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Fractional Calculus for Space-Fractional Thermoelasticity

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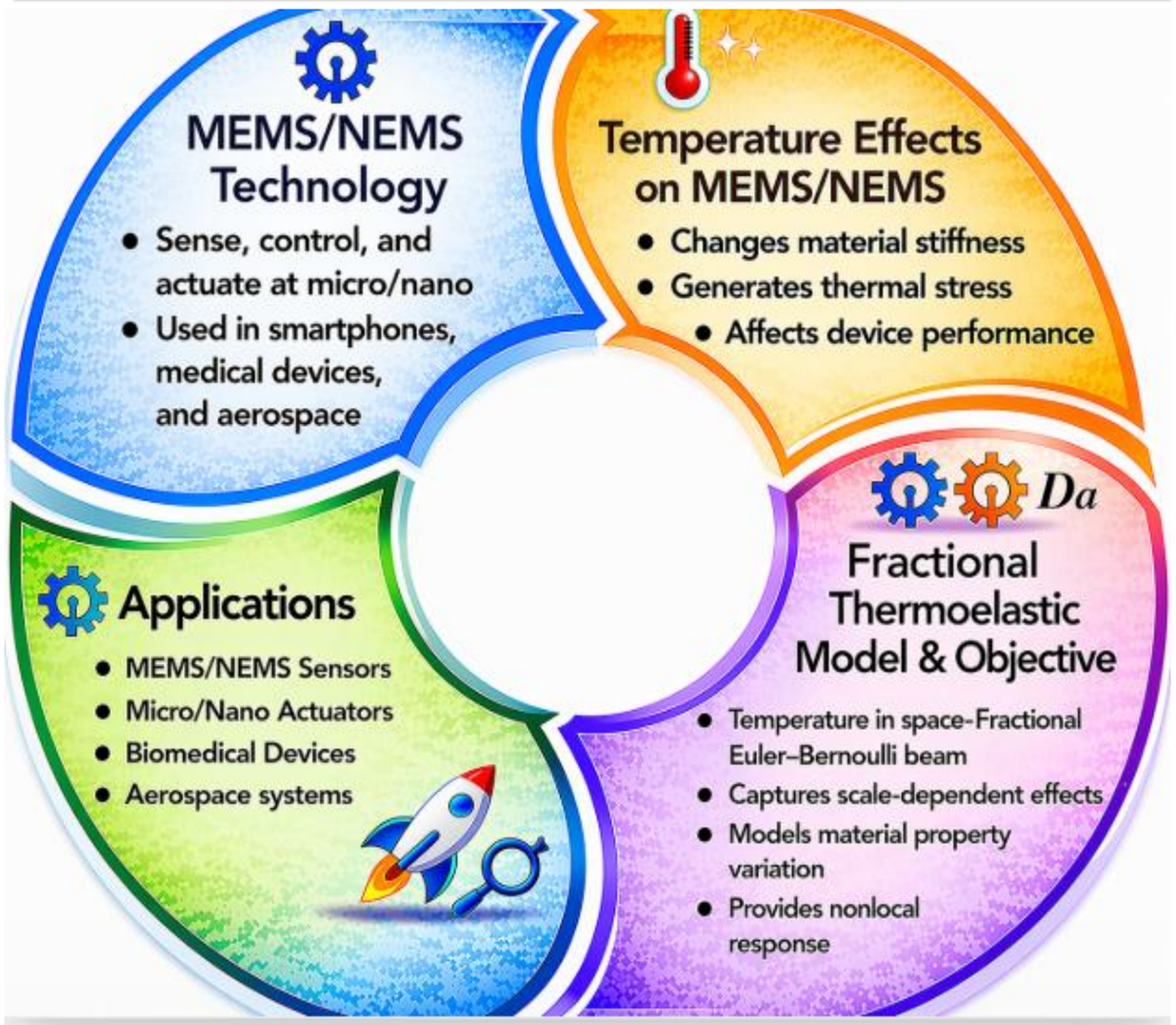
INTRODUCTION & AIM



Method

Space-fractional Euler–Bernoulli beam [1] is extended by including **temperature-dependent parameters** $\boldsymbol{E(T), \alpha(T)}$ **and** $\boldsymbol{l_f(T)}$**:**

$$D_x^{\alpha}\{l_f^{\alpha-1}D_x^{\alpha}[l_f^{2\alpha-2}D_x^{\alpha}(l_f^{\alpha-1}D_x^{\alpha}\boldsymbol{u})]\}EI = p(x)$$

Special Case:

For $\alpha = 1$, the model reduces to the classical Euler–Bernoulli beam:

$$\frac{d^4u}{dx^4}\mathbf{EI} = \mathbf{p(x)}$$

Boundary Conditions:

Propped cantilever beam:

$$\begin{cases} u_0(x_0)=0,\ \varphi_0(x_0)=0, \\ u_n(x_n)=0,\ M_n(x_n)=0 \end{cases}$$

Solution Approach:

1. Finite Difference Method.
2. Trapezoidal rule [1] (for approximating Caputo derivative).

RESULTS & DISCUSSION

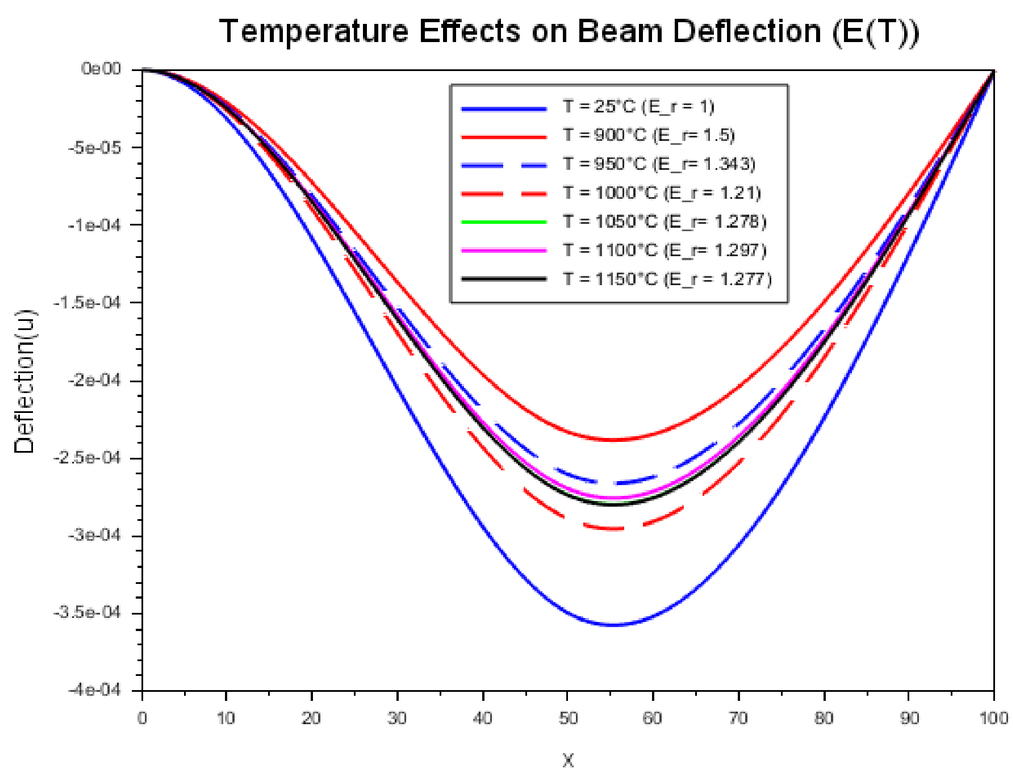


Figure 1: The effect of increasing temperature on deflection, with varying Young's modulus E(T) while the grain size remains constant.

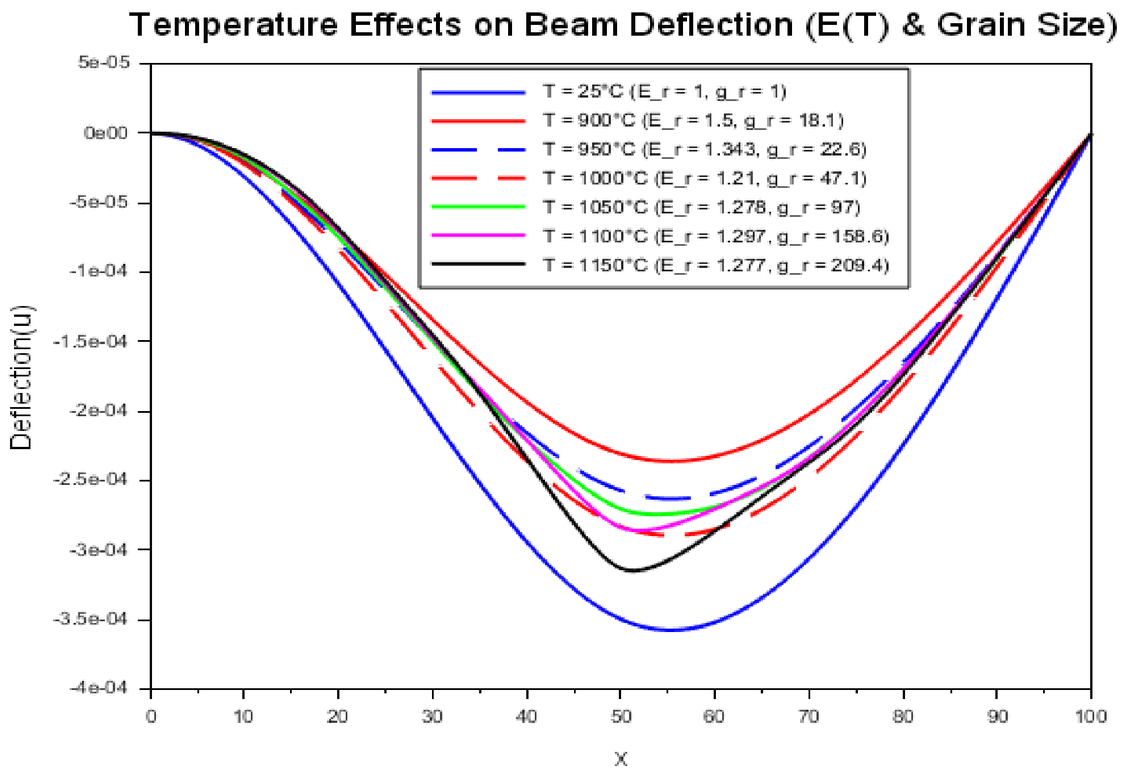


Figure 2: The effect of increasing temperature on deflection, with varying Young's modulus E(T) and grain size.

CONCLUSION

- For a constant grain size, an increase in temperature leads to a reduction in Young's modulus, which results in enhanced beam deflection.
- Moreover, deflection increases further due to the combined influence of decreasing Young's modulus and increasing grain size with rising temperature.

REFERENCES

1: Stempin P, Sumelka W. Space-fractional Euler–Bernoulli beam model – Theory and identification for silver nanobeam bending. Int J Mech Sci 2020;186:105902.