

MATHEMATICAL MODELING OF THIN-LAYER DRYING OF *OPUNTIA FICUS-INDICA* PEELS

Aymen DHAOUADI¹, Nadia SMIRANI¹, Souhir BOUAZIZI¹ and Moktar HAMD¹

Affiliation 1: Laboratory of Microbial Ecology and Technology, The National Institute of Applied Science and Technology, University of Carthage, BP 676,1080 Tunis

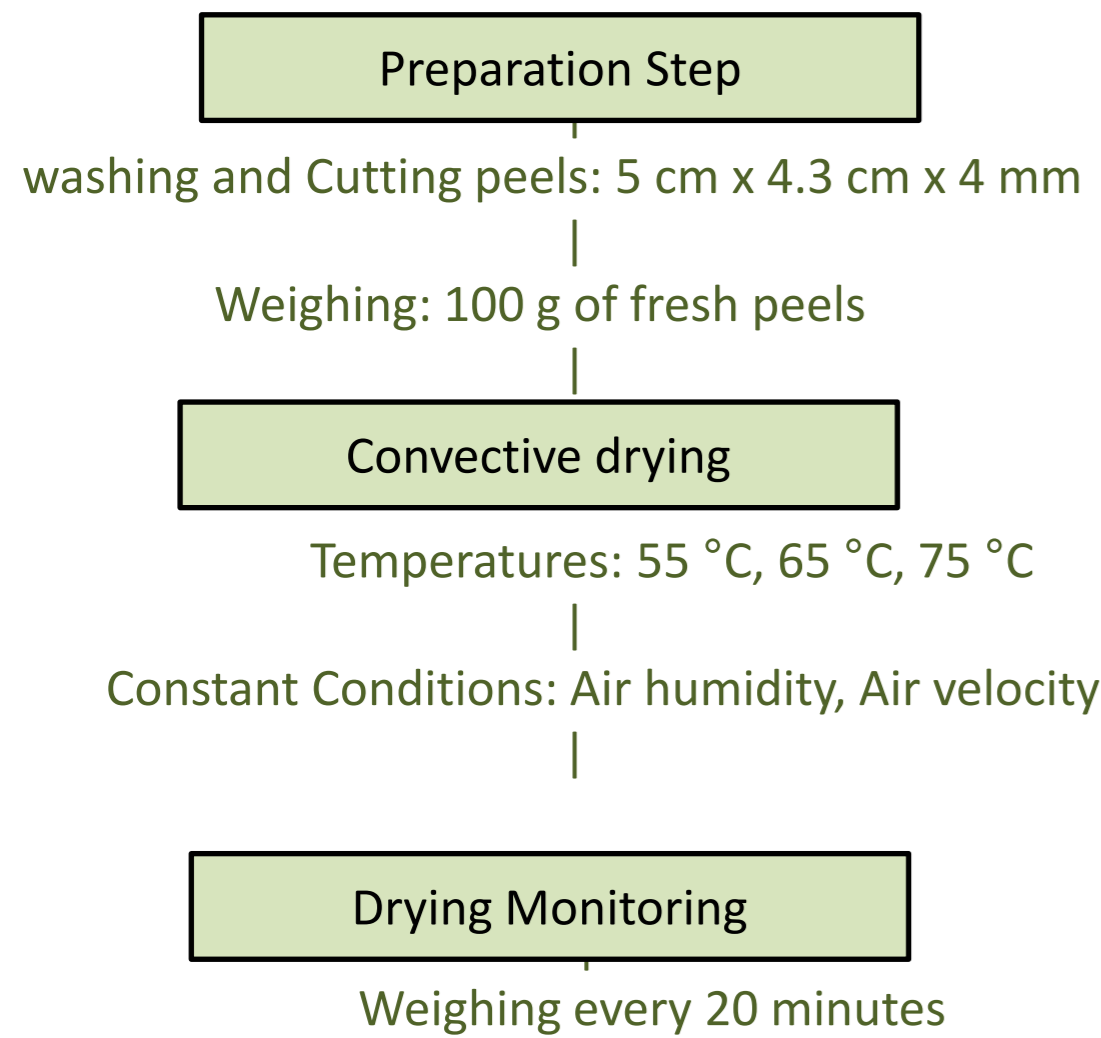
INTRODUCTION & AIM

Drying is commonly used unit operation for removing water from moist products to improve their shelf life. It is a heat and mass transfer process that takes place both within the product and at its interface with the drying environment. Due to the complexity of those mechanisms and the diversity of the products, an extensive understanding of fundamental data obtained from mathematical models of drying kinetics is required.

This work focuses on the convective drying of *Opuntia ficus indica* peels, based on the influence of temperature. The objectives of this research are to analyze the kinetics of drying; identify an appropriate model that describes drying process; quantify the moisture diffusion coefficient that characterizes moisture flow; and calculate the activation energy useful in describing how drying kinetics vary with temperature. Experimental runs illustrate how such parameters could affect the drying processes for effective modelling and dryer design.

METHOD

Opuntia ficus indica peels drying experiments



Mathematical empirical models used to simulate the thin layer drying

Model name	Model équation
