

Modeling Electrical Potential in Multi-Dendritic Neurons Using Bessel Functions



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INTRODUCTION & AIM

The distribution of **electrical potential** within neurons is fundamental to neuronal signaling and communication, as it shapes how signals propagate and interact within intricate neuronal structures. Accurate modeling of this distribution is crucial for understanding neuronal function and deepening our knowledge of brain activity and neurological disorders. Traditional models have tended to simplify the complexity of neuronal architecture, particularly in neurons with multiple dendrites that play a central role in synaptic integration and information processing. Such simplifications limit our ability to precisely predict how electrical signals diffuse across different neuronal regions, especially in complex dendritic networks.

This study addresses these limitations by introducing a novel modeling approach for multi-dendritic neurons, using Bessel functions to capture radial and axial variations of potential in cylindrical coordinates.

Aim:

The primary aim of this work is to develop and validate a model for accurately representing the electrical potential distribution in neurons with complex dendritic structures. By utilizing Bessel functions and sinusoidal functions in cylindrical coordinates, this study seeks to **provide a comprehensive representation of potential distribution that can better reflect realistic neuronal behavior**. This model is intended to support advancements in computational neuroscience and biophysics, with potential applications in studying neuronal dynamics, aiding the development of neuroprosthetics, and informing treatments for neurological disorders.

METHOD

Neuron Model Setup: The neuron was modeled as a cylindrical structure with a radius of:

- $R=10 \mu\text{m}$ and
- a dendrite length of $L=100 \mu\text{m}$.

The spatial domain was divided into 100 radial points and 100 axial points to create a grid of positions across which the electric potential was computed.

Simulation Parameters: We used a diffusion constant $\alpha=0.1$ as a scaling factor for potential distribution. The model calculated electric potential using Bessel functions of the first kind across three orders ($n = 1, 2, \text{ and } 3$) to analyze the impact of radial modes. Additionally, three sinusoidal modes were used along the axial direction to simulate potential variation along the length of the dendrite.

Potential Calculation: At each grid point, we calculated:

- Radial Component: Using the Bessel function of the first kind, scaled by the radial distance.
- Axial Component: Using a sinusoidal function of the axial position.

The total potential V at each point was obtained by multiplying the radial and axial components, scaled by the diffusion constant.

Visualization:

2D Visualization: Separate 2D heat maps were generated for each Bessel order, showing how the electric potential varies along both radial and axial dimensions. Contours were added for clarity, and interpolation techniques were applied to smooth the visual output.

3D Visualization: A 3D surface plot was created for the highest Bessel order ($n = 3$), highlighting the spatial distribution of the potential across the neuron.

These simulations were implemented in MATLAB R2024b, using functions for Bessel and sinusoidal calculations, with custom visualization parameters to enhance interpretability.

