IOCFF 2026 Conference
The 1st International Online Conference on Fractal and Fractional
13–15 April 2026 | Online
MDPI

Analysis of Multivariable Control Loops in Distillation Columns Using Variable-Order Fractional Calculus

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INTRODUCTION & AIM

The study addresses the complexities of **Multi-Input Multi-Output (MIMO)** control in industrial processes, specifically focusing on systems with cross-coupling control loops and time delays.

- **Objective:** To design and evaluate a multivariable control strategy for a binary distillation column (the Wood & Berry model).
- **Goal:** To manage/reduce loop interaction and provide more robust performance than traditional integer or constant-order controllers by utilizing **variable-order fractional calculus**.

METHOD

- **Model:** The classical Wood & Berry distillation column model, relating reflux flowrate R(s) and reboiler heat (St(s)) to distillate (XD(s)) and bottoms (XB(s)) compositions.

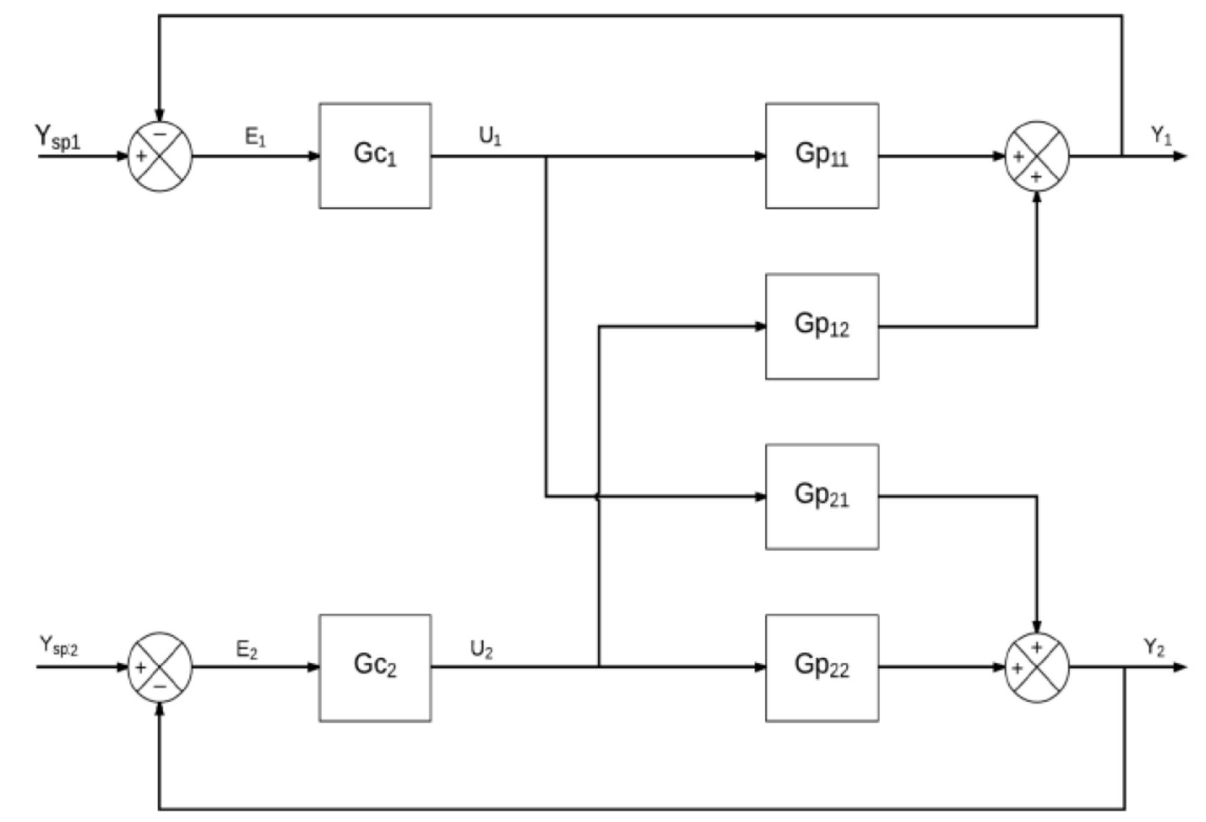
$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12{,}8 \cdot e^{-s}}{16{,}7 \cdot s+1} & \frac{-18{,}9 \cdot e^{-3 \cdot s}}{21 \cdot s+1} \\ \frac{6{,}6 \cdot e^{-7 \cdot s}}{10{,}9 \cdot s+1} & \frac{-19{,}4 \cdot e^{-3 \cdot s}}{14{,}4 \cdot s+1} \end{bmatrix} \cdot \begin{bmatrix} R(s) \\ St(s) \end{bmatrix}$$

- **Controller Design:** Fractional Order PI controllers (Gc1 and Gc2) based on the **Scarpi variable-order fractional integral**.
- **Adaptive Strategy:** The fractional order alpha(t) is not static; it is modeled as a **time-varying exponential function** to adapt the controller's aggressiveness dynamically. More especifically, alpha(t)=a1+(a1 - a2)*exp(-c*t).
- **Computation:** To solve the Laplace domain equations in the time domain, the **de Hoog et al. numerical inversion method** was used via the Python MpMath library.
- **Optimization:** Parameters were tuned by minimizing the **Integral of Squared Error (ISE) of both loops**.

