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A novel decision approach for the performance analysis of gamma type double piston Stirling engine †

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Abstract: Stirling engines represent a category of external heat transfer engines that demonstrate versatility by harnessing various heat sources, including solar energy, bio-mass, conventional fuel, and nuclear power. Achieving high thermal efficiency in power production has been a paramount concern driving researchers across the globe to focus on developing Stirling engines. A gamma-type double-piston Stirling engine has been carefully selected for detailed analysis in this research endeavour. A polytropic model is employed in the investigation to gain deeper insights into the engine's behaviour. The outcomes derived from the polytropic analysis are subsequently compared with classical adiabatic analysis. Remarkably, the polytropic approach significantly outperforms the classical adiabatic analysis in enhancing the overall performance of the Stirling engine. The results reveals that power and efficiency obtained by ideal polytropic analysis found 90.30 W which is very close to ideal adiabatic and experimental efficiency. The results hold significant promise for advancing the efficiency and practical application of Stirling engines, reinforcing their position as a prominent contender in pursuing sustainable and highly efficient power generation technologies.

Keywords: Stirling engine; Thermodynamics; Power; Efficiency; Polytropic

1. Introduction

Stirling engines represent a form of external combustion engines capable of utilizing various heat sources like solar, bio-fuel, and nuclear energy [1,2]. These engines operate within a closed cycle, employing a gaseous working fluid. The cycle involves two working spaces - compression and expansion - along with three heat exchangers: a cooler, heater, and regenerator [3]. In these engines, the working fluid is circulated between the spaces through two pistons, namely the displacer and power pistons. The displacer facilitates the distribution of working gas, while the power piston generates power. Alternatively, another type, alpha Stirling engines, is characterized by two separate expansion and compression spaces and two power pistons without a displacer [4]. Their appeal lies in their potential for high thermal efficiency, quiet operation, and compatibility with diverse thermal energy sources, making them an attractive option for power generation or refrigeration worldwide.

Among various thermal methods, numerical models offer the most accuracy. Consequently, several numerical models based on thermodynamic principles have been considered for analyzing and predicting the performance of Stirling engines. For instance, Urieli and Berchowitz [5] attempted to develop an adiabatic model for Stirling engines using numerical calculations. Their work involved creating a computer code to predict the engine's performance based on the adiabatic model.

In this research work, the performance of the Stirling engine is analyzed using the polytropic method presented by Babaelahi and Sayyaadi [6]. This method was applied for the performance of beta type Stirling engine. In this work the first time this method is applied to gamma type Stirling engine. The performance of this model is analysed on the Stirling engine without losses and compared with other classical thermodynamic models and experimental work. The power and efficiency determined by this new model is 90.30 W and 20.90 %

2. Engine data and Polytropic analysis

The engine under study is a gamma-type double-piston Stirling engine. In this engine, the exhaust gas of the diesel engine is used as the heat source. To heat the heater tubes of the Stirling engine, especially the cap is made for the Stirling engine. Further details of Engine data are given in Table 1. This approach involves treating Stirling engines as polytropic processes and introduces a methodology to assess polytropic indexes using the engine's operational parameters. A novel numerical thermal model was proposed by Babaelahi and Sayyadi et al. [6] employing polytropic analysis. This model replaces the polytropic expansion and compression in the working spaces with isothermal or adiabatic models. In this research work, only the performance of the Stirling engine is presented without losses. For the details description of the equations, readers are referred to as [6].

Table 1. Engine data of gamma type double piston Stirling engine [3].

Engine Data	Values	Engine Data	Values
Cooler temperature	294	Pressure (bar)	3.58
Heater temperature	424	Rotational speed (rpm)	882
Cooler volume	223×10^{-6}	Phase angle (degree)	88°
Heater volume	87.28×10^{-6}	Regenerator volume (m³)	308.93
Expansion swept volume (m³)	221×10^{-6}	Compression swept volume (m³)	194×10^{-6}
Expansion clearance volume (m³)	24×10^{-6}	Compression clearance volume (m³)	35×10^{-6}

Table 2. Polytropic Equations [6]

