarios with simultaneous unknown input actuator and sensor faults, necessitating integrated control and diagnostic systems. For TS models, several state and unknown input estimation methods have been developed. These include PI observers for decoupled Multiple Models [5], UI-PI observers with measurable premise variables [6], [7], and a TS multi-model based PI observer for simultaneous state and input estimation [9]. However, PI observers have limitations with time-varying inputs, leading to the development of PMI observers, like the Thau-Luenberger observer [8], capable of assessing all unknown input derivatives. This research aims to refine PMI-based unknown input observers for TS-model systems, focusing on convergence conditions as linear matrix inequalities.

The paper's structure is as follows: Section 2 introduces the TS fuzzy model and constructs a fictitious system with a fault, with the design of a PMI observer, ensuring

check for updates

**Citation:** Lastname, F.; Lastname, F.; Lastname, F. Title. *Eng. Proc.* **2023**, *1*, 0. https://doi.org/

Published: 15 November 2023

**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

estimation error stability and $H\infty$ performance. In Section 3, we validate the approach with simulation results. Conclusions are drawn in Section 4.

## 2. Problem Statement

Consider the Takagi-Sugeno model described by the system of equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{Q} \mu_i(x(t))(A_i x(t) + B_i u(t) + E_i f_a(t)) + Mw(t) \\ y(t) = Cx(t) + Ff_a + Hw(t) \end{cases} \tag{1}$$

Here, $x(t) \in \mathbb{R}^n$ represents the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector, $f_a(t) \in \mathbb{R}^{n_{f_a}}$ signifies the unknown input vector including actuator and sensors faults, $w(t)$ is the disturbance, and $y(t) \in \mathbb{R}^{n_y}$ corresponds to the output vector.

Consider the matrices $A_i$, $B_i$, $E_i$, $M$, $C$, $F$, and $H$, which are constants with appropriate dimensions. The activation functions $\mu_i(x(t))$, dependent on the system's state, adhere to the following convexity properties [10][11]:

$$\begin{cases} \sum_{i=1}^{Q} \mu_i(x(t)) = 1 \\ \quad \forall i \in \{1, \dots, Q\} \end{cases} \tag{2}$$

The scalar $Q$ designates the number of local models.

**Hypothesis 1**: The unknown input $f_a(t)$ satisfies:

$$f_a^{(q)}(t) = 0 \tag{3}$$

Generally, the sequences ${f_a}^{(1)}(t)$, ${f_a}^{(2)}(t)$, $\dots$, ${f_a}^{(q-1)}(t)$ denote the continuous derivatives of $f_a(t)$, expressed as:

$$\begin{bmatrix} \dot{f}_a(\mathrm{t}) \\ \dot{f}_{a1}(t) \\ \vdots \\ \dot{f}_{a_{q-1}}(t) \end{bmatrix} = \begin{bmatrix} f_{a1}(t) \\ f_{a2}(t) \\ \vdots \\ f_{a_q}(t) \end{bmatrix} \tag{4}$$

The system (1) can be expressed as a perturbed system with weighting functions $\mu_i$ based on the estimated state, where the signals $u(t)$, $f_a(t)$, and $w(t)$ are bounded.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{Q} \mu_i(\hat{x})(A_i x + B_i u + E_i f_a + Mw(t) + \delta(t)) \\ y(t) = Cx(t) + Ff_a + Hw(t) \end{cases} \tag{5}$$

where:

$$\delta(t) = \sum_{i=1}^{Q} (\mu_i(x) - \mu_i(\hat{x}))(A_i x + B_i u + E_i f_a + Mw(t))$$

Then, system (1) can be articulated as:

$$\begin{cases} \dot{x}_a(t) = \sum_{i=1}^{Q} \mu_i(\hat{x}(t))(\bar{A}_i x_a(t) + \bar{B}_i u(t) + \bar{\sigma}_i \bar{\Omega}(t)) \\ y(t) = \bar{C} x_a(t) + \bar{F} \bar{\Omega}(t) \end{cases} \tag{6}$$

$$x_a = \begin{bmatrix} x \\ f_a \\ f_{a1} \\ \vdots \\ f_{a_{q-1}} \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & E_i & 0 & \dots & 0 & 0 \\ 0 & 0 & I_{n_{f_a}} & \dots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & I_{n_{f_a}} \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \bar{\sigma}_i = \begin{bmatrix} \sigma_i^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{7}$$

$$\bar{\Omega}(t) = \begin{bmatrix} \sigma(t) \\ w(t) \end{bmatrix}, and\ \bar{C} = \begin{bmatrix} C & F & 0 & \dots & 0 & 0 \end{bmatrix} \tag{8}$$

where: $\sigma_i = \begin{bmatrix} I_n & M \end{bmatrix}, and \bar{F} = \begin{bmatrix} 0 & H \end{bmatrix}$,

