

# DRT, diffusive representation and infinite state description: application to fractional behaviours analysis and modelling

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## INTRODUCTION & AIM

Given the various limitations and drawbacks now associated to fractional models and recently described in the literature, these models should be used with caution, only when justified or better yet, replaced by other classes of models.

This work thus proposes to use the DRT concept as:

- a model discriminator
- a new modelling tool for fractional behaviours.

## DRT DEFINITION

The first explicit integral formulation of Distribution of Relaxation Time (DRT), appeared in the 1940s

$$H(j\omega) = \int_0^{\infty} \frac{G(\tau)}{1 + j\omega\tau} d\tau$$

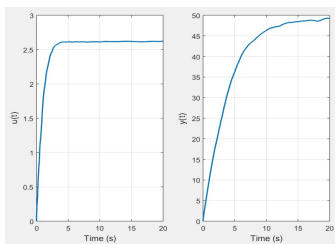
Relation linking the input signal  $u(t)$  and the output signal  $y(t)$  of a system

$$y(t) = \int_0^{\infty} G(\tau) \left( \frac{1}{\tau} \int_0^t u(t - \xi) e^{-\xi/\tau} d\xi \right) d\tau$$

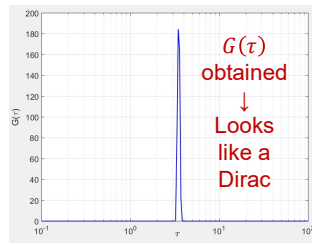
## DRT AS A MODEL DISCRIMINATOR

### Fractional modelling of a solar panel

Ebead R., Abo-Zalam B., Nabil E., System identification of photovoltaic system based on fractional-order model. Journal of Computational Electronics, Vol. 22, pp. 471–484, 2023

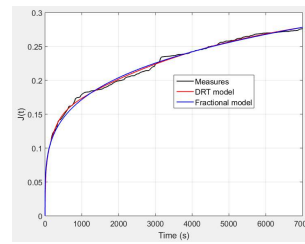


DRT

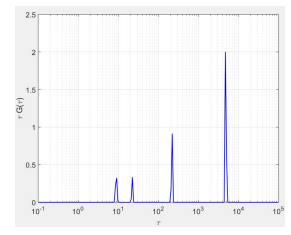


### Fractional modelling of sandstone under axial stress

Ding X., Zhang G., Zhao B., Wang Y., Unexpected viscoelastic deformation of tight sandstone: Insights and predictions from the fractional Maxwell model, Scientific Reports, Vol. 7, No 11336, 2017



DRT



✗ A fractional model is unnecessary, a first-order model is enough

✓ A fractional model leads to a small number of parameters but introduces several limitations.

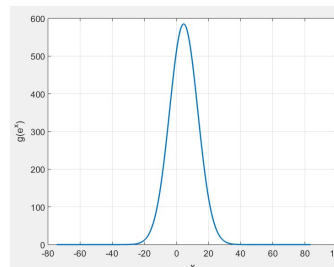
## DRT AS AN ALTERNATIVE TO FRACTIONAL MODELS FOR FRACTIONAL BEHAVIOURS

The models used are based on distribution of the logarithm of the time constants and defined by

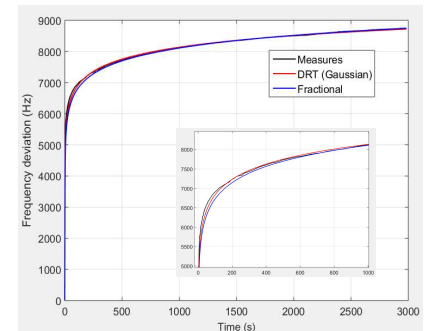
$$H(j\omega) = \int_{\tau_{min}}^{\tau_{max}} \frac{g(\tau)}{1 + j\omega\tau} d\ln(\tau)$$

where  $g(\tau)$  follows a symmetric log-normal distribution with three independent parameters ( $a, \mu, \sigma$ )

$$g(\tau) = ae^{-\frac{(\ln(\tau) - \mu)^2}{2\sigma^2}}$$



### Application to a gas sensor



**Interest in relation to fractional models :** same accuracy, same number of parameters, limitations of fractional models have been overcome, physical interpretation permitted with DRT.

## CONCLUSION

The work analyses the interest of Distribution of Relaxation Times (DRT) in the context of fractional dynamic or kinetic behaviours. It shows the link (not discussed here) between DRT and diffusive representations (and infinite-state formulation). It demonstrates on real data the interest of DRT to justify (or to avoid) a fractional modelling approach and as an alternative (with similar number of parameters and accuracy) to fractional models due to their limitations while allowing physical interpretations