

Analysis of the Stress Concentration Factor at the Circular Holes Near Materials Border Assessed by the Extended Finite Element Method

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INTRODUCTION & AIM

The XFEM was developed when the minimum finite meshing method for the discontinuous boundary problem was introduced. In the extended finite element method, a discontinuous function and a two-dimensional displacement field are added to the finite element approximation to solve the discontinuity problem using the partition of unity method. This allows the region around a discontinuous boundary such as a void to be modeled by a finite element without requiring complex meshing. The finite element mesh is independent of the pore geometry, which is a significant advantage in computation compared to the traditional FEM. The pore geometry is characterized by level set functions and then the signed distance function are used to compute the enrichment functions appearing in the displacement field based on the approximation finite element. Discontinuous enrichment functions are added to the finite element approximation to account for the presence of voids. This method allows voids to be arbitrarily positioned within the mesh. Due to its outstanding advantages over FEM, the XFEM method was chosen as a tool to analyze the cracking problem within the scope of this research [1-3].

In this paper, we present the XFEM for calculating stress concentration factor around the circular in a rectangular isotropic plate with two different materials.

METHOD

1. Level set method for the model of the hole

To compute the value of the normal level set function ϕ , we consider Γ as the boundary of a hole in the body. From any point x on the body, we can find a point x_Γ on the hole boundary such that the distance $\|x - x_\Gamma\|$ is the smallest. The distance function considering the sign ϕ can be represented as follows:

$$\phi(x) = \pm \min_{x_\Gamma} \|x - x_\Gamma\| \quad (1)$$

Thus, the occurrence of holes can be completely determined by the level set function ϕ . Inside the object, $\phi < 0$ at any position inside the region bounded by the boundary Γ and $\phi > 0$ at any location outside the boundary.

$$\phi = \|x - x_c\| - r_c \quad (2)$$

where x_c and r_c are the center and radius of the circular hole.

In XFEM, the level set function ϕ can be interpolated inside the element using functions of the following form:

$$\phi(x) = N_i(x)\phi_i \quad (3)$$

where ϕ_i is the value of the nodes with the corresponding level set value

2. Apply XFEM to a rectangular isotropic plate with two different materials with a hole near boundary

For Mode I:

$$\begin{cases} \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \\ \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \\ \sigma_{zz} = \begin{cases} 0 & \text{for plane stress} \\ \nu(\sigma_{xx} + \sigma_{yy}) & \text{for plane strain} \end{cases} \\ u_x = \frac{K_I}{2\mu_{tip}} \sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left[\kappa_{tip} - 1 + 2\sin^2\left(\frac{\theta}{2}\right)\right] \\ u_y = \frac{K_I}{2\mu_{tip}} \sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left[\kappa_{tip} + 1 - 2\sin^2\left(\frac{\theta}{2}\right)\right] \end{cases} \quad (4)$$

where K_I is the SIF for mode I. μ_{tip} is shear modulus at the crack tip and $\mu_{tip} = E/2(1 + \nu_{tip})$; $\kappa_{tip} = (3 - \nu_{tip})/(1 + \nu_{tip})$ for plane stress and $\kappa_{tip} = 3 - 4\nu_{tip}$ for plane strain.

RESULTS & DISCUSSION

Considering a rectangular isotropic plate with two different materials are tied together. The hole is near the material boundary. Material 1 has elastic modulus $E_1 = 10000\text{N/m}^2$ and Poisson's coefficient $\nu = 0.45$. Material 2 has elastic modulus $E_2 = 1000\text{N/m}^2$ and Poisson's coefficient $\nu = 0.3$. The geometrical dimensions are given as shown in Fig. 1 (easy to see the hole near boundary of two materials than the bottom edge of plate 1). The bottom edge and the top edge of plate subjected to tensile stress is

The stress concentration factor (SCF) is defined as follows:

$$SCF = \frac{\sigma_{yy\max}}{\sigma_{nom}} \quad (6)$$

Fig. 2 show the enrichment elements around a hole (see Fig. 2a), for every nodes at a hole (see Fig. 2b) by XFEM for a hole near boundary of two materials (see Fig. 1) with the dimensions $L=8\text{m}$, $r=1\text{m}$, $D=4\text{m}$ same this model (see Fig. 2).

The results displacement u_y of a hole near boundary of two materials follow y-axis show in Fig. 3 by FEM (see Fig. 3a) and XFEM (see Fig. 3b). Maximum displacement by FEM $\text{Max}(u_y) = 0.00059997\text{(m)}$, by XFEM $\text{Max}(u_y) = 0.0005539\text{(m)}$, and %Error=7.43% (see Fig. 3 and Table 1). The results show that XFEM is a reliable method when compared with the traditional FEM.