RESULTS & DISCUSSION

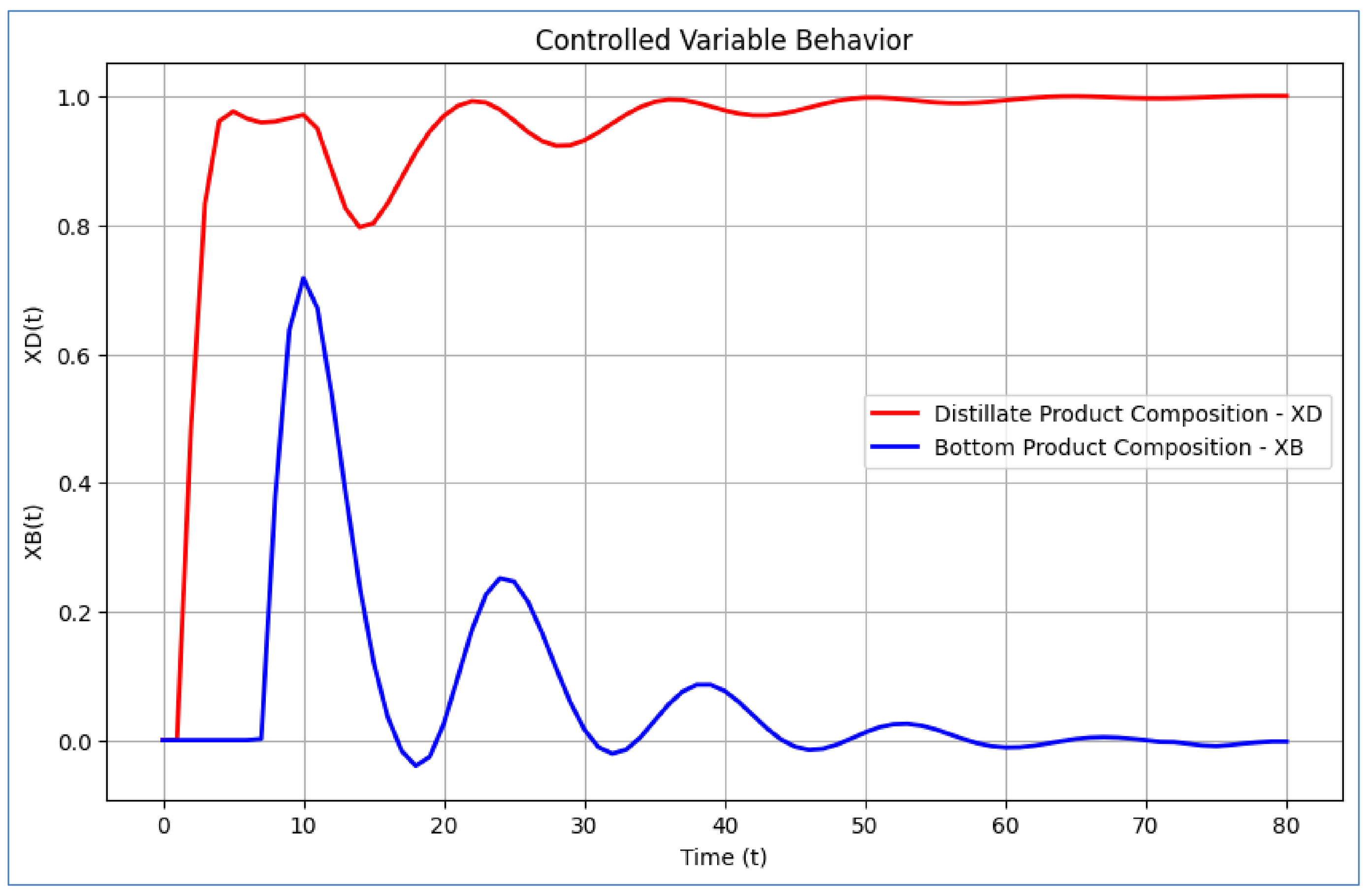
$$FOBJ = \int_0^\infty e_1(t)^2\,dt + \int_0^\infty e_2(t)^2\,dt = \int_0^\infty (y1(t) - y_{sp1}(t))^2\,dt + \int_0^\infty (y2(t) - y_{sp2}(t))^2\,dt$$

1,2 : NUMBER OF THE CONTROL LOOP, XD or XB composition control;
$e_i(t)$: error of the controlled Variable in loop i
y : Controlled Variable ; y_{sp}: Setpoint of the controlled variable



The variable-order approach demonstrated regulatory capabilities:

- **Performance Metric:** The optimization yielded a total objective function (FOBJ) value of **4.04**.
- **Efficiency:** The system achieved a settling time of approximately **60 time units**.
- **Key Outcome:** The strategy successfully managed the loops interaction.



CONCLUSION

The variable-order fractional controller is an effective tool for robust disturbance rejection and manage decoupling in complex industrial MIMO systems.

FUTURE WORK / REFERENCES

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