Pressure and pressure variation

$$M = \left\{ \frac{P\left(\dfrac{V_c}{T_c} + \dfrac{V_k}{T_k} + \dfrac{V_r}{T_r} + \dfrac{V_h}{T_h} + \dfrac{V_e}{T_e}\right)}{R} \right\} - \left\{ \pi D \frac{P+P_{buffer}}{4RT_g}\left(u_p J - \frac{J^3}{6\mu}\frac{P+P_{buffer}}{L}\right) \right\}$$

$$dp = \frac{-\gamma\left[\left[\dfrac{\left(\dfrac{PdV_e + \dfrac{V_e}{\gamma}dP}{RT_{he}}\right) - \left(\dfrac{C_{ne}}{C_p}\dfrac{m_e}{T_{he}}dT_c\right) + \left(\dfrac{Q_{shuttle}}{C_p T_{he}}\right)}{B_1}\right] + \left[\dfrac{\left(\dfrac{PdV_c + \dfrac{V_c}{\gamma}dP}{RT_{ck}}\right) - \left(\dfrac{C_{nc}}{C_p}\dfrac{m_c}{T_{ck}}dT_c\right) - \left(\dfrac{Q_{shuttle}}{C_p T_{ck}}\right)}{B_2}\right] + Rm_{leak}\right]}{\dfrac{V_c}{T_{ck1}B_1} + \dfrac{V_e}{T_{ck1}B_2} + \gamma\left(\dfrac{V_{k_1}}{T_k} + \dfrac{V_{k_2}}{T_k} + \dfrac{V_r}{T_r} + \dfrac{V_h}{T_h}\right)}$$

where $B_1 = \left[\left(\dfrac{C_{ne}}{C_p}\right)\left(\dfrac{T_0-T_c}{T_{he}}\right)+1\right], B_2 = \left[\left(\dfrac{C_{nc}}{C_p}\right)\left(\dfrac{T_0-T_c}{T_{ck}}\right)+1\right]$

$$m_i = \frac{P_i V_i}{R T_i}$$ where i= k, r, h (cooler, regenerator and heater)	Mass
$$dm_c = \frac{\left[\left(\frac{P dV_c + \frac{V_c}{\gamma} dP}{R T_{ck}}\right) - \left(\frac{C_{nc}}{C_p}\frac{m_c}{T_{ck}} dT_c\right) - \left(\frac{Q_{shuttle}}{C_p T_{ck}}\right)\right]}{\left[\left(\frac{C_{nc}}{C_p}\right)\left(\frac{T_0 - T_c}{T_{ck}}\right) + 1\right]}$$ $$dm_e = \frac{\left[\left(\frac{P dV_e + \frac{V_e}{\gamma} dP}{R T_{he}}\right) - \left(\frac{C_{ne}}{C_p}\frac{m_e}{T_{he}} dT_c\right) + \left(\frac{Q_{shuttle}}{C_p T_{he}}\right)\right]}{\left[\left(\frac{C_{ne}}{C_p}\right)\left(\frac{T_0 - T_e}{T_{he}}\right) + 1\right]}$$ $$dm = \frac{dP}{R}\frac{V_i}{T_i}$$ where i= k, r, h (cooler tube, regenerator, and heater)	Mass accumulation
$$m_{ck} = -dm_c$$ $$m_{kr} = m_{ck} - dm_k$$ $$m_{rh} = m_{kr} - dm_r$$ $$m_{he} = m_{rh} - dm_h$$	Mass flow
If mass m_{he}>0 then T_{he}=T_h else if T_{he}= T_e If mass m_{ck}>0 then T_{ck}=T_c else if T_{ck}= T_k	Conditional temperature
$$dT_e = T_e\left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e}\right)$$ $$dT_c = T_c\left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}\right)$$ $$T_r = \frac{T_h - T_k}{\ln\left(\frac{T_h}{T_k}\right)}$$	Temperature
$$dQ_h = \frac{V_h dP c_v}{R} - c_p\left(T_{rh} m_{rh} - T_{he} m_{he}\right)$$ $$dQ_r = \frac{V_r dP c_v}{R} - c_p\left(T_{kr} m_{kr} - T_{rh} m_{rh}\right)$$ $$dQ_k = \frac{V_k dP c_v}{R} + c_p\left(T_{ck} m_{ck} - T_{kr} m_{kr}\right)$$ $$dW_e = P dV_e$$ $$dW_c = P dV_c$$ $$dW = dW_e + dW_c$$ $$\eta = \frac{W}{Q_h}$$	Energy and Efficiency

The equation for the expansion and compression space volume calculation is given below. Whereas V_{DE}, V_{DC}, V_{SE}, V_{SC}, $\emptyset$ are the expansion dead volume, compression dead volume, expansion swept volume, compression swept volume and phase angle respectively.