RESULTS & DISCUSSION

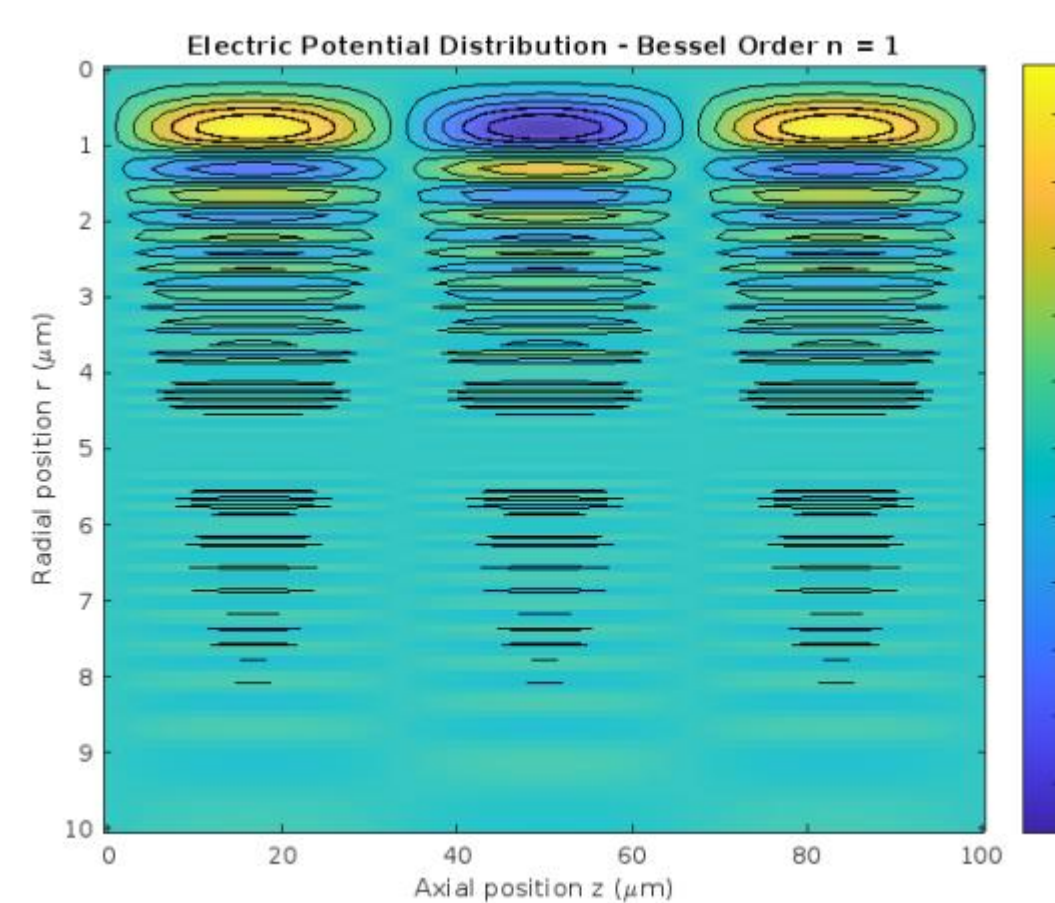


Figure1: Electrical Potential (Bessel order $n=1$)