Newton	$MR = \exp(-k t)$
Page	$MR = \exp(-k t^n)$
Henderson and Pabis	$MR = a \exp(-k t)$
Logarithmic	$MR = a \exp(-k t) + c$
Wang and Singh	$MR = 1 + at + bt^2$
Midilli Kucuk	$MR = a \exp(-k t) + bt$
Diffusion approach	$MR = \exp(-k t)$

Moisture ratio is defined as:

$$MR = \frac{X_t - X_e}{X_0 - X_e}$$

X_t : moisture content
 X_0 : initial moisture content
 X_e : equilibrium moisture content

Estimation of moisture diffusion coefficients (D_{eff})

Fick's second law of diffusion was used to determine the moisture diffusion coefficient:

$$MR = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} \exp\left[-(2n+1)^2 \frac{\pi^2 D_{eff} t}{4L^2}\right]$$

where D_{eff} is the moisture diffusion coefficient ($m^2 \cdot s^{-1}$); L is half the material thickness (m); n is the number of experimental groups; t is the drying time (s).

When the drying time is long, it can be simplified as:

$$MR = \frac{8}{\pi^2} \exp\left[-\frac{\pi^2 D_{eff} t}{4L^2}\right]$$

Taking the natural logarithm of both sides of

$$\ln MR = \ln \frac{8}{\pi^2} - \left[\frac{\pi^2 D_{eff} t}{4L^2}\right]$$

The result is a straight line with a slope given by

$$a = \frac{\pi^2 D_{eff}}{4L^2}$$

Estimation of activation energy (E_a)

$$D_{eff} = D_0 \exp\left(\frac{-E_a}{RT}\right)$$

where E_a is the activation energy, ($kJ \cdot mol^{-1}$); R is the universal gas constant $R = 8.31451(J \cdot mol^{-1} \cdot K^{-1})$; D_0 is the integral constant ($m^2 \cdot s^{-1}$); and T is the drying temperature ($^{\circ}C$).

