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Mechanical Homogenization of TPMS Architectures: A Comparison between Finite Element and Mechanics of Structure Genome Approaches

S. Mouman[1,2], Y. Koutsawa[1], L. Binsfeld[1], L. Kirkayak[1], J. Yang[2], and G. Giunta[1]

[1]Luxembourg Institute of Science and Technology, 5 Avenue des Hauts-Fourneaux, L-4362 Esch-sur-Alzette, LU.

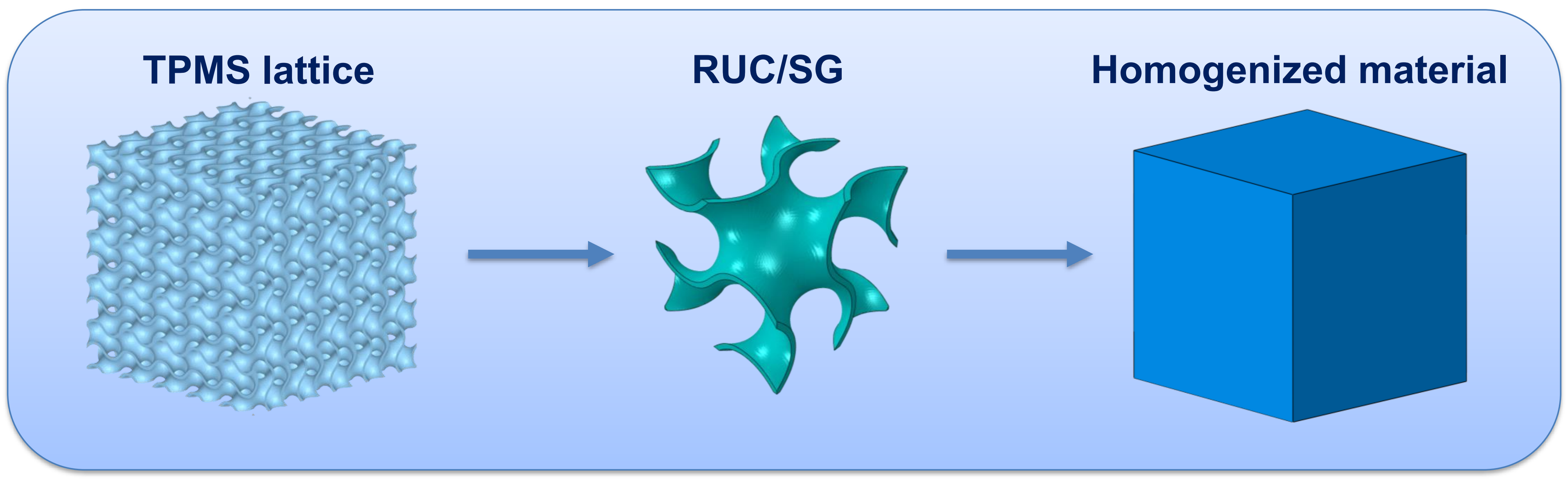
[2]Faculty of Mechanical Engineering, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.

INTRODUCTION & AIM

Triply Periodic Minimal Surface (TPMS)-based metamaterials have gained interest due to their tunable properties. However, accurately predicting their **effective mechanical behaviour** remains challenging. While **FEM homogenization** provides detailed insights, it can be computationally expensive. Another approach is that of the **Mechanics of Structured Genome (MSG)**, which offers a semi-analytical alternative. This study aims to demonstrate that MSG can predict the effective mechanical properties of TPMS metamaterials with FEM-level accuracy and higher computational efficiency.

METHOD

To find the effective mechanical properties of the heterogenous metamaterial, homogenization is carried on the **Representative Unit Cell (RUC)** for FEM and on the **Structure Genome (SG)** for MSG. They represent the smallest part of the lattice that contains all the necessary constitutive information.