**Hypothesis 2**: The pairs $\left(\overline{A_i}, \bar{C}\right)$ are observable for all $i$.

The Proportional Multiple Integral (PMI) observer can be described by:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{Q} \mu_i(\hat{x}(t))\Big(A_i\hat{x}(t) + B_i u(t) + E_i\hat{f}_a(t) + L_{Pi}(y(t) - \hat{y}(t))\Big) \\ \dot{\hat{d}}_j(t) = \sum_{i=1}^{Q} \mu_i(\hat{x}(t))\Big(\hat{f}_{a_{j+1}} + L_{Ii}^{j}(y(t) - \hat{y}(t))\Big), \quad j = 1, \dots, q-1 \\ \dot{\hat{d}}(t) = \sum_{i=1}^{Q} \mu_i(\hat{x}(t))\Big(\hat{f}_{a1} + L_{Ii}(y(t) - \hat{y}(t))\Big) \\ \hat{y}(t) = C\hat{x}(t) + F\hat{f}_a \end{cases} \tag{9}$$

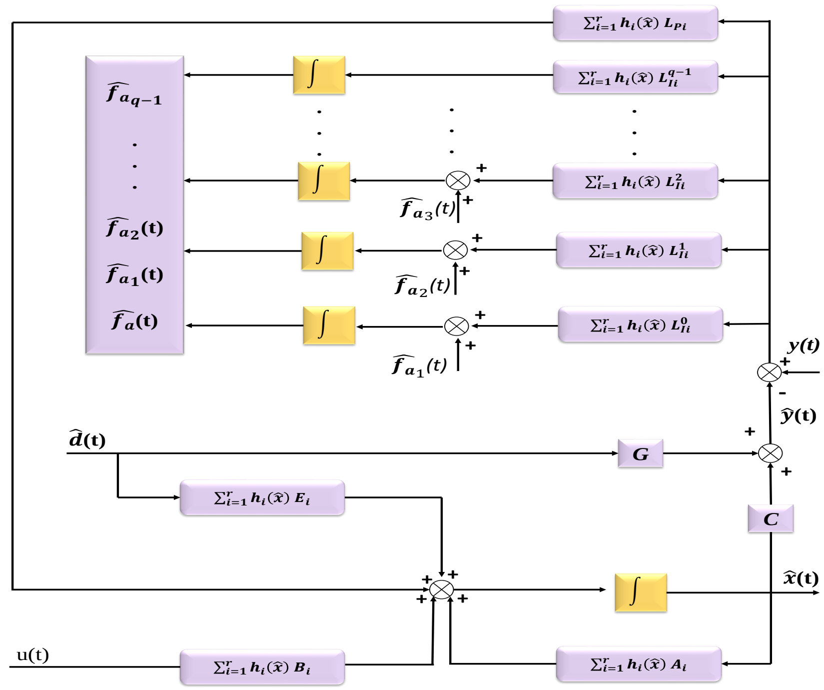


**Figure 1.** Principle of the PMI Observer.

With the above, the augmented PMI observer that incorporates an output error into its activation functions is articulated.

$$\begin{cases} \hat{x}_a(t) = \sum_{i=1}^{Q} \mu_i(\hat{x}(t))(\bar{A}_i\hat{x}_a(t) + \bar{B}_i u(t) + \bar{L}_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = \bar{C}\hat{x}_a(t) \end{cases} \tag{10}$$

Where $\quad \bar{L}_i = \begin{bmatrix} L_{Pi}^T & L_{Ii}^T & L_{Ii}^{1^T} & \dots & L_{Ii}^{q-2^T} & L_{Ii}^{q-1^T} \end{bmatrix}^T$,

The dynamic of the state estimation error is presented in the following augmented format:

$$\dot{\bar{e}} = \sum_{i=1}^{Q} \mu_i(\hat{x})((\bar{A}_i - \bar{L}_i\bar{C})\bar{e} + (\bar{\sigma}_i - \bar{L}_i\bar{F})\bar{\Omega}) \tag{11}$$

