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Baryon Asymmetry as a Stochastic Result and Implications for the Time of Baryogenesis

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Abstract: We try to explain the known asymmetry between baryons and antibaryons in the observable universe as the result of random fluctuations in the number of baryons and anti-baryons within the particle horizon. The establishment of this asymmetry occurs at a poorly constrained time, before the epoch of baryon/anti-baryon annihilation between 10^{-32} and $10^{-1°}$ s. The low initial gravitational entropy of the universe is due to this initial matter/anti-matter asymmetry. The observed ratio of cosmic microwave background photons to baryons (~ ten billion) is a measure of this asymmetry. Different horizons render different probability distributions for the baryon asymmetry, all variants of a Gaussian centered at zero. Using the current level of baryon asymmetry as an estimate of the one sigma width of the Gaussian, we estimate the time of baryogenesis to be on the order of $10^{-1°}$s, near the time of reheating, within established constraints.

Keywords: baryogenesis; cosmology; stochastic process

1. Introduction

Baryon asymmetry has escaped conclusive explanation since the prediction and experimental verification of antimatter's existence. Standard models of particle physics predict the production of equal amounts of matter and antimatter, though this contradicts observation [1]. In an inflationary scenario, baryogenesis occurs at a poorly constrained energy, between 100 GeV (electroweak scale) $< E < 10^{12}$

GeV (sphaleron processes) [2]. These energy levels constrain the time of known baryogenesis models between the end of inflation and the electroweak phase transition, $10^{-32} < t < 10^{-10}$s. The baryon asymmetry parameter is

$$\eta = \frac{N_B - N_{\bar{B}}}{N\gamma} \approx 6 * 10^{-6} \tag{1}$$

As one averages baryon asymmetry (eq. 1) over larger volumes, does total baryonic charge average to zero [3]?

