The entanglement generation in a system of two nonlinear quantum oscillators

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The system's ability to produce maximally or almost maximally entangled states (MES) is crucial for developing quantum information theory and its applications. Such states are usually applied in quantum communication, quantum cryptography, or quantum calculations problems. As a source of MES, we consider the system consisting of two mutually interacting anharmonic quantum oscillators. Both oscillators are externally driven by a series of ultra-short coherent pulses. For such a system, we discuss the influence of the form of excitations on the generation of entangled states. We show that the efficiency of the creation of MES strongly depends on the time between two subsequent pulses and the scheme of excitations [1, 2].

The model

We consider a system of two coupled anharmonic quantum oscillators (subsystems) labeled as 1 and 2. The oscillators interact with each other via the linear process, and they are independently excited by the external fields (each by a different field's mode). The excitations have a form of a series of ultra-short pulses that are modeled by Dirac-delta functions.

The effective Hamiltonian in the interaction picture has the following form:

$$\hat{H} = \hat{H}_{eff} + \hat{H}_{int}, \tag{1}$$

where $\hat{H}_{eff}$ describes the system of two nonlinear oscillators themselves:

$$\hat{H}_{eff} = \frac{\chi_1}{2}(\hat{a}_1^\dagger)^2\hat{a}_1^2 + \frac{\chi_2}{2}(\hat{a}_2^\dagger)^2\hat{a}_2^2 + \chi_{12}\hat{a}_1^+\hat{a}_1\hat{a}_2^+\hat{a}_2 + \varepsilon\hat{a}_1^+\hat{a}_2 + \varepsilon^\star\hat{a}_1\hat{a}_2^+. \tag{2}$$

The parameters χ_1, χ_2, and χ_{12} appearing here are self- and cross-Kerr-type nonlinearities, whereas ε is the amplitude of linear coupling between the two oscillators.

The Hamiltonian $\hat{H}_{int}$ describes external ultra-short excitations with strengths equal to β_1 and β_2:

$$\hat{H}_{int} = (\beta_1\hat{a}_1^+ + \beta_1^\star\hat{a}_1)\sum_k^\infty \delta(t - kT_1) + (\beta_2\hat{a}_2^+ + \beta_2^\star\hat{a}_2)\sum_n^\infty \delta(t - nT_2), \tag{3}$$

where T_1 and T_2 are the times between subsequent excitations of oscillators 1 and 2, respectively; whereas k and n are the numbers of applied pulses.

We shall assume that the system initially is in the vacuum state, and we will consider two cases: when the self-nonlinearity is neglected ($\chi_{12} = 0$), and both: self- and cross-nonlinearities are assumed to be different from zero.

The entanglement

In our studies, we concentrate on finding such conditions for which MES are generated with high efficiency. As a measure of the entanglement, we apply the negativity defined as the absolute sum of eigenvalues of the matrix ρ^{T_A} [3, 4]

$$N(\rho) = \frac{1}{2}\sum_i |\lambda_i| - \lambda_i \tag{4}$$

where ρ^{T_A} is a partial transpose of the density matrix ρ, calculated with respect to one of the subsystems $A = 1$ or 2. In the presented here results, we analyze only the maximal values of the negativity N_{0110} that is defined in the space of four two-mode states: $|0\rangle|0\rangle$; $|0\rangle|1\rangle$; $|1\rangle|0\rangle$ and $|1\rangle|1\rangle$.

Simultaneous excitations

First, we analyze the influence of the strength of pulses and the time of free evolution on the generation of MES in the simultaneous excitation scenario. For such the case, the times between two subsequent pulses are identical for the two pumping modes – we assume that $T_1 = T_2 = T = \pi$. For simultaneous excitations, as is seen in Fig. 1, the formation of MES depends on the excitation strength β. However, only for $\chi_{12} = 0$ (Fig. 1(a)), the negativity takes its highest possible value. Such a situation can be observed when we assume that $\beta \approx 0.025$. On the other side, the nonzero cross-nonlinearity in the simultaneous excitations regime (Fig. 1(b)) does not allow for the production of MES.

