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Performance Analysis of Ambiguity Resolution with Combined GPS/BDS Observations

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Abstract: BDS regional system has been officially announced to provide positioning services over the Asia-pacific region at present. We investigate the performance of ambiguity resolution with combined GPS/BDS observations under this condition. Using observation data of the different-length short baselines and PANDA software analyze the success rate of ambiguity-fixing of GPS single system, BDS single system and GPS/BDS combined system, we find that the ambiguity of GPS/BDS combined system relative to each single system can be quickly fixed even if with L1 observation. Combine the theory of ambiguity resolution to analyze experimental results, and interpret the experimental conclusions in theory.

Keywords: GPS; BDS; Combined positioning; Ambiguity resolution; System bias

1. Introduction

High precision positioning using multiple navigation systems has been focused on the combination of GPS and GLONASS observations. Due to the similar signal structure and frequencies between GPS and BDS systems, BDS has the potential to provide a much better performance than GLONASS when integrating with GPS. A number of investigations of BDS have been conducted such as precise orbit determination and clock estimation [1, 2, 3, 4, 5, 6], Precise Point Positioning (PPP) and relative positioning [1, 5, 7, 8, 9].

This paper focuses on the analysis of the performance improvement of ambiguity resolution by combining GPS and BDS observations for precise positioning. The resultant improvement in precision and reliability would create new applications such as it is feasible to establish geological disasters monitoring network using only low-cost single-frequency receivers.

2. Ambiguity Resolution with GPS/BDS combined Observations

In the process of solve double-difference carrier phase observation equation of baselines using GPS and BDS combined observation, in addition to solve the station coordinate parameters, ambiguity must also be solved. The double-difference carrier phase observation equation and the corresponding floating-point least-squares solution are given by the following expressions:

$$l = Ax + By + v \qquad E(v) = 0 \qquad E(vv^T) = D_\varphi \tag{1a}$$

$$\begin{bmatrix} A^T D_\varphi^{-1} A & A^T D_\varphi^{-1} B \\ B^T D_\varphi^{-1} A & B^T D_\varphi^{-1} B \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} A^T D_\varphi^{-1} l \\ B^T D_\varphi^{-1} l \end{bmatrix} \tag{1b}$$

$$\hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} \\ Q_{yx} & Q_{yy} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = QW, \quad D = \hat{\sigma}_0^2 Q \tag{1c}$$

where *x* and *y* denote station coordinate parameters and double-difference ambiguity parameters, D_φ is the variance/covariance matrix of the double-difference carrier phase observations, $\hat{\sigma}_0^2$ is the a posteriori variance of unit weight, Q is the cofactor matrix of the estimated parameters and *D* is the variance/covariance matrix of the estimated parameters. A and B are the corresponding design matrices for the station coordinates and the double-difference ambiguity parameters. A augmented by B is an $n \times t$ matrix where *n* is the number of observations and *t* is the total number of unknowns.

Ambiguity-fixing is the key issues to precise positioning. LAMBDA is one of the well-known generic methods. In this method, the candidate integer ambiguity parameters are constrained by the following inequalities [10]:

$$P\left(\frac{(\bar{y}-\hat{y})^T Q_{\hat{y}\hat{y}}^{-1}(\bar{y}-\hat{y})}{t_y \hat{\sigma}_0^2} < F(t_y, n-t, \alpha_F)\right) = 1 - \alpha_F \tag{2}$$

Where, $F(t_y, n-t, \alpha_F)$ is the tail value of the *F* distribution with t_y and *n-t* as the first and the second degrees of freedom under the significance level α_F. $\bar{y}_\mathrm{i}$ and $\hat{y}$ are candidate integer ambiguity parameters and the floating-point solution of ambiguity parameters respectively. Q_{yy} is the cofactor matrix of the ambiguity parameters.

