Average Path Profile of Atmospheric Temperature and Humidity Structure Parameters from a Microwave Profiling Radiometer

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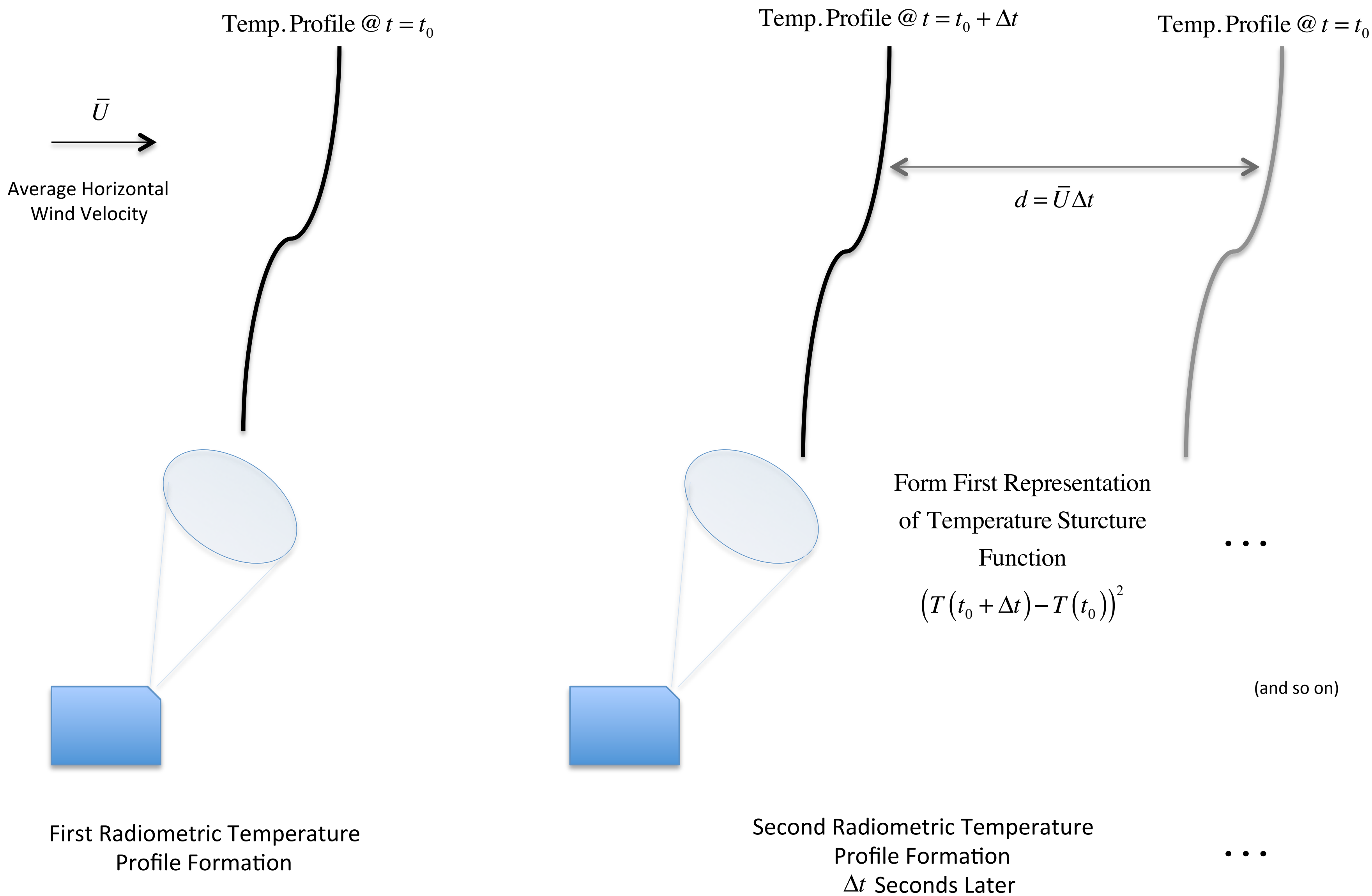
The problem examined is essentially this: Can a microwave profiling radiometer be used (with a 40 second integration time) to form structure functions of the atmospheric temperature and water-vapor fields from which the associated structure parameters C_T^2 and C_Q^2 can be obtained?

Introduction



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Introduction (cont'd)

- After N such representation formations, perform a moving average to get the temperature structure function

$$D_T\left(\bar{U}\Delta t\right) = \frac{1}{N}\sum_{i=0}^{N}\left(T\left(t_i + \Delta t\right) - T\left(t_i\right)\right)^2$$

- Can $D_T\left(\bar{U}\Delta t\right)$ be used to find the corresponding C_T^2 using something similar to the 2/3 Law, i.e., $D_T\left(\bar{U}\Delta t\right) = C_T^2\left(\bar{U}\Delta t\right)^{2/3}$ even though $\Delta t \sim 40\text{ sec}$ and so $\bar{U}\Delta t \geq 200\text{ m}$?