Taking the natural logarithm of both sides of

$$\ln D_{eff} = \ln D_0 + \left(\frac{-E_a}{RT}\right)$$

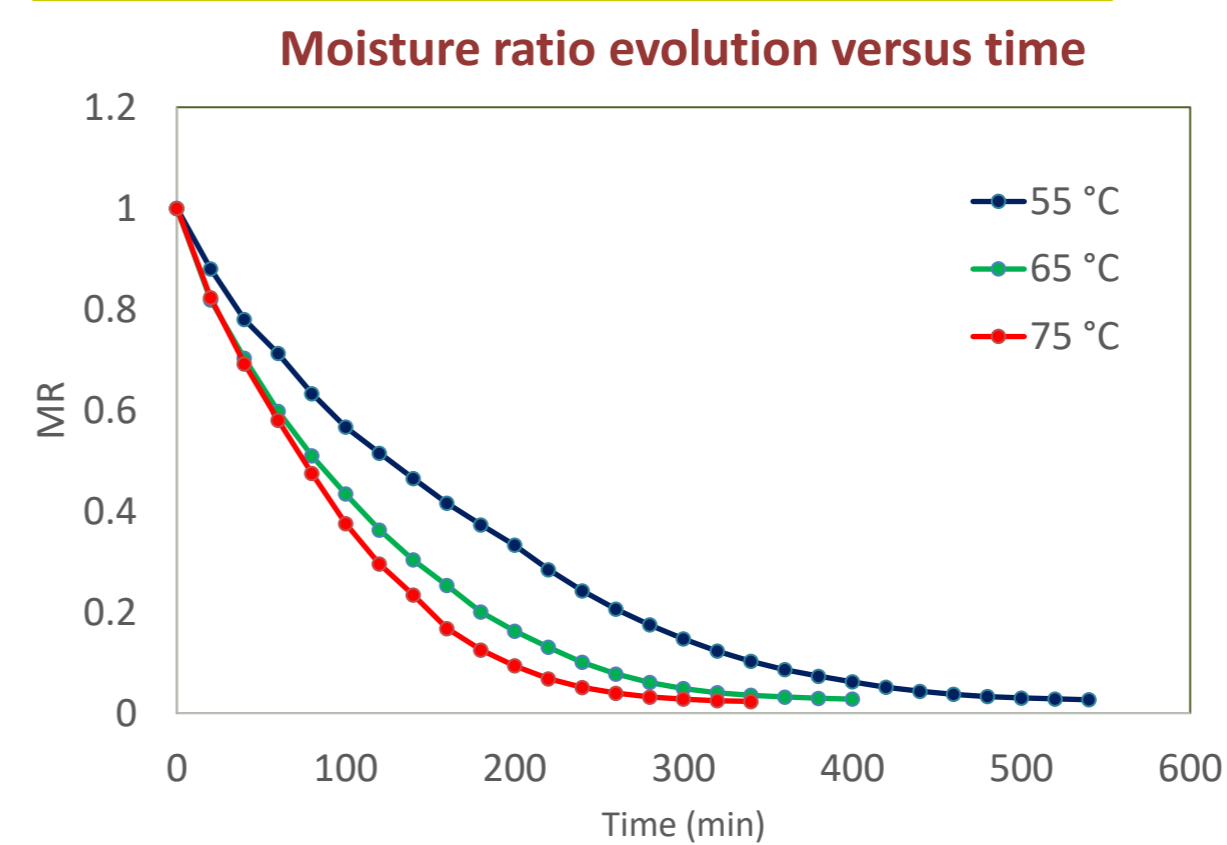
E_a is calculated using the slope derived from

$$\ln D_{eff} = f\left(\frac{1}{T}\right)$$

$$Slope = \left(\frac{-E_a}{R}\right)$$

RESULTS & DISCUSSION

Opuntia ficus indica peels drying kinetics



-The drying is taken place in falling rate period.
-The drying is determined by internal moisture diffusion.

Results of fitting of drying kinetics of *Opuntia ficus indica* peels

Drying temperatures	Model	Constants	R ²	χ^2
75 °C	Page	k = 0,00497 ; n = 1,15438	0,99834	1,57E-4
65 °C	Diffusion	a = 1,60731E11 ; b = 1 ; k = 0,01193	0,99835	1,399E-4
55 °C	Page	k = 0,00357 ; n = 1,09695	0,99647	2,961E-4

The Page model is the best with 75 °C and 55 °C while the diffusion model is the most effective at 65 °C.