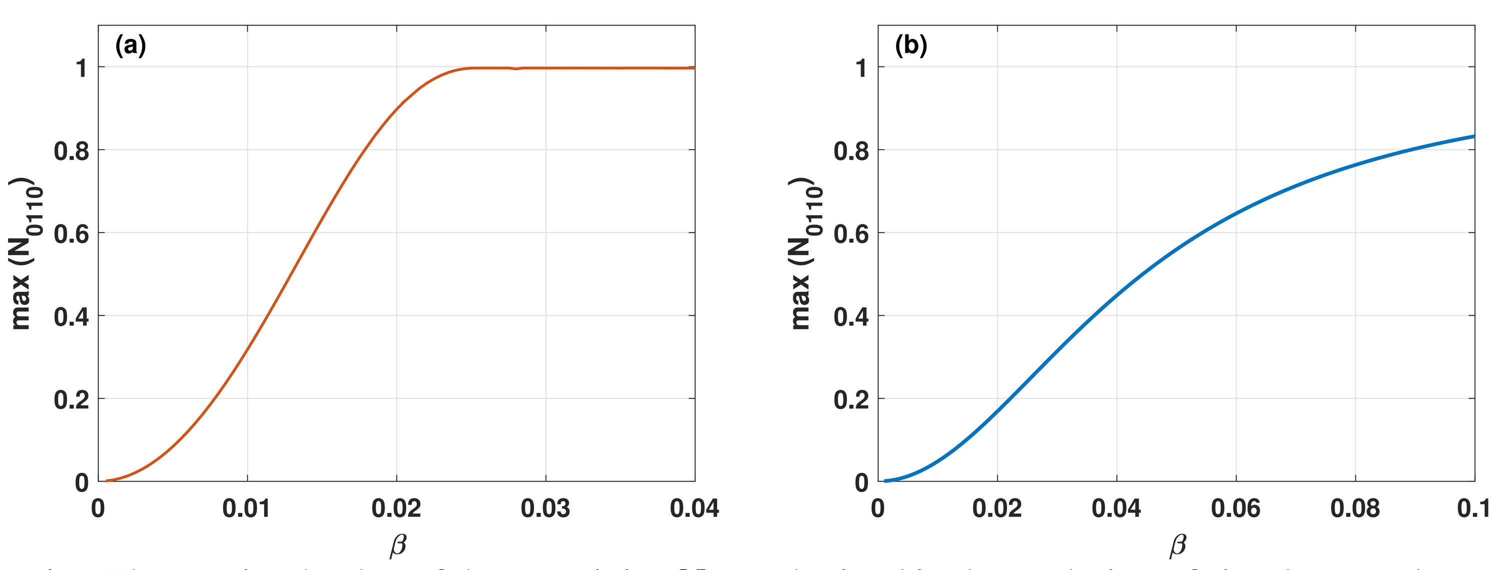

Figure 1: The maximal value of the negativity N_{0110} obtained in the evolution of simultaneously excited nonlinear oscillators versus the amplitude of external excitation β ($\beta_1 = \beta_2$). All parameters are scaled in χ units, $\chi_1 = \chi_2 = \chi = 1$ We assume that the time between pulses $T = \pi$ and $\epsilon = 1/25$. In (a) we assume $\chi_{12} = 0$ and in (b) $\chi_{12} = \chi$.

In Fig. 2, we can see that MES's creation process depends on the time T of free evolution between two subsequent external pulses. For the considered here values of β and ϵ, the MES can be produced only for narrow ranges of the value of T. So it is necessary to control the free evolution time T carefully.