**Theorem 1.** *The PMI Observer, as defined by (9), designed to Simultaneously estimate the state and the unknown inputs of the fuzzy system represented in (1). This is achieved while minimizing the $\mathcal{L}_2$-gain $\bar{\gamma}$ from the unknown inputs to the augmented state estimation error $\bar{e}$. This can be* *obtained by determining a positive definite matrix $P$, matrices $M_i$, and a positive scalar $\bar{\gamma}$ that satisfy the following LMI constraints for $i = 1, \ldots, r$:*

$$\begin{bmatrix} \bar{A}_i^T P + P\bar{A}_i - M_i\bar{C} - \bar{C}^T M_i^T + I & P\bar{\sigma}_i - M_i\bar{F} \\ \bar{\sigma}_i^T P - \bar{F}^T M_i^T & -\bar{\gamma} I \end{bmatrix} < 0 \tag{12}$$

*where $\gamma = \sqrt{\bar{\gamma}}$.*

*The observer gains: $\bar{L}_i = P^{-1} M_i$*

**Proof.** Where System (1) stands as a paragon of stability and all preceding signals remain bounded, a transformative revelation unfolds. By invoking Lemma of Perturbation attenuation and satisfying the condition $\|\bar{e}(t)\|_2 < \gamma\|\tilde{\omega}(t)\|_2$ yields an enigmatic Linear Matrix Inequality (LMI):

$$\begin{bmatrix} \bar{A}_i^T P + P\bar{A}_i - P\bar{L}_i\bar{C} - \bar{C}^T \bar{L}_i^T P + I & P\bar{\sigma}_i - P\bar{L}_i\bar{F} \\ \bar{\sigma}_i^T P - \bar{F}^T \bar{L}_i^T P & -\gamma^2 I \end{bmatrix} < 0 \tag{13}$$

The essence of Theorem 1 obtain through variable transformations:

$$M_i = P\bar{K}_i, \quad \bar{\gamma} = \gamma^2$$

□

## 3. Practical Example

As a demonstration of the methodology, we examine a nonlinear CSTR system depicted by a multi-model with unmeasurable premise variables. This multi-model comprises two local models, each with three states [3].

Consider a thoroughly mixed CSTR where the multi-component chemical reaction $A \rightleftharpoons B \rightarrow C$ takes place. The Schematic of the Continuous Stirred Tank Reactor (CSTR) is shown in Figure 2, illustrating the concentrations of species $A, B$, and $C$ as $C_A, C_B$, and $C_C$ respectively. The reactor temperature is denoted by $T$, with $C_{Af}$ and $T_f$ representing the feed's actual concentration and temperature. The flow rate and temperature of the cooling water are indicated by $q_c$ and $T_c$. The system's nonlinear dynamics can be represented as:

$$\dot{x} = \begin{bmatrix} -4 & 0.8796 & 0 \\ 3 & -3.6388 & 0 \\ 0 & 1.7592 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0.5x_2^2 \\ -1.5x_2^2 \\ x_2^2 \end{bmatrix}, \tag{14}$$

where $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ denotes the concentrations of species $A, B$, and $C$ respectively.

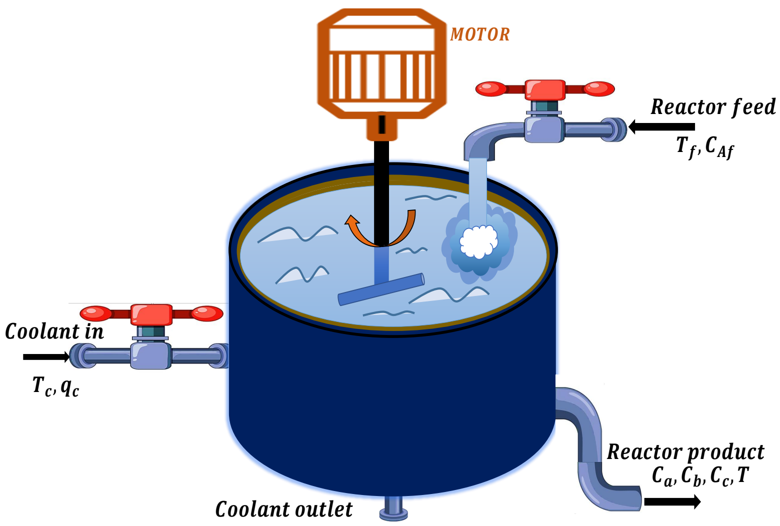


**Figure 2.** Schematic of CSTR.

To evaluate the effectiveness of our proposed approach, we introduce a fault and disturbance to the dynamics, leading to:

$$\begin{cases} \dot{x} = \begin{bmatrix} -4 & 0.8796 + 0.5x_2 & 0 \\ 3 & -3.6388 - 1.5x_2 & 0 \\ 0 & 1.7592 + x_2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} f_a + \begin{bmatrix} 0.6 \\ 0.05 \\ 0.03 \end{bmatrix} w(t) \\ y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix} f_a + \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix} w(t) \end{cases} \tag{15}$$