These constraints define a confidence ellipsoid. The lengths of axis of confidence ellipsoid are determined by the eigenvalues of variance/covariance matrix of the ambiguity parameters, and the search region and the lengths of axis of confidence ellipsoid are closely related, so we can judge the search region size based on the eigenvalues.

Both ambiguity resolution speed and success rate by combining GPS and BDS observations should be improved in theory, and we can regard the eigenvalues of double difference ambiguities of combined system should lower than single system according to above analysis. In order to verify this hypothesis, we should compare the eigenvalues of GPS single system, BDS single system and GPS/BDS combined system. The process of comparison is described in the following section.

Before comparison, we need consider the following two aspects:

(1) In the process of parameter salvation, the noise level of GPS is not consistent with BDS, which leads to the difference between the variance of unit weight of GPS single system, BDS single system

and GPS/BDS combined system. So we can't make a comparison between the eigenvalues of GPS single system or BDS single system and the eigenvalues of GPS or BDS in the combined system directly. In order to solve this problem, we don't calculate eigenvalue based on the variance/covariance matrix of the ambiguity parameters of each system directly, but calculate eigenvalue based on cofactor matrix of each system firstly, and take the variance of unit weight of GPS single system or BDS single system as uniform variance of unit weight of three system (GPS, BDS and GPS/BDS). The eigenvalues which can be compared between combined system and single systems are calculated by the following equalities:

$$\lambda_{\mathrm{Di}} = \lambda_{\mathrm{Qi}} \mathrm{D}_0 \qquad \mathrm{i} = (\mathrm{GPS}, \mathrm{BDS\ or\ GPS/BDS}) \qquad (3)$$

where λ_{Di} is the comparable eigenvalues of variance/covariance matrix of a system, λ_{Qi} is the eigenvalues of cofactor matrix of a system and D_0 is the uniform variance of unit weight.

(2) We can find that $(\bar{y} - \hat{y})^2$ is proportional to Q_{yy} in Eq. (2). We calculate eigenvalues of Q_{yy} using the orthogonal decomposition of matrix Q_{yy}, which shows that the square of ambiguity search radius is proportional to the eigenvalues of Q_{yy}. The comparison to the square root of eigenvalue of variance/covariance matrix can be more intuitive to reflect changes of ambiguity search radius relative to the comparison to the eigenvalue of variance/covariance matrix. So we use the comparison to the square root of eigenvalue instead of the comparison to the eigenvalue.

Specific flow of comparison is described as follow.

(1) Solve the parameters of GPS single system, get the cofactor matrix of ambiguity parameters of GPS single system (Q_G) and the variance of unit weight of GPS single system (D_{0G}), calculate the eigenvalues of Q_G (λ_{Q_G}) using the orthogonal decomposition of matrix Q_G, take D_{0G} as uniform variance of unit weight of three system (D_0), calculate the eigenvalues of ambiguity parameters of GPS single system (λ_G) according to D_0 and λ_{Q_G}, and calculate the square root of eigenvalues used for the comparison ($\sqrt{\lambda_G}$) according to λ_G.

(2) Use same method to get the eigenvalue of cofactor matrix of ambiguity parameters of BDS single system ($\lambda_{\mathrm{Q_B}}$), calculate the square root of eigenvalue used for the comparison ($\sqrt{\lambda_{\mathrm{B}}}$) according to D_0 and $\lambda_{\mathrm{Q_B}}$.

(3) Break down the cofactor matrix of ambiguity parameters of GPS/BDS combined system (Q_{GB}) to get the cofactor matrix which is concerned only with ambiguity parameters of GPS ($\mathrm{Q}_{\mathrm{GB(G)}}$) and BDS ($\mathrm{Q}_{\mathrm{GB(B)}}$).