$$V_E = \frac{V_{SE}}{2}\left[1-\cos(\alpha)\right]+V_{DE} \tag{1}$$

$$V_C = \frac{V_{SE}}{2}\left[1+\cos(\alpha)\right]+\frac{V_{SC}}{2}\left[1-\cos(\alpha-\phi)\right]+V_{DC} \tag{2}$$

3. Results and Discussion

4.1. Model Validation

The PV diagram is drawn below for the model validation of ideal polytropic analysis, and the PV curve is compared with the experimental, adiabatic, isothermal and ideal polytropic curves. The curve of the new isothermal model is within the range of the experimental curve, as shown in Figure 1. Table 3 compares work, power and efficiency with the isothermal, adiabatic, experimental and ideal polytropic methods.
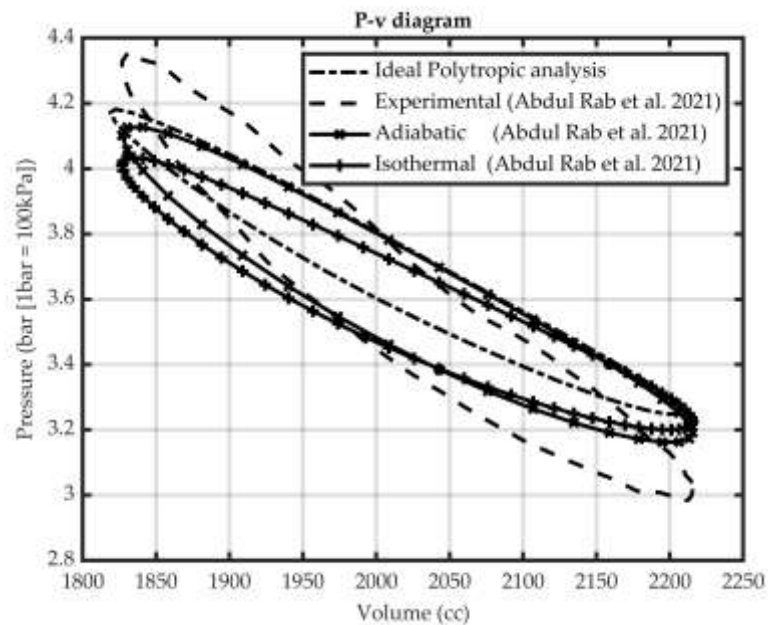

Figure 1. Comparison of the Polytropic model with other thermodynamic models.

Table 3. Summary of thermodynamic models

Parameters	Isothermal [3]	Adiabatic [3]	Experimental [3]	Ideal Polytropic
Work (J)	9.08	9.70	7.50	6.14
Power (W)	133.8	143.75	111.43	90.30
Efficiency (%)	30.70	30.90	24.70	20.90

The dotted line in the p-V diagram represents the experimental curve presented by Abdul Rab et al. [3]. The power and efficiency of the isothermal method is 133.8 W and 30.70 % [3], while the experimentally determined power is 111.43 W and 24.70 % thermal efficiency [3]. The power and efficiency obtained through the polytropic model is 90.30 W and 20.90 %, closely related to experimental work. Moreover, the Stirling engine's volume variation concerning expansion, compression, and the overall volume associated with the crank angle is a crucial factor in its operation. Analyzing the volume variation can determine the minimum and maximum levels of expansion and compression that occur during the engine's cycle. The minimum volume variation, representing the smallest volume attained during the engine's operation, is calculated to be 1.8267×10^{-3} m³ [3]. On the other hand, the maximum volume variation, corresponding to the highest vol-

ume attained during the engine's cycle, is determined to be 2.2155×10-3 m^3 [3]. These values hold significant implications for the engine's performance and efficiency, as they influence the engine's power output and overall operational characteristics. Understanding the volume variation trends as a function of the crank angle is essential for optimizing the Stirling engine's design and ensuring its smooth and reliable operation.

4. Conclusion

The Stirling engine has been renowned as the best externally heated machine. This research focuses on analysing a Stirling engine's performance using the polytropic method, initially presented by Babaelahi and Sayyaadi [6]. While this method has been applied to assess the performance of beta-type Stirling engines before, this study marks the first application to a gamma-type Stirling engine.

1. The model's performance is examined on a Stirling engine operating without losses and compared with other classical thermodynamic models and experimental data.
2. The power and efficiency determined by this new model are 90.30 W and 20.90%, respectively. These results are very close to ideal adiabatic and experimental results.
3. The findings shed light on the engine's capabilities and efficiency using this novel approach, contributing to a deeper understanding of Stirling engine performance and potential applications.

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