Assuming the concentration of species $B$ is dimensionless, it is given by $x_2 \in [-1, 1]$. This allows the definition of two membership functions using Takagi-Sugeno (TS) rules as:

$$h_1 = \frac{1 - x_2}{2} \quad \text{and} \quad h_2 = \frac{1 + x_2}{2}. \tag{16}$$

With this, the corresponding local linear TS matrices can be established as:

$$A_1 = \begin{bmatrix} -4 & 0.8796 - 0.5 & 0 \\ 3 & -3.6388 + 1.5 & 0 \\ 0 & 1.7592 - 1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} -4 & 0.8796 + 0.5 & 0 \\ 3 & -3.6388 - 1.5 & 0 \\ 0 & 1.7592 + 1 & -1 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, E_1 = E_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, F = \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix} \tag{17}$$

Given that these TS fuzzy system matrices comply with all the preconditions, the TS fuzzy observer( 9) is constructible.

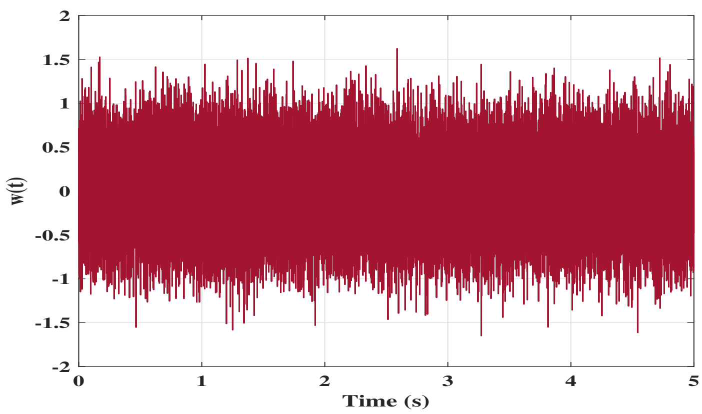


**Figure 3.** Disturbance w(t).

For the simulation setup, parameters and input signals were chosen as follows: $u = \sin(t)$, with initial conditions $x_0 = \begin{bmatrix} 0.15 & 0.2 & 0.1 \end{bmatrix}^T$ and $\hat{x}_0 = \begin{bmatrix} 0.15 & 1 & 3 \end{bmatrix}^T$. The disturbance profile depicted in Figure 3.

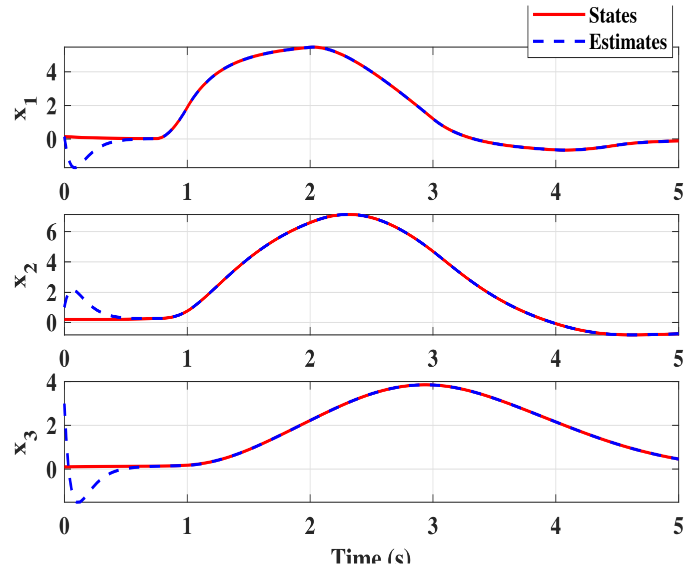


**Figure 4.** States and their estimates.

The Linear Matrix Inequality (LMI) outlined in the provided theorem is addressed using the Matlab Yalmip toolbox, leading to the determination of the observer's gains. The results of state and fault estimation are visually depicted in Figure 4 and Figure 5.

In Figure 4, it is evident that the estimated states converge rapidly to their true signals right from the beginning. Figure 5 displays a robust estimation of the fault. These simulation outcomes distinctly demonstrate that the proposed observer not only guarantees accurate state estimation but also effectively handles concurrent fault detection, even in the presence of unknown disturbances.