(4) Likewise, calculate $\lambda_{\mathrm{Q_{GB}}}$, $\lambda_{\mathrm{Q_{GB(G)}}}$ and $\lambda_{\mathrm{Q_{GB(B)}}}$ according to Q_{GB}, $\mathrm{Q}_{\mathrm{GB(G)}}$ and $\mathrm{Q}_{\mathrm{GB(B)}}$ respectively, and calculate $\sqrt{\lambda_{\mathrm{GB}}}$, $\sqrt{\lambda_{\mathrm{GB(G)}}}$ and $\sqrt{\lambda_{\mathrm{GB(B)}}}$ according to D_0, $\lambda_{\mathrm{Q_{GB}}}$, $\lambda_{\mathrm{Q_{GB(G)}}}$ and $\lambda_{\mathrm{Q_{GB(B)}}}$ respectively.

(5) Compare $\sqrt{\lambda_{\mathrm{G}}}$ and $\sqrt{\lambda_{\mathrm{B}}}$ to $\sqrt{\lambda_{\mathrm{GB}}}$, $\sqrt{\lambda_{\mathrm{GB(G)}}}$ and $\sqrt{\lambda_{\mathrm{GB(B)}}}$ respectively.

3. Performance Analysis of Ambiguity Resolution with Combined GPS/BDS Observations

3.1. Data collection

We collected observation data in static mode with different-length baselines of 1km, 5km, 10km and 15km. All baselines use the same reference stations which locate on the campus of Liaoning Technical University, and other observation stations are selected according to the requirement of distance. All the stations are equipped with the UR240 dual-frequency and BeiDou/GPS dual-system receivers and the UA240 antennas manufactured by the UNICORE Company in China. The session length of each baseline is about 3 hours. The data collection began on Feb. 20, 2013, was finally accomplished on Feb. 26, 2013. During this period, there were 5 GEO, 5 IGSO, 4 MEO satellites on orbit.

3.2. Data processing

The PANDA (Positioning and Navigation Data Analyst) software is used for the data processing [11]. The atmospheric delay can be eliminated or weakened by the double difference for short baseline. So use L1, L2, B1 and B2 original carrier phase double difference observations to solve parameters. Normally, Phase Center Offset (PCO) and Phase Center Variation (PCV) of satellite and receiver correction should be considered in the process of precise GNSS data processing [12]. Because PCO and PCV corrections of BDS satellites and UNICORE antennas are not available now, PCO and PCV corrections are neglected in this paper. Ambiguity parameters are fixed by LAMBDA method.

3.3. Processing Scheme

We use the experiment data to solve GPS/BDS combined positioning solution. To compare GPS/BDS combined system solution with single system solution, we simultaneously solve their single positioning solution. In order to analyze the performance of GPS/BDS combined positioning on ambiguity-fixing, we calculate the success rate of ambiguity-fixing based on different baseline, different system, different session length, and different observation respectively. We divided the experiment data into 70 sessions, and every length of session is 160 seconds. The processing schemes of every session can be described as follow: positioning solution includes GPS single system positioning solution, BDS single system positioning solution and GPS/BDS combined positioning solution; session length includes 2, 4, 8, 16 and 32 epochs; observation includes L1 and L1 + L2. The total number of those processing schemes is 30, and every of them include 70 solutions.

3.4. Data analysis and results

We solve baselines of 1km, 5km, 10km and 15km, and calculate the success rate of ambiguity-fixing based on different baseline, different system, different session length and different observation. The statistical results of 1km and 15km are listed in Table 1~2.

Table 1 The relationship between the success rate of ambiguity-fixing and session length for 1km baseline

session length (epoch)	L1 success rate (%)			L1+L2 success rate (%)		
	GPS	BDS	GPS+BDS	GPS	BDS	GPS+BDS
2	5	20	97	94	89	98
4	13	28	97	96	89	100
8	24	36	98	98	88	100
16	46	50	98	100	88	100
32	82	72	98	100	92	100

Table 2 The relationship between the success rate of ambiguity-fixing and session length for 15km baseline

session length (epoch)	L1 success rate (%)			L1+L2 success rate (%)		
	GPS	BDS	GPS+BDS	GPS	BDS	GPS+BDS
2	0	2	47	53	68	93
4	2	3	57	63	67	96
8	10	5	57	70	70	96
16	21	6	67	78	72	96
32	27	6	81	87	85	96

According to all above tables, we can find that GPS/BDS combined system can significantly improve success rate of ambiguity-fixing with L1 observation. Under the condition of dual-frequency observation, the success rate of ambiguity-fixing of GPS/BDS combined system with L1+L2 observation all can be higher than 90% when session length is only 2 epoch for 1km, 5 km, 10km and 15 km baseline, all ambiguity almost can be fixed.

