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Modeling and Analysis of Fractional Maxwell Fluid in Intra-Articular Drug Injection Flow

Zifeng Wei, Yuyuan Yang, Chunyan Liu

School of Science, Beijing University of Civil Engineering and Architecture, Beijing 100044, China

Beijing Key Laboratory of Functional Materials for Building Structure and Environment Remediation, Beijing 100044, China

INTRODUCTION & AIM

Osteoarthritis (KOA) is a chronic degenerative joint disorder highly prevalent among middle-aged and elderly populations. It severely impairs joint function and substantially reduces quality of life. Intra-articular drug injection serves as a primary therapeutic intervention for KOA patients across all disease stages.

However, the efficacy evaluation of current injection-based therapies relies excessively on clinicians' empirical experience and patients' subjective symptom feedback, lacking objective evidence derived from fluid dynamic analyses of drug distribution and diffusion patterns within the joint cavity.

Thus, establishing research methodologies tailored to individual patient characteristics to analyze intra-articular drug distribution is critical for formulating precise injection treatment strategies and optimizing clinical efficacy.

METHOD

1. CT-based construction of a patient-specific knee joint cavity model for fluid flow simulation.

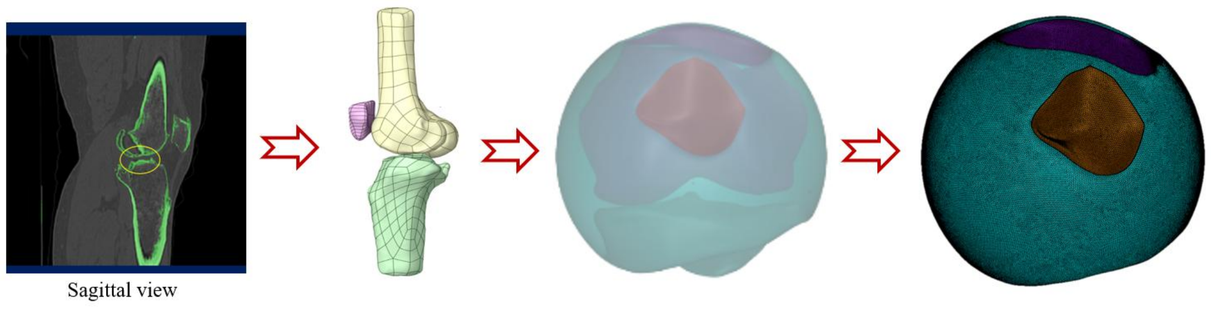

Figure 1. Schematic of the personalized knee joint cavity model construction
A. Patient CT imaging data.
B.3D bone-joint model.
C.3D joint cavity model.
D.Joint cavity outer mesh.

2. The injected drug exhibits non-instantaneous, memory-dependent viscoelastic relaxation behavior; therefore, the fractional-order Maxwell constitutive equation is adopted to accurately characterize its mechanical response.

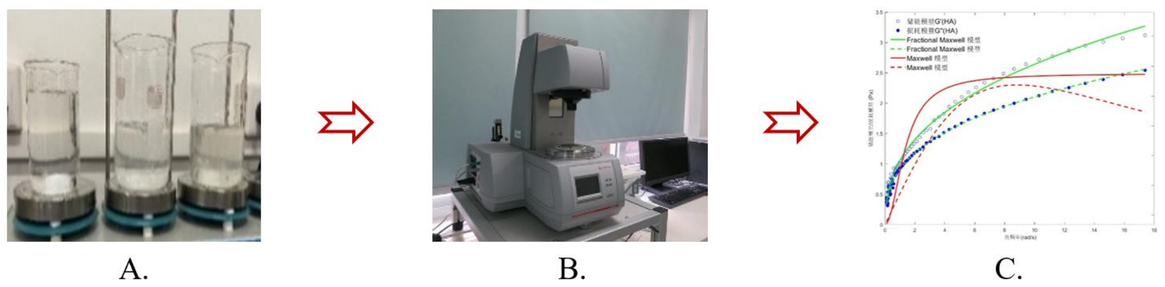

Figure 2. Schematic of the experimental workflow for rheological characterization of the injected drug.
A. Preparation of the injectable drug.
B. Rheological experiment of the injectable drug.
C. Rheological characterization using the constitutive equation.

3. This constitutive equation is incorporated into the hydrodynamic governing equations, which are then discretized and solved numerically using the finite volume method combined with the L1 algorithm. The L1 algorithm is given below:

$$D^{\alpha}\tau(t) \approx \frac{1}{\Gamma(2-\alpha)} \sum_{k=0}^{n-1} \left[(k+1)^{1-\alpha} - k^{1-\alpha} \right] \frac{\tau(t_{n-k}) - \tau(t_{n-k-1})}{\Delta t^{\alpha}} \qquad (1)$$

RESULTS & DISCUSSION

Numerical simulations systematically investigate the effects of characteristic relaxation time and fractional order on intra-articular drug distribution.