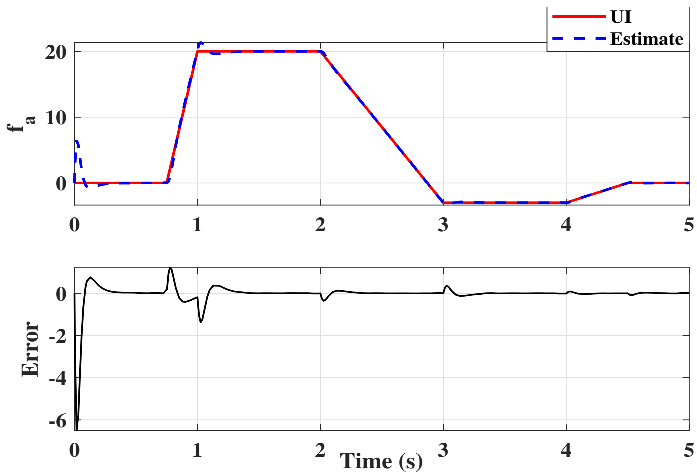


**Figure 5.** Unknown input and the State estimation error.

## 4. Conclusions

In this study, we focus on estimating states and unknown inputs in Takagi-Sugeno systems that have unmeasurable premise variables, utilizing a Proportional Multiple Integral (PMI) observer. We define the stability conditions using Linear Matrix Inequalities to ensure robust performance. To validate the PMI observer's efficacy, we conduct a simulation using the Continuous Stirred Tank Reactor (CSTR) system as a practical example.

## References


1. HAMDI, Wail, HAMMOUDI, Mohamed Yacine, et HAMIANE, Madina. Proportional Multi-Integral Observer Design for Takagi-Sugeno Systems with Unmeasurable Premise Variables: Conservatism Reduction via Polyquadratic Lyapunov Function. *European Journal of Control*, **2023**, p. 100915.
2. Bzioui, Sana, and Rafik Channa. Robust Tracking Control for the Non-isothermal Continuous Stirred Tank Reactor. *International Journal Bioautomation* **2020**, *24(2)*, 131–142.
3. Bzioui, Sana, and Rafik Channa. An adaptive observer design for nonlinear systems affected by unknown disturbance with simultaneous actuator and sensor faults. application to a CSTR. **2021**.
4. Asadi, Samira, Mehrdad Moallem, and G. Gary Wang. Simultaneous Sensor and Actuator Fault Reconstruction by Using a Sliding Mode Observer, Fuzzy Stability Analysis, and a Nonlinear Optimization Tool. *Sensors* **2022**, *22(18)*, 6866.
5. Orjuela, Rodolfo, et al. Conception d'un observateur PI pour un multimodèle découplé. In Proceedings of 8ème conférence internationale des Sciences et Techniques de l'Automatique, STA'**2007**.
6. Pasand, MM Share, and H. D. Taghirad. Disturbance retrieving unknown input proportional integral observer for generalized linear systems: Application to fault diagnosis.In Proceedings of ICCAS, IEEE **2010**.
7. Hadi, Ali Saad, and Montadher Sami Shaker. A new estimation/decoupling approach for robust observer-based fault reconstruction in nonlinear systems affected by simultaneous time varying actuator and sensor faults. *Journal of the Franklin Institute* **2020**, *357 (13)*, 8956–8979.
8. Ichalal, Dalil, et al. "Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi Sugeno model with unmeasurable premise variables. In Proceedings of 17th Mediterranean conference on control and automation. **2009**, *IEEE*.
9. BOUKHLOUF, A., HAMMOUDI, M. Y., SAADI, R., et al. Hardware-in-the-loop implementation of an unknown input observer for synchronous reluctance motor. *ISA transactions* **2023**, *133*, 485–494.
10. Mimoune, K., Hammoudi, M. Y., Hamdi, W., & Mimoune, S. M. Observer design for Takagi–Sugeno fuzzy systems with unmeasured premise variables: Conservatism reduction using line integral Lyapunov function. *ISA transactions* **2023**.
11. MIMOUNE, Khalida, HAMMOUDI, Mohamed Yacine, SAADI, Ramzi, et al. Real-Time Implementation of Non Linear Observer Based State Feedback Controller for Induction Motor Using Mean Value Theorem. *Journal of Electrical Engineering & Technology*, **2023**, *18 (1)*, 615–628.