Take the result of 1km baseline as an illustration, Figure 1 shows the specific tendency of Ratio value for different-length short baselines with single frequency observation and session length of only 2 epochs, and dose comparison between BDS single system, GPS single system and GPS/BDS combined system. We can find that the Ratio value of the combined system is significantly higher than the single system for all experimental baselines with single frequency observation and session length of 2 epochs. We can give a conclusion that the ambiguity of GPS/BDS combined system can be quickly fixed even if with L1 observation.

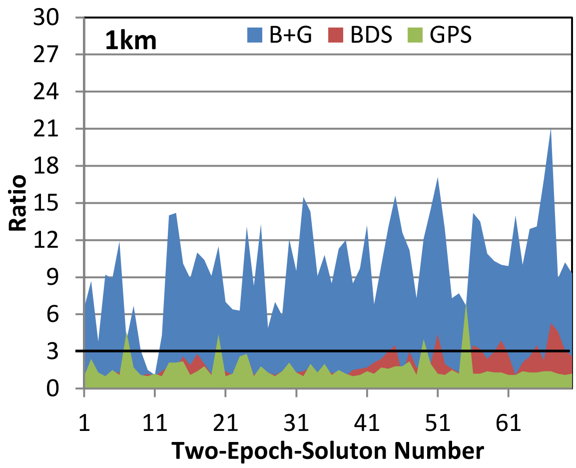

Fig. 1 sequence charts of Ratio value of different system positioning solutions for different-length short baselines with single frequency observation and session length of 2 epochs

Use above flow in section 2 to process 1km experimental data on Feb. 26, 2013. The start time is 7500s, and the session length is 2 epochs. Figure 2~4 show the comparison of the square root of eigenvalue between single system and combined system with single frequency observation.

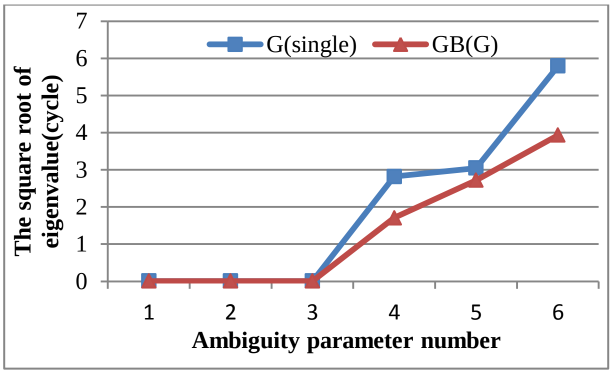

Fig. 2 Comparison between $\sqrt{\lambda_G}$ and $\sqrt{\lambda_{GB(G)}}$ with single frequency observation

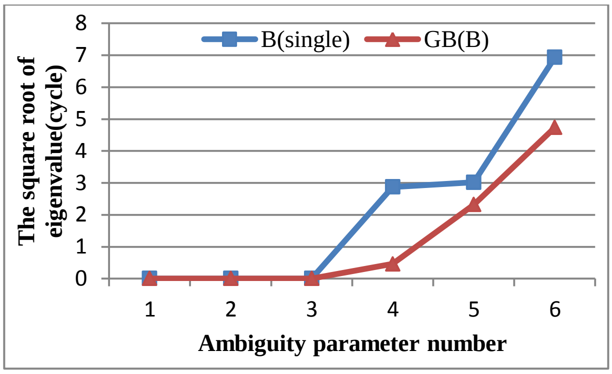

Fig. 3 Comparison between $\sqrt{\lambda_B}$ and $\sqrt{\lambda_{GB(B)}}$ with single frequency observation