$$B = \int_V \eta \, d^3x = 0 \tag{2}$$

If it does, what might a probability density plot of eq. (1) for any particular volume look like? This question envisions random fluctuations in eq. (1) as baryon/antibaryon levels vary in a volume V_1 existing within an arbitrarily larger volume over which eq. (2) holds (Figure 2). We expect it to be centred at $\eta = 0$, where averaging η over larger horizon volumes reveals global symmetry. The conceptual models in Figure 1 disagree about eq. (2). If there is some 'global' baryon symmetry, looking at a number of distinct horizon volumes should reveal a Gaussian distribution function of η. If there is some fixed global *asymmetry*, then we might expect delta functions at the observed η or at a different η for different inflationary bubbles. A distinct but related question is concerned with the size of of matter/antimatter dominated regions and whether they are fit for observed large-scale structure formation. Omnes has attempted to answer these questions while assuming a globally baryon-symmetrical universe [4]. His mechanism of baryongenesis is a phase separation between matter and antimatter at temperatures of $10^{12} - 10^{13}K$ to create distinct matter/antimatter dominated regions that partially annihilate but go on later to coalesce into structures roughly the size of galaxies, with the expected baryonic asymmetry to be $\eta \sim 10^{-12} - 10^{-8}$. We use a simpler mechanism to ensure baryogenesis in random η fluctuations within a commoving volume. The following analysis makes use of the time-dependent baryonic density of the observable universe, and the changing size of the horizon volume. We expect the observed η to be within a bound that demarcates a statistical fluctuation. Hence we allow the observed asymmetry to equal σ of the derived (time-dependent) probability density function $\Pr(\eta)$. From this we find the time t_0 at which η for our current horizon became fixed if baryon asymmetry was a stochastic result.
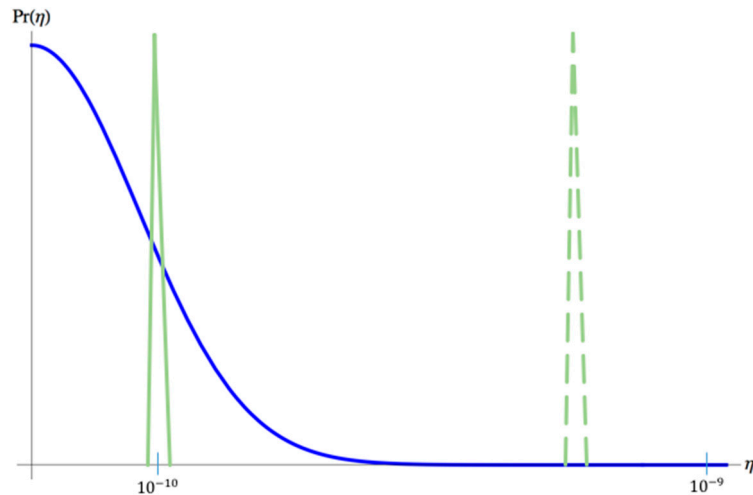
Figure 1. (a) At the heart of this analysis is the question of whether eq. (2) holds or not. If it does, then the result is (i) blue: a statistically expected distribution of η for a given horizon size in a baryon-symmetric universe, and if not, then (ii) green: delta functions at either the observed asymmetry of $\sim 10-1°$ or at another asymmetry (green dashed) in a horizon volume distinct from ours in the same inflationary bubble.
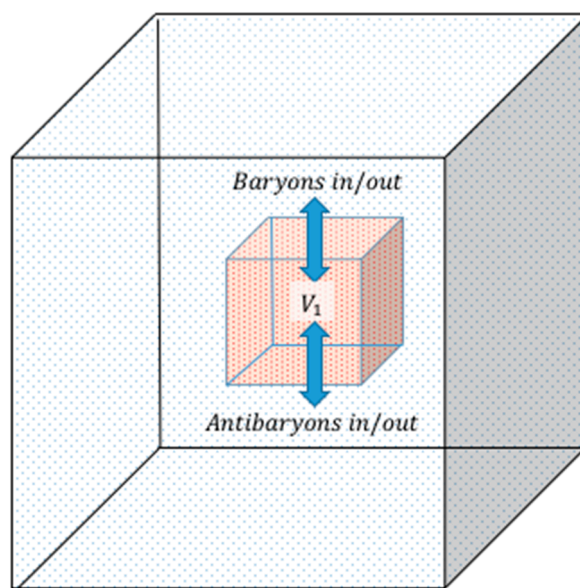

Figure 2. The asymmetry parameter described by Eq. (1) fluctuates, or 'randomly walks' in the sub-volume V_1 around a global baryon-symmetric value of $\eta = 0$. V_1 represents the current size of our particle horizon whose baryon asymmetry is postulated to be a stochastic result in a globally baryon-symmetric universe.

1.1. Observational constraints

Observed baryon asymmetry is calculated under the ΛCDM model by either observing abundances of light elements in the intergalactic medium (IGM), or from temperature fluctuation spectrum in the cosmic microwave background (CMB). Each measure baryon asymmetry at different points in the history of the universe, and serve to qualify the constancy η. The observe range of value varies between

$5.80 - 6.20 \times 10^{-10}$ [5]. The AMS (Alpha Magnetic Spectrometer) datasets constrain models which posit large amounts of antimatter in the observable universe, either as a homogeneous matter-antimatter mixture, or as distinct co-existing matter and anti-matter dominated regions of space. AMS-01 results failed to find antihelium neutrons with an upper limit of accuracy of $\sim \frac{N_{\overline{He}}}{N_{He}} = 10^{-6}$ [6]. Large-scale antimatter structures are unobserved and 'large' antimatter deposits greater than those synthesized at CERN are unobserved in our Solar System.