FEM-based Homogenization

$$\chi_i(y_j^+) = \chi_i(y_j^-)$$

To represent an infinite periodic lattice, **Periodic Boundary Conditions (PBCs)** are applied.

$$\bar{\sigma}_{ij} = \langle \sigma_{ij} \rangle = \frac{1}{V}\int_V \sigma_{ij} dV$$

Six independent linear elastic simulations are solved, each applying a single macroscopic unit strain. The resulting **stress field** is **averaged** over the unit cell **volume**.

$$\bar{\sigma}_{ij} = C^*_{ijkl}\bar{\epsilon}_{kl}$$

Each averaged stress vector forms one column of the stiffness matrix, providing the **complete anisotropic elastic response** of the TPMS architecture.

MSG-based Homogenization

$$\left[C_{ijkl}\left(\bar{\epsilon}_{kl} + \chi_{k,l}\right)\right]_{,j} = 0$$

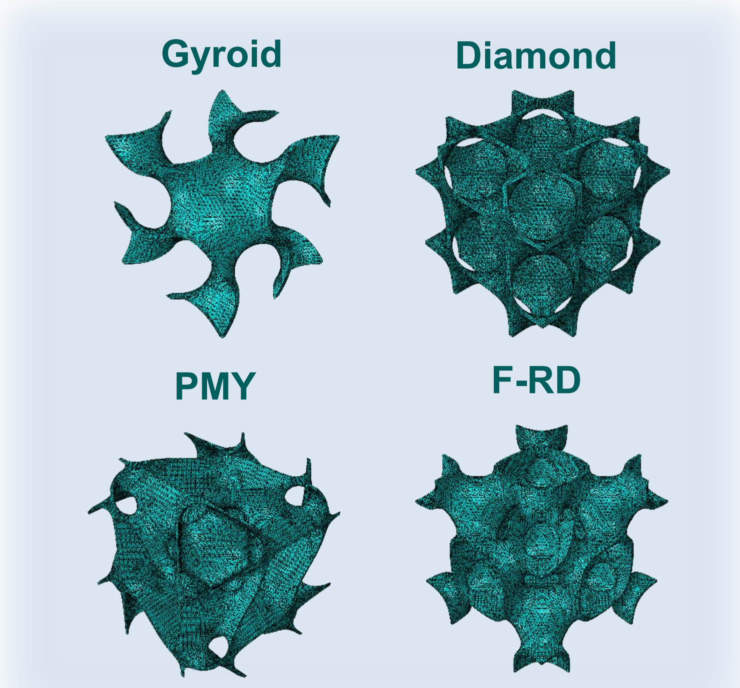
MSG is based on the **principle of minimum information loss**: it ensures that the homogenized model **preserves the strain energy**. PBCs are also applied.

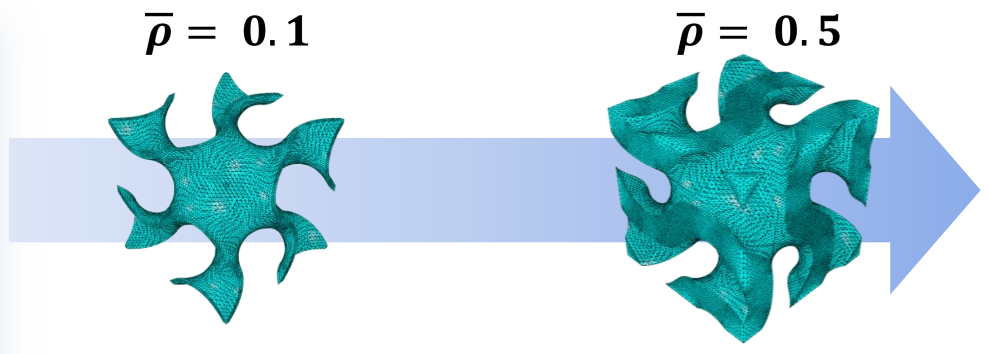
The homogenized **stiffness tensor** is obtained by **volume-averaging** the **strain energy density**.

$$C^*_{ijkl} = \langle C_{ijmn} + C_{ijkl}H^{mn}_{k,l} \rangle$$

MSG derives the stiffness tensor analytically from a **single, energy-consistent model** of the structure genome.