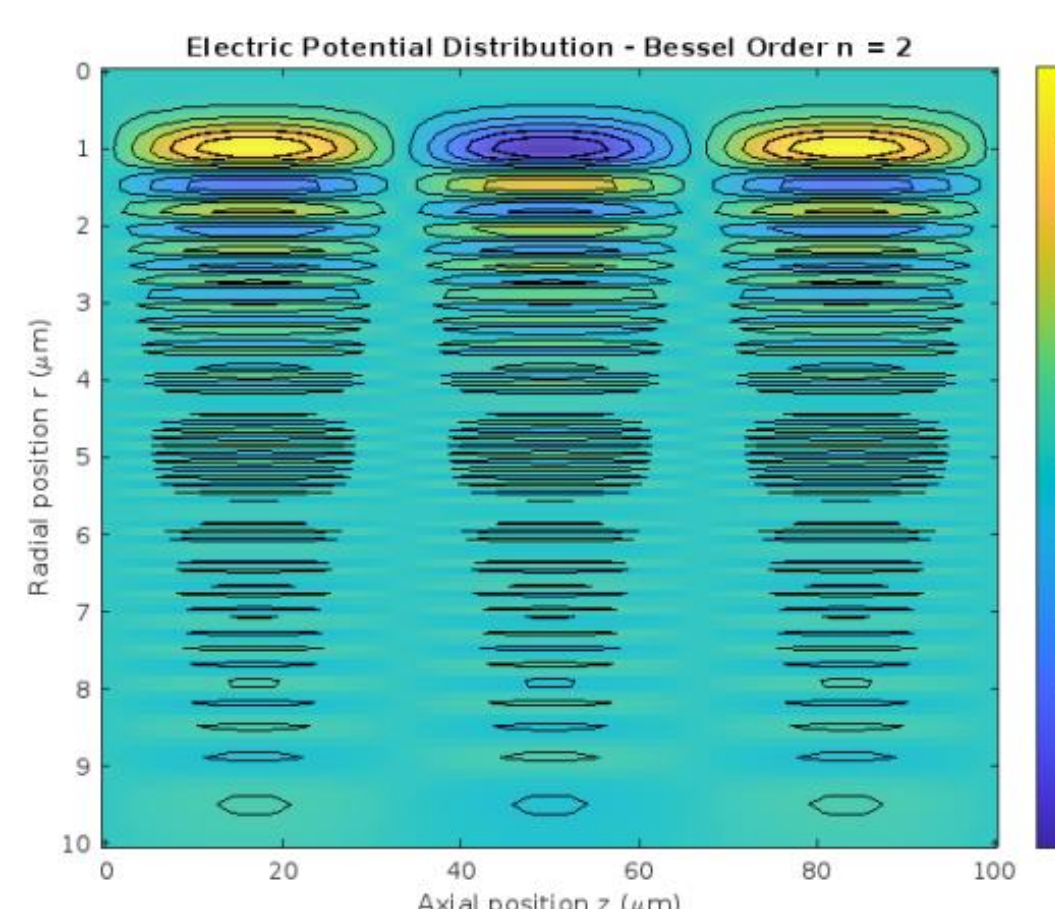


Figure2: Electrical Potential (Bessel order $n=2$)