2. Results and Discussion

Consider a horizon volume V_1 existing within a larger volume V (Figure 1). Flipping a two sided coin, let a heads result in the addition of a baryonic particle to V_1; tails, the addition of an antibaryonic particle. The number of flips of the coin N will refer to the total number of particles in the volume itself, since each flip adds one particle:

$$N = \rho(a(t)) \cdot V_1(a(t)) \tag{3}$$

where $\rho(a)$ and $V_1(a(t))$ is the scale-factor dependence density and particle horizon volume.
As flips are undertaken, we expect eq. (4) to undertake a random walk, either increasing or decreasing by one depending on the flip. The stochastic fluctuations described in Figure 1 are represented by the 'randomness' of the coin flips. The statistics of a random walk are well known [7].

$$\xi = N_B - N_{\bar{B}} \tag{4}$$

Given that $\eta = \frac{\xi}{N_\gamma}$, the probability density function will look different [8]. More specifically, $N_\gamma = 411 \; cm^{-3}$. In the coin-flipping situation described above, ξ effectively undergoes a random walk around an initial point of zero (Figure 2). Given that both heads and tails are equally likely, we expect the average value of ξ (and hence η) to be zero for large enough N. The probability of encountering a particular value of ξ for a given value of N is given by the binomial distribution:

$$P_\xi = \frac{N!}{\left[\frac{1}{2}(N + \xi)\right]! \left[\frac{1}{2}(N - \xi)\right]!} \left(\frac{1}{2}\right)^N \tag{6}$$

The normalised normal approximation to the binomial distribution is

$$P_\xi \cong \sqrt{\frac{1}{2\pi N}} e^{-\left(\frac{\xi^2}{2N}\right)} \tag{7}$$

Changing parameters from ξ to η, by using

$$P_\eta(\eta)d\eta = P_\xi(\xi)d\xi \tag{8}$$

where $P_\eta(\eta)$ and $P_\xi(\xi)$ are probability density functions and $\eta(\xi)$ is well-defined.

$$P_\xi \cong N_\gamma \sqrt{\frac{1}{2\pi N}} e^{-\left(\frac{\eta^2 N_\gamma^2}{2N}\right)} \tag{9}$$

The time-dependence of the PDF can now be included. In eq. (2) density of dust $\rho(a) \sim \frac{1}{a(t)^3}$ where we assume that baryogenesis occurs between the end of inflation and the beginning of reheating where the scalefactor $a(t) \sim t^{\frac{2}{3}}$ is of a matter-dominated era [8]. particle horizon volume is given by

$$V(t) = a(t)c \int_0^t \frac{dt'}{a(t')} = 3 \cdot c \cdot t \tag{10}$$

$\Omega_m = \frac{\rho(a(t))}{\rho_c} = \frac{0.27}{a^3(t)}$ where critical density $\rho_c = \frac{3H_0^2}{8\pi G}$ and $H_0 \sim 10^{-10}$, meaning

$$\rho(a(t)) = 0.27 \cdot \frac{\rho_c}{a^3(t)} \Rightarrow \rho(t) \approx \frac{10^{-31}}{t^2}$$

Combining these results into a time-dependent number of flips N:

$$N \approx \frac{10^{-23}}{t}$$

Eq. (8) becomes

$$P_\xi \cong N_\gamma \sqrt{\frac{10^{23} t}{2\pi}} e^{-\left(\frac{10^{23} \eta^2 t \cdot N\gamma^2}{2}\right)} \tag{11}$$

Letting one standard deviation $\sigma = 6 * 10^{-10}$ – the postulate that the observed asymmetry $\eta \sim 6 * 10^{-10}$ is within acceptable bounds of our probability density function (a result of baryon/antibaryon fluctuations) the time of must be set to:

$$t_0 \sim 10^{-10} \, s$$

3. Conclusions

We use a model based on random fluctuations of baryon/antibaryon levels in a given horizon volume in a globally baryon-symmetric universe to compute the time at which our present baryon asymmetry $\eta \approx 6 \times 10^{-10}$ becomes fixed. This time is on the order of $10^{-10} \, s$, at the upper limit the previously constrained range of $10^{-32} < t < 10^{-1 \circ}$ s.

Conflicts of Interest

The authors declare no conflict of interest.

References and Notes

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