The results stress σ_{xx} of a hole near boundary of two materials follow x-axis show in Fig. 4 by FEM (see Fig. 4a) and XFEM (see Fig. 4b). Maximum stress by FEM $\text{Max}(\sigma_{xx}) = 0.61128\text{(Pa)}$, by XFEM $\text{Max}(\sigma_{xx}) = 0.59110\text{(Pa)}$, and %Error=3.30% (see Fig. 4 and Table 2). The results show that XFEM is a reliable method when compared with the traditional FEM.

The results stress σ_{yy} of a hole near boundary of two materials follow y-axis show in Fig. 5 by FEM (see Fig. 5a) and XFEM (see Fig. 5b). Maximum stress by FEM $\text{Max}(\sigma_{yy}) = 2.305\text{(Pa)}$, by XFEM $\text{Max}(\sigma_{yy}) = 2.105\text{(Pa)}$, and %Error=8.68% (see Fig. 5 and Table 3). The results show that XFEM is a reliable method when compared with the traditional FEM.

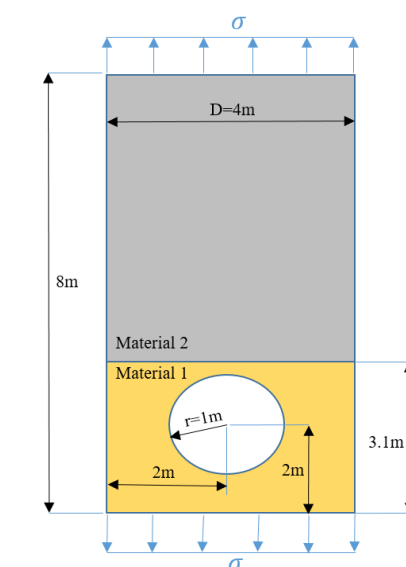


Figure 1. Isotropic plate with a hole located near the material boundary

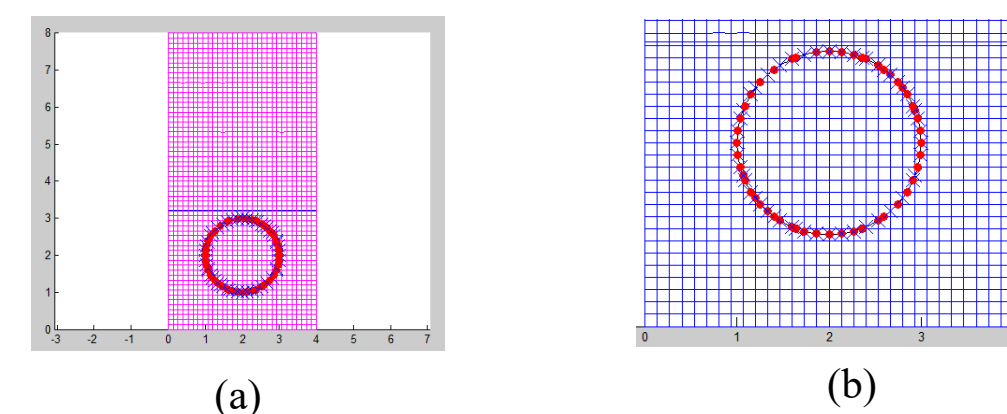


Figure 2. The enrichment elements by XFEM: a) around a hole ;b) for every nodes at a hole