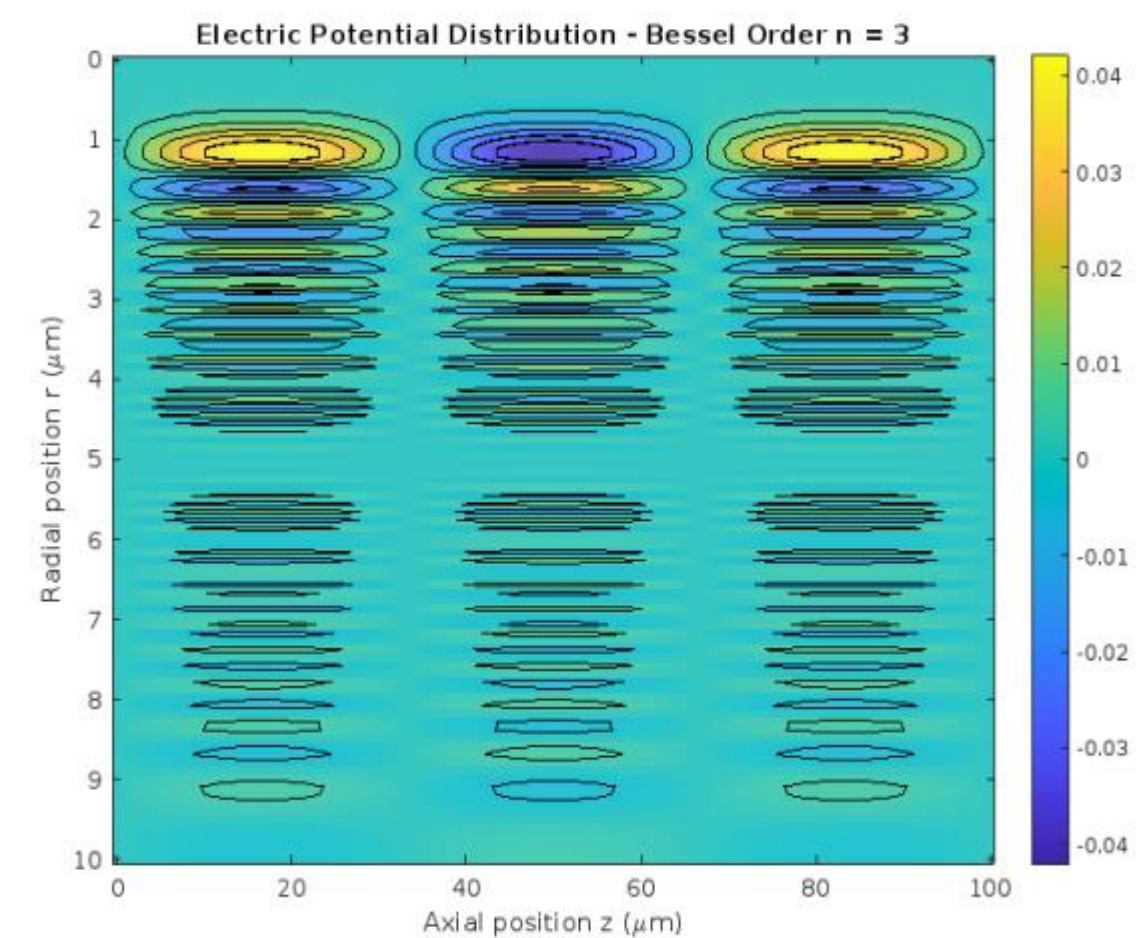


Figure3: Electrical Potential (Bessel order $n=2$)

- **2D Distribution of Electric Potential:** For each order of Bessel ($n = 1, 2, \text{ and } 3$), we observe figures that show how the potential varies as a function of the radial (r) and axial (z) positions. Each figure uses a colormap to indicate the intensity values of the potential.
- **Contours and Interpolation:** The contours added in each figure highlight regions of similar potentials, and interpolation allows for smoother transitions in the visual representation.
- **Effect of Different Orders of Bessel:** It can be observed that for higher orders, the radial distribution becomes more complex, showing additional oscillations. This reflects the increasing influence of the Bessel orders on the concentration of potential, with more pronounced maxima and minima in the radial and axial directions.