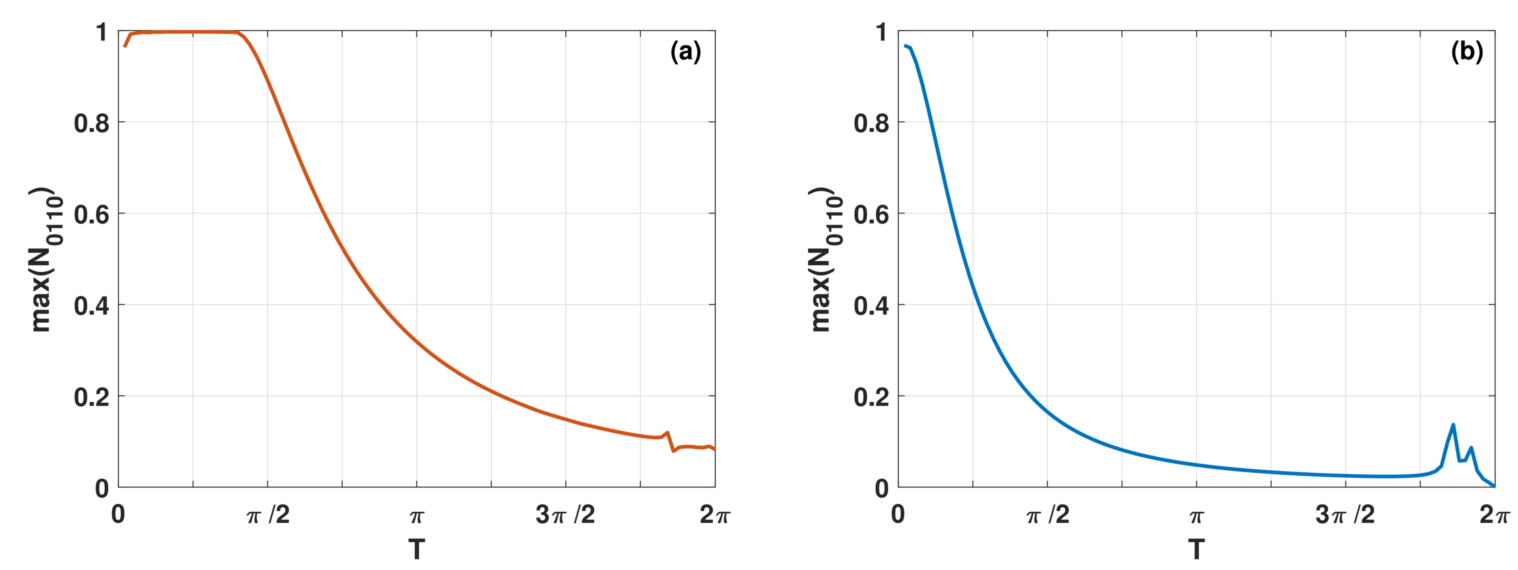

Figure 2: The maximal value of negativity N_{0110} reached in the evolution of simultaneously excited nonlinear oscillators versus the time of free evolution T. All parameters are scaled in χ units, $\chi_1 = \chi_2 = \chi = 1$, $\beta = 1/100$. The remaining parameters are identical to those for Fig. 1.

Timings variations in impulse excitations

Next, we will consider the timings variations of pulse excitations and show that such changes in the energy delivery can significantly increase the probability of obtaining MES relative to the excitations in the form of simultaneous pulses. We assume that the time of free evolution of the first oscillator is constant while the other is excited several times in parallel. The scheme of such timings is given in Fig. 3.

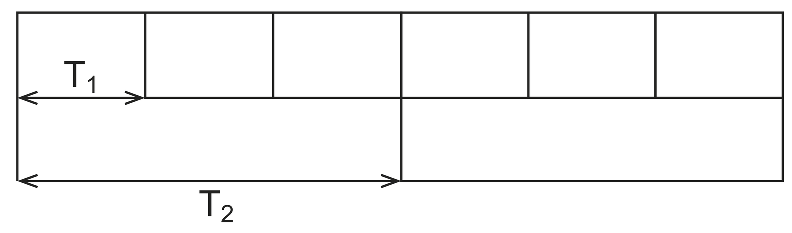

Figure 3: Scheme depicting the timings of the appearance of external pulses in the two modes. The time-interval T_1 corresponds to the time of the free evolution of the first oscillator, whereas T_2 refers to the second oscillator.