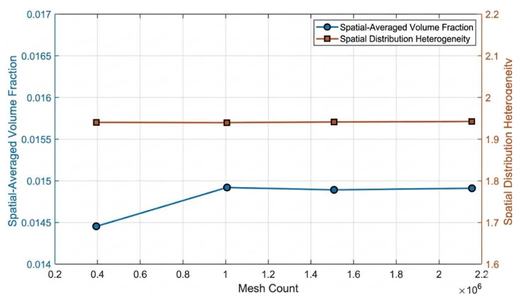

Figure 3. Grid independence test

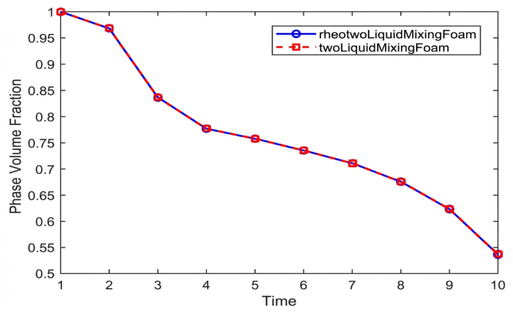

Figure 4 Validation of the rheotwoLiquidMixingFoam

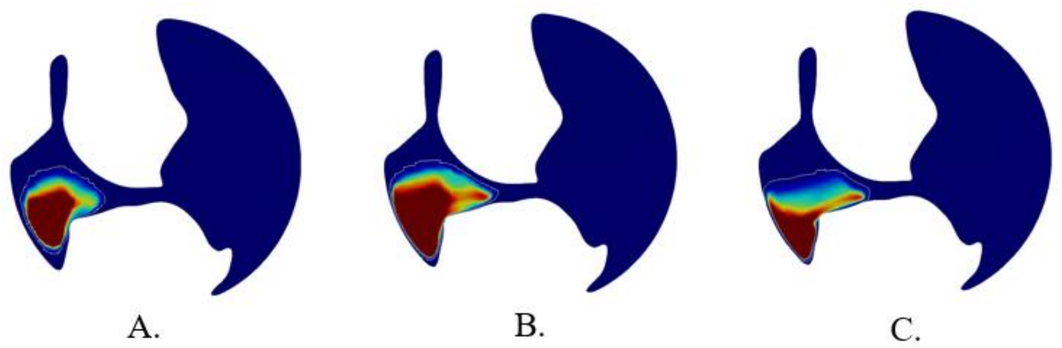

Figure 5. Sagittal section views highlighting the patellofemoral region. (A) Distribution of the injected drug at 5 seconds post-injection; (B) Distribution at 15 seconds post-injection; (C) Distribution at 40 seconds post-injection.

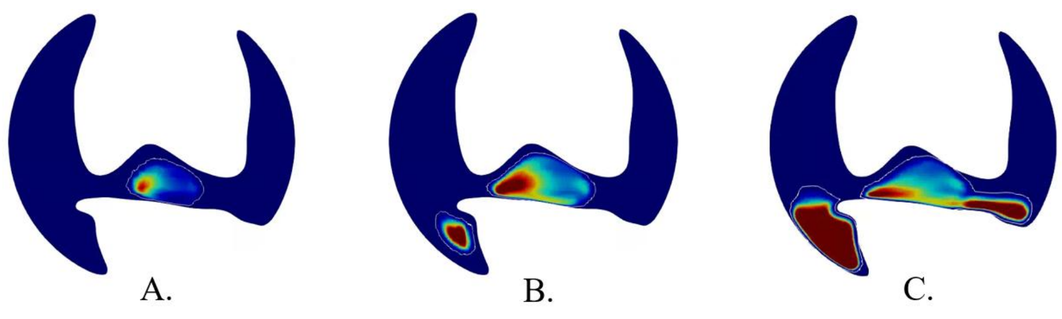

Figure 6. Coronal section views highlighting the tibiofemoral region. (A) Distribution of the injected drug at 5 seconds post-injection; (B) Distribution at 15 seconds post-injection; (C) Distribution at 40 seconds post-injection.

CONCLUSION

- The grid independence test and validation of the self-compiled *rheotwoLiquidMixingFoam* solver collectively demonstrate that the established fractional-order viscoelastic fluid numerical model can accurately simulate the transport and distribution of the injected drug within the knee joint cavity.
- The injected drug exhibits distinct spatiotemporal distribution patterns between the patellofemoral and tibiofemoral regions: it initially accumulates near the injection site, then gradually diffuses toward the peripheral regions of the joint cavity over time, ultimately achieving widespread coverage of the cavity.

FUTURE WORK / REFERENCES

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