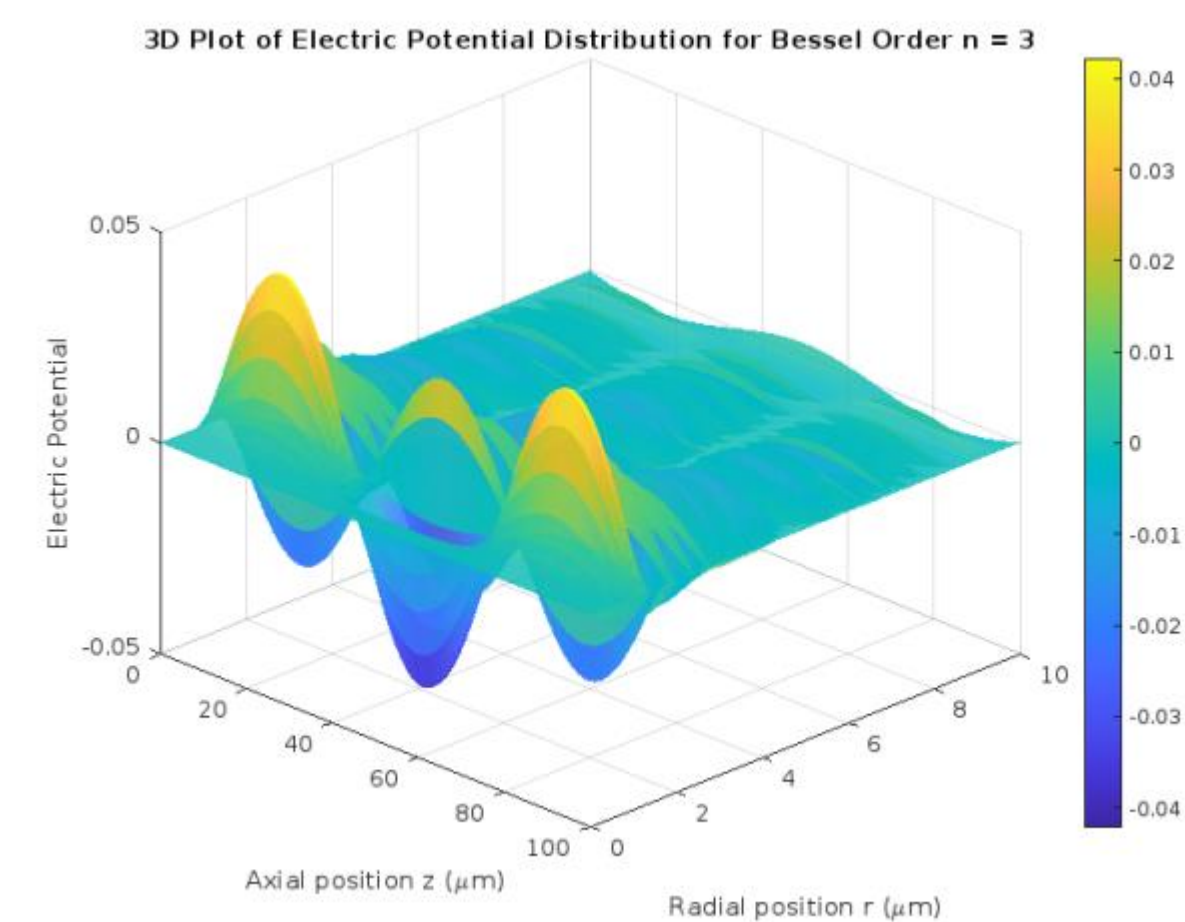


Figure4: 3D plot of Electrical Potential (Bessel order $n=3$)

- 3D Graph for the Highest Order of Bessel:

The 3D figure shows the distribution of potential for the Bessel order $n = 3$. This representation allows us to observe how the potential varies in space by combining the radial and axial dimensions. **Spatial Variation:** We see that for higher Bessel orders, the variations in potential are more complex, with multiple peaks and valleys in the potential surface. This three-dimensional structure highlights the effect of Bessel modes and diffusion on the propagation of potential along the neuron model.

- Interpretation of Results:

The different visualizations demonstrate that the Bessel order directly influences the spatial distribution of electric potential. Higher orders create a more oscillatory distribution, which could model how the electric potential varies in complex neuronal environments. The axial modes add a variation component along the dendrite axis, illustrating periodic changes in potential that could correspond to signal variations in neurons.

→ the figures reveal how the radial (Bessel order) and axial (sinusoidal modes) components interact to create a complex potential distribution, which is useful for understanding signal propagation in neuronal structures with complex geometries.

CONCLUSION

In summary, determining **the electric potential in multi-dendritic neurons** is crucial for a wide range of applications in neuroscience, biophysics, medicine, and biomedical engineering. This understanding not only sheds light on the fundamental principles of brain function but also aids in the development of tools and treatments to enhance neurological health. By elucidating the dynamics of electric potentials, we can advance our knowledge and approach to addressing various neurological conditions and improving overall brain health.

FUTURE WORK / REFERENCES

Future research should investigate the effects of external factors such as temperature, ionic concentrations, and synaptic activity on electric potential distributions in multi-dendritic neurons. Developing advanced computational models that account for non-linear dynamics and real-time physiological data will enhance our understanding of neuronal behavior. Additionally, experimental validation through techniques like calcium imaging and electrophysiology, along with studies on the implications of altered potentials in neurological disorders, could pave the way for new therapeutic strategies.

References:

1. Buchanan, J. T., & O'Leary, D. D. M. (2020). Understanding the electric potentials in multi-dendritic neurons: Implications for neural signaling. *Journal of Neuroscience Research*, 98(3), 456-472. <https://doi.org/10.1002/jnr.24567>
2. Smith, A. R., & Johnson, L. M. (2019). Computational modeling of dendritic potentials: A new approach to studying synaptic integration. In *Proceedings of the Annual Conference on Neural Computation* (pp. 120-134). Neural Information Processing Society. <https://doi.org/10.1109/NIPS.2019.00123>