Figs. 4(a) and 4(b) show that applying a proper number of excitations in the one mode when the second oscillator evolves freely may increase the effectiveness of the MES creation process. It is especially well pronounced, when we are dealing with situations when simultaneous excitations give relatively small negativities (Fig. 4(a) – the case when $N_{0110} \approx 0.32$), or are almost negligible (Fig. 4(b) – $N_{0110} \approx 0.05$).

From Fig.4, one can see that the number of pulses should be appropriately chosen to achieve the desired effectiveness of the MES generation. Especially when we assume that $\chi_{12} \neq 0$, the system becomes even more sensitive to the proper choice of the number of pulses k. It is relevant, particularly as we have only a limited range of the values of k for which the creation of almost maximally entangled states is possible. Thus, further Increasing the value of k may even deteriorate the conditions allowing for the entanglement generation – see Fig. 4(b).

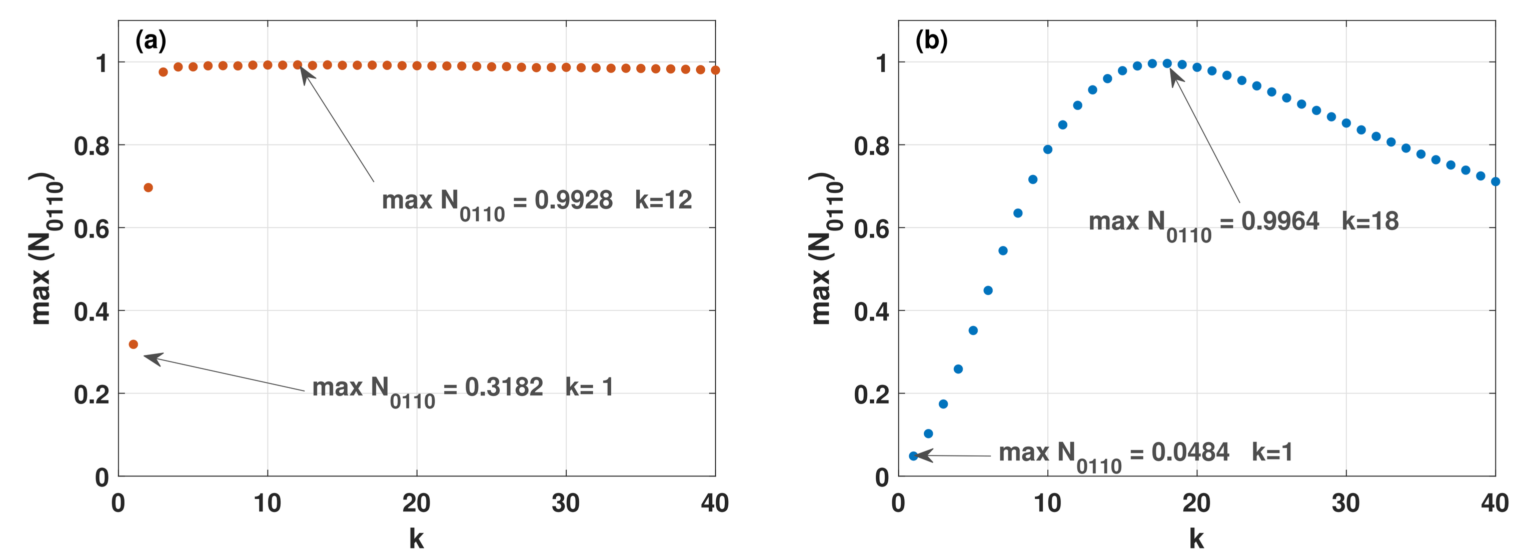

Figure 4: The maximal values of negativity N_{0110} as a function of k – the number of excitations applied to the second oscillator during the time $T_2 = \pi$ of free evolution of the second oscillator. We assume that $T_2/T_1 = k$, $\beta = 1/100$. Remaining parameters are identical as those in Fig. 1.

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