RESULTS & DISCUSSION

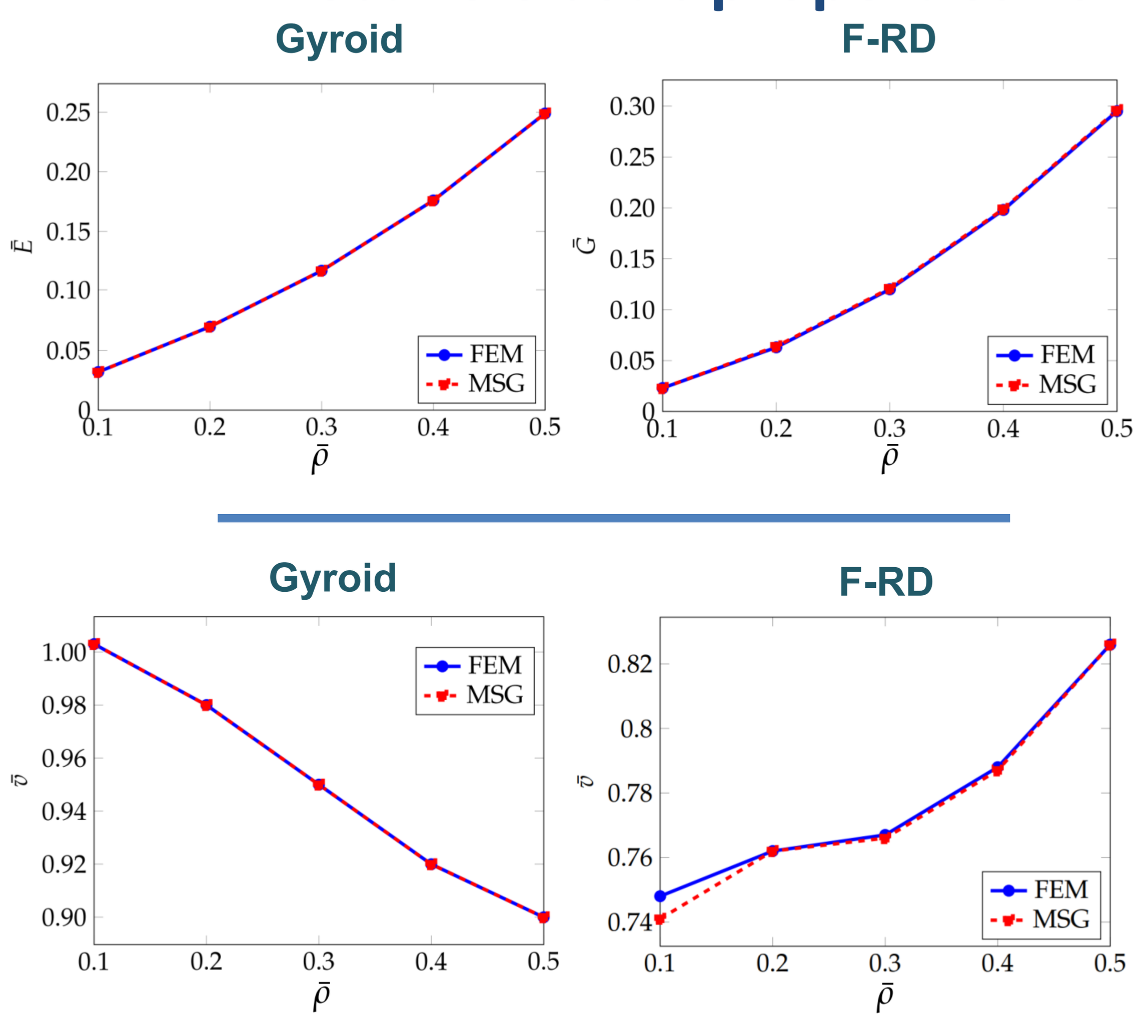




A comparative study was carried out on:

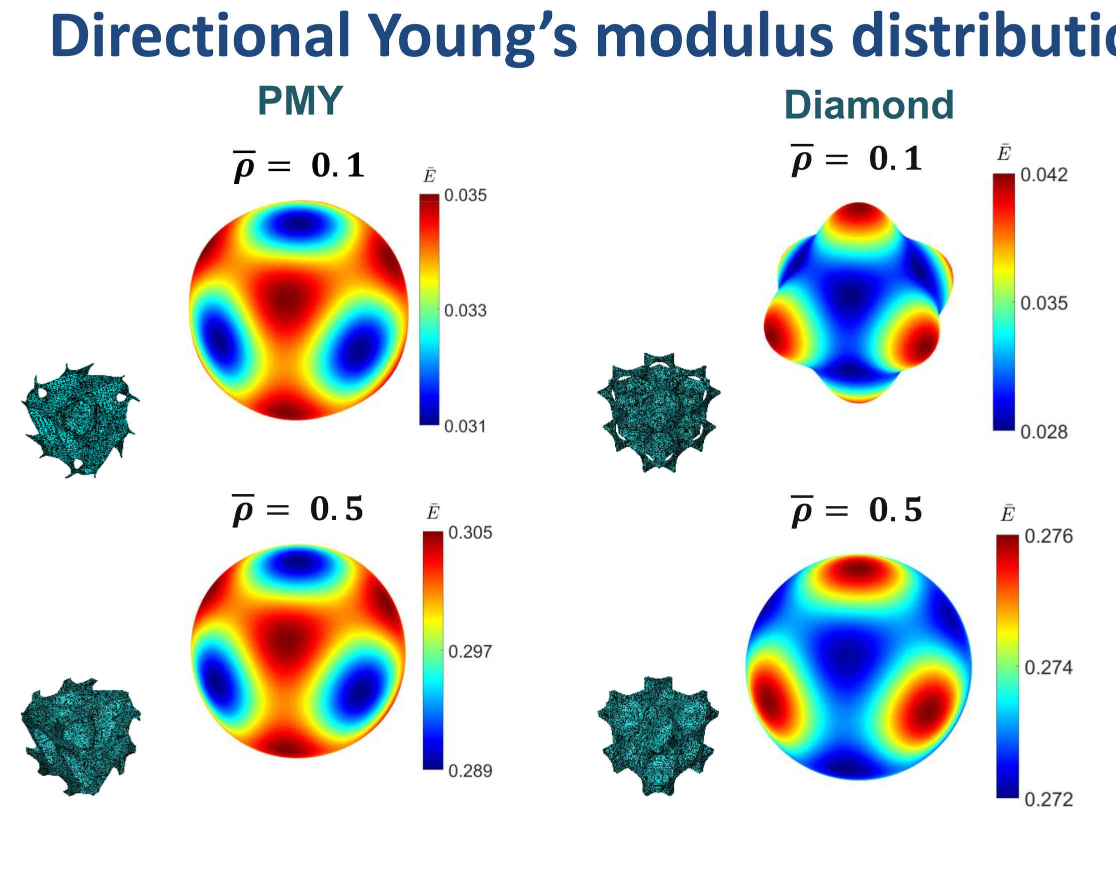
- Four **distinct TPMS** topologies
- Relative **densities ranging** from 10% to 50%

Effective elastic properties vs relative density



- Both Young's and shear moduli **increase** with relative density for all TPMSs.
- Except for F-RD, Poisson's ratio **decreases** at low densities, but grows again above 50%.
- The MSG method matches FEM results within 1% error.

Directional Young's modulus distribution vs relative density



- **Gyroid & PMY:** Highly isotropic across all densities; stiffest along the diagonal.
- **Diamond & F-RD:** Anisotropic at low densities; isotropy increases with relative density; stiffest along principal axes.

CONCLUSION

The mechanical properties of TPMS metamaterials depend strongly on both the topology and density of the representative unit cell. Mechanical homogenization using the FEM and MSG methods produced nearly identical results, confirming MSG as an efficient and reliable alternative to FEM.

FUTURE WORK / REFERENCES

Future work:

- Use MSG to study the thermal properties of metamaterials.
- Investigate other topologies.
- Validate simulation results through experiments.

References:

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- Koutsawa, Y.; Tiem, S.; Yu, W.; Addiego, F.; Giunta, G. A micromechanics approach for effective elastic properties of nano-composites with energetic surfaces/interfaces. Compos. Struct. **2017**, 159, 278–287.