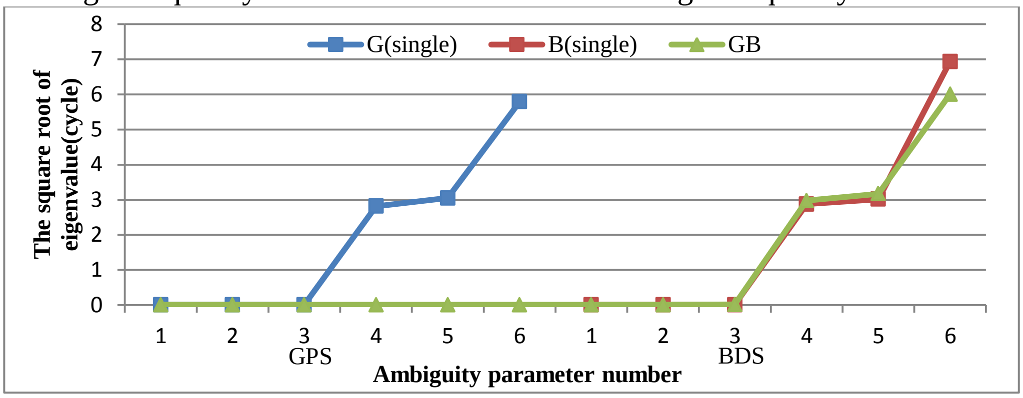

Fig. 4 Comparison of the square root of eigenvalue between single system (GPS and BDS) and combined system (GPS/BDS) with single frequency observation

According to above results, we can conclude at least:

(1) $\sqrt{\lambda_{GB(G)}}$ and $\sqrt{\lambda_{GB(B)}}$ are smaller than $\sqrt{\lambda_G}$ and $\sqrt{\lambda_B}$, which show that combination of GPS and BDS influences the variance/covariance matrix of each single system, and can reduce the search region of each single system.

(2) A part of the square root of eigenvalues of every system is very small, for example, even smaller than 0.1 cycle. We can regard that these ambiguity parameters don't need search and can be fixed directly, and the success rate of ambiguity-fixing will be very high.

(3) Compare $\sqrt{\lambda_{GB}}$ to $\sqrt{\lambda_G}$ and $\sqrt{\lambda_B}$, in the case of $\sqrt{\lambda_{GB}}$ are not significantly higher than $\sqrt{\lambda_G}$ and $\sqrt{\lambda_B}$, the number of $\sqrt{\lambda_{GB}}$ with large value is half of the sum of the number of $\sqrt{\lambda_G}$ with large value and the number of $\sqrt{\lambda_B}$ with large value, for example, with single frequency observation, the number of higher than 0.1 cycle in $\sqrt{\lambda_{GB}}$ is 3, however, the number of higher than 0.1 cycle in $\sqrt{\lambda_G}$ and $\sqrt{\lambda_B}$ is 3 respectively. So although the number of ambiguity of combined system relative to each single system is increased, the number of the square root of eigenvalues with large value is falling off, the proportion of the number of ambiguity which can be fixed directly is increased. So the whole size of search region is decreased, and the success rate of ambiguity-fixing will be improved.