The suitability of the chosen is evaluated using two statistical parameters: determination coefficient (R^2) and Chi square (χ^2).

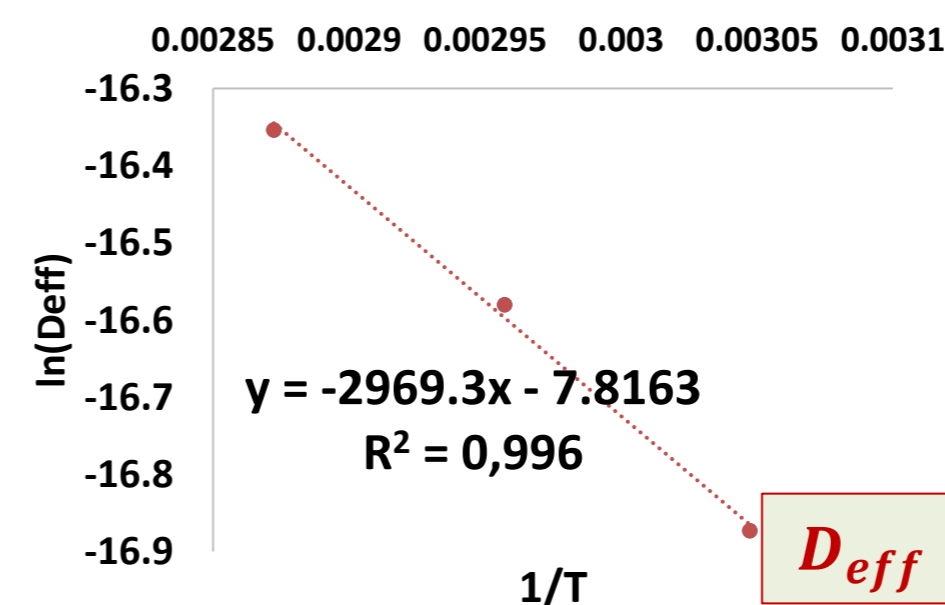
Moisture diffusion coefficients of *Opuntia ficus indica* peels (D_{eff})

Drying temperatures	Moisture diffusion coefficient ($m^2 \cdot s^{-1}$)
55 °C	4,7E-08 ± 4,6E-10
65 °C	6,3E-08 ± 1,4E-10
75 °C	7,9E-08 ± 9,2E-11

The diffusion coefficient depends on temperature. D_{eff} is ranging from 10^{-12} to 10^{-8} as it reported for food materials.

Activation energy of *Opuntia ficus indica* peels (E_a)

Arrhenius equation



The E_a values for food and agricultural products range from 12 to 130 $kJ \cdot mol^{-1}$, which is consistent with our study, where it is equal to 24.688 $kJ \cdot mol^{-1}$. The Arrhenius relationship according to our study is:

$$D_{eff} = 0,000403 \exp\left(-\frac{24688}{8.31451 \cdot T}\right)$$

CONCLUSION

The moisture diffusion coefficient of *Opuntia ficus indica* peels ranged from 4.7×10^{-8} to $7.9 \times 10^{-8} m^2 \cdot s^{-1}$ and increased with air temperature at constant velocity and relative humidity. Page and diffusion models accurately predicted the peels' behaviour during convective drying. Among the measured temperatures, 75 °C had the quickest drying time. Overall, the results demonstrated air temperature effect on drying efficiency and kinetics.

FUTURE WORK / REFERENCES

Analysis will be carried out to determine the influence of drying temperature on the physicochemical characteristics and bioactive substances of peels in order to give a value-added opportunity.

References

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- M. S., Sarker, M. S. H., Hasan, S. M. K., Ahmed, M., & Wazed, M. A. (2023). Comparison on drying characteristic, efficiency, unit drying cost and quality of maize dried by a novel multi-crop mobile dryer, existing industrial dryer and sun drying method. *Journal of Agriculture and Food Research*, 14, 100804.