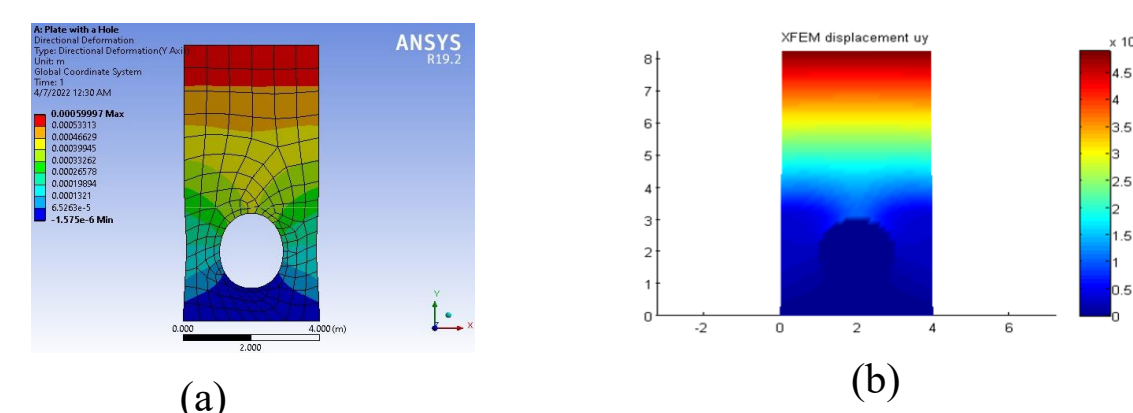


Figure 3. The results displacement u_y : a) by FEM ;b) by XFEM

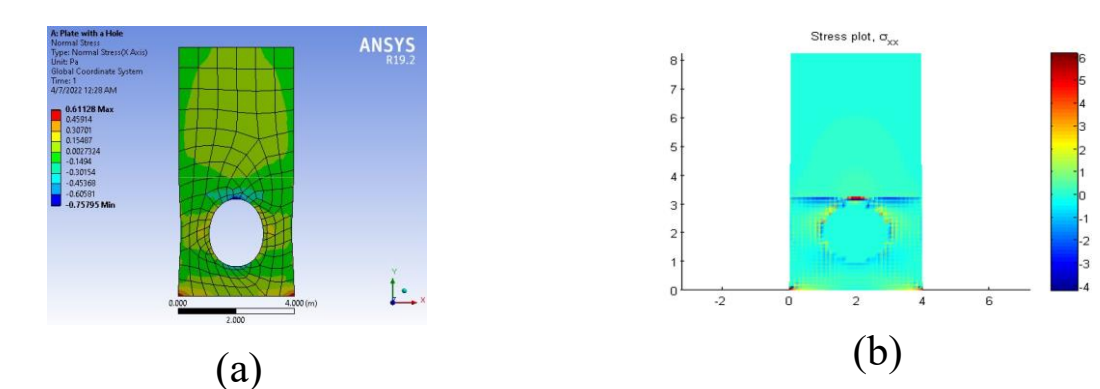


Figure 4. The results stress σ_{xx} : a) by FEM ;b) by XFEM

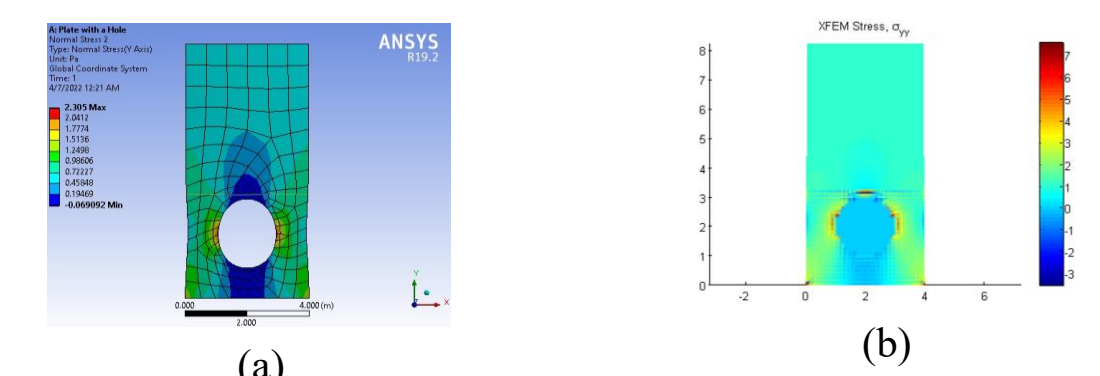


Figure 5. The results stress σ_{yy} : a) by FEM ;b) by XFEM

Table 1. Comparing the displacement results between XFEM and FEM from Fig. 3

	FEM	XFEM	%Error
Max(u_y)	0.00059997(m)	0.0005539 (m)	$\frac{0.00059997-0.0005539}{0.00059997} \cdot 100\% = 7.43\%$

Table 2. Comparing the stress σ_{xx} results between XFEM and FEM from Fig. 4

	FEM	XFEM	%Error
Max(σ_{xx})	0.61128(Pa)	0.59110 (Pa)	$\frac{0.61128-0.59110}{0.61128} \cdot 100\% = 3.30\%$

Table 3. Comparing the stress σ_{yy} results between XFEM and FEM from Fig. 5

	FEM	XFEM	%Error
Max(σ_{yy})	2.305(Pa)	2.105 (Pa)	$\frac{2.305-2.105}{2.305} \cdot 100\% = 8.68\%$

The results stress concentration factors (SCF) of a hole near boundary of two materials show in Eq.(7). SCF by FEM $SCF_{FEM} = 2.305$, by XFEM $SCF_{XFEM} = 2.105$, and %Error SCF by FEM and XFEM compare with exact method: $\text{Error}_{FEM} = 14.11\%$, $\text{Error}_{XFEM} = 4.19\%$. The results show that XFEM is a better method than the traditional FEM when compared with the exact method.

CONCLUSION

The contribution to this research was to study the stress concentration around the circular hole near the border of two isotropic plates. According to the analytical results of reference for an isotropic flat plate, there is a circular hole and is pulled at both ends. Calculating stress concentration for the circular hole of an isotropic flat plate with two materials joined together, this new method stress calculation approach gives satisfactory results for both XFEM and FEM methods compared with analytical results. In particular, the XFEM method gives very good errors compared to the analytical method.

REFERENCES

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- [2] Nguyen, H.-D.; Huang, S.-C. Using the Extended Finite Element Method to Integrate the Level-Set Method to Simulate the Stress Concentration Factor at the Circular Holes Near the Material Boundary of a Functionally-Graded Material Plate. *J. Mater. Res. Technol.* **2022**, *21*, 4658–4673.