4. Conclusions

Ambiguity-fixing is the key issues to precise kinematic positioning. This paper investigates the performance of ambiguity resolution with combined GPS/BDS observations under the condition that

BDS regional system has been officially announced to provide positioning services over the Asia-pacific region. Using observation data of the short baselines of 1km, 5km, 10km, 15km and PANDA software analyze the success rate of ambiguity-fixing of GPS single system, BDS single system and GPS/BDS combined system, and draw the following conclusion at last:

(1) The ambiguity of GPS/BDS combined system can be quickly fixed even if with L1 observation.
(2) For 1km, 5 km, 10km, 15 km baseline, the success rate of ambiguity-fixing of GPS/BDS combined system with L1+L2 observation all can be higher than 90% when session length is only 2 epoch, all ambiguity almost can be fixed.
(3) The combination of GPS and BDS influences the variance/covariance matrix of each single system, and can reduce the search region of each single system.
(4) Although the number of ambiguity of combined system relative to each single system is increased, the number of the square root of eigenvalues with large value is falling off, the proportion of the number of ambiguity which can be fixed directly is increased, and lead to the success rate of ambiguity-fixing is increased at last.

We only analyze the performance of ambiguity resolution with combined GPS/BDS observations under short baseline condition in this paper, and want to expand which to long baseline relative positioning and precise point positioning, because they all require a longer initial time of ambiguity. If the conclusions of this paper can be expand to above two positioning mode, which will inevitably improve their operation efficiency significantly and widen their application area.

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Conflicts of Interest

The authors declare no conflict of interest.

References and Notes

1. Montenbruck, O.; Hauschild, A.; Steigenberger, P.; Hugentobler, U.; Teunissen, P.; Nakamura S. Initial assessment of the COMPASS/BeiDou-2 regional navigation satellite system. GPS Solut. 2012, doi: 10.1007/s10291-012-0272-x.
2. Steigenberger, P.; Hauschild, A.; Montenbruck, O.; Hugentobler, U. Performance Analysis of COMPASS Orbit and Clock Determination and COMPASS-Only PPP. In Proceedings of IGS Workshop, Olsztyn, Poland, 23–27 July, 2012.

3. Shi, C.; Zhao, Q.; Li, M.; Tang, W.; Hu, Z.; Lou, Y.; Zhang, H.; Niu, X.; Liu J. Precise orbit determination of BeiDou Satellites with precise positioning. Sci. China Earth Sci. 2012, 55, 1079–1086.
4. Zhao, Q.; Guo, J.; Li, M.; Qu, L.; Hu, Z.; Shi, C.; Liu, J. Initial results of orbit and clock determination for COMPASS navigation satellite system. J. Geod. 2013, doi:10.1007/s00190-013-0622-7.
5. Ge, M.; Zhang, H.P.; Jia, X.L.; Song, S.L.; Wickert, J. What Is Achievable with the Current COMPASS Constellation? In Proceedings of the 25th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2012), Nashville, Tennessee, 17–21 September 2012; pp. 331–339.
6. He, N.; Ge, M.; Wang, J.; Wickert. J; Schuh, H. Experimental Study on Precise Orbit Determination of BeiDou Navigation Satellite System. Sensors 2013, 13, 2911–2928.
7. Li, M. Precise Point Positioning with BeiDou Navigation Satellites System. The third China Satellite Navigation Conference, Guangzhou, China, May 16–19, 2012.
8. Cao, W.; O'Keefe, K.; Cannon, ME. Evaluation of COMPASS ambiguity resolution performance using geometric-based techniques with comparison to GPS and Galileo. In Proceedings of the 21th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS 2008), Savannah, Georgia, 16–19 September, 2008.
9. Shi, C.; Zhao, Q.; Hu, Z.; Liu, J. Precise relative positioning using real tracking data from COMPASS GEO and IGSO satellites. GPS Sol. 2013, 17, 103–119.
10. Teunissen, PJG. The least squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. J Geod 1995, 70, 65-82.
11. Shi, C., Zhao, Q., Lou, Y., Li, M. Comprehensive Processing Software for Satellite Navigation Systems and Its Research Progress. Spacecraft Engineering 2009, 18, 64-70.
12. Schmid, R.; Steigenberger, P.; Gendt, G.; Ge, M.; Rothacher, M. Generation of a consistent absolute phase-center correction model for GPS receiver and satellite antennas. J. Geod. 2007, 81, 781–798.