- Characteristic turbulence sizes (~ 200 m) and acquisition times (~40 sec) are not conventional quantities to prevail within the inertial sub-range of the turbulence spectrum in which the 2/3 law holds.

- Can a theory for large-scale atmospheric turbulence within, e.g., the buoyancy sub-range, be obtained from which one can obtain a similar law

$$D_T\left(\bar{U}\Delta t\right) = C_T^2 F\left(\bar{U}\Delta t\right)$$

where the function $F\left(d\right)$, $d = \bar{U}\Delta t$, replaces the 2/3 law ?

The derivation of this constitutes much of the text since it forms the fundamental basis of the entire remote-sensing method

Turbulence Theory for the Basis of Radiometer Profiling of Structure Parameters

A 'Back-to-Basics" theory is developed for large-scale turbulence (ignoring molecular diffusion effects) that gives various turbulent spectra $\Phi_{TT}(x')$ for a passive additive (in this case temperature but also can be water vapor concentration) as a function of atmospheric stability and buoyancy:

$$4x^4 = \left\{1 \pm \frac{\left(K\Gamma_U^2\right)^{1/2}}{2}\left(1 + \frac{K\Gamma_U^2}{4} + KM^{1/2}\right)^{-1/2}\right\}\frac{1}{K^3}\left(1 + \frac{M^{-1/2}}{2}H_T\right),$$

$$H_T \equiv 1 \pm \left(K\Gamma_T^2\right)^{1/2}\left(1 + \frac{K\Gamma_T^2}{4}\right)^{-1/2}$$

Use upper sign for $d\bar{U}/dz > 0$
Use lower sign for $d\bar{U}/dz < 0$

Use upper sign for $d\bar{T}/dz > 0$
Use lower sign for $d\bar{T}/dz < 0$

where

$$\Gamma_U = \left|\frac{d\bar{U}}{dz}\right|\left(bN\beta^2\right)^{-1/2}\varepsilon^{1/2}, \qquad \Gamma_T = \left|\frac{d\bar{T}}{dz}\right|\left(N\beta\right)^{-1}\varepsilon$$

introduces the prevailing wind shear and lapse rate of the atmosphere.

The parametric equation given above is then solved, within specific approximations, for the kinematic viscosity $K = K(x)$ from which one forms

$$\frac{dK(x)}{dx} = -\frac{1}{2K(x)x^2}\Phi(x)$$

to obtain the spectrum $\Phi(x)$ of velocity fluctuations and

$$2x^2\Phi_{TT}(x) = \frac{1}{4K^3x^2}\left(1 + H_T\right)\Phi(x)$$

to get an expression for the associated spectrum of temperature (or humidity) fluctuations $\Phi_{TT}(x)$

This general model of large-scale turbulence can be applied to many atmospheric scenarios involving stable as well as unstable cases.

Only two extreme cases are considered here but others should be examined to attempt to capture spectral transitions as atmospheric conditions evolve.

Turbulence Theory … (cont'd)

Backup for Previous Vu-Graph

$$x = \frac{k}{k_0}, \qquad \Phi = \frac{\phi}{\phi_0}, \qquad \Phi_{TT} = \frac{\phi_{TT}}{\phi_{TT,0}}$$

are dimensionless variables and functions where

$$k_0 = \gamma^{1/2}\left(bN\beta^2\right)^{3/4}\varepsilon^{-5/4}, \quad \phi_0 = \gamma^{-3/2}\left(bN\beta^2\right)^{-5/4}\varepsilon^{11/4}, \quad \phi_{TT,0} = \gamma^{-3/2}b^{-9/4}N^{-1/4}\beta^{-5/2}\varepsilon^{7/4}$$

ε is the energy dissipation due to viscosity
N is the dissipation of temperature fluctuations by thermal conductivity
$\beta \equiv g/\overline{T}$ is the buoyancy parameter
b is the ratio (~1) of thermal diffusivity to kinematic viscosity
γ a constant (~1) entering into the spectrum of the kinematic viscosity
k_0 characteristic spatial frequency
ϕ_0 characteristic spectral amplitude of velocity fluctuations
$\phi_{TT,0}$ characteristic spectral amplitude of temperature fluctuations

Turbulence Theory … (cont'd)

Look at two extremes:

1) $K\Gamma_T^2 << 1, K\Gamma_U^2 << 1$; ***No atmospheric stratification or shear***

NOTE: Other intermediate cases could be considered but, for purposes of a demo of the possibilities of using a microwave microwave profiling radiometer for turbulence sensing, these two extremes are a good initial point from which to begin.

Solving the parametric equation in the approximation $H_T \approx 1$ and $M \approx K^{-1}$

$$K(x) \approx \left(\frac{1}{4}\right)^{1/3} x^{-4/3}, \quad \Phi(x) \approx \left(\frac{8}{3}\right)\left(\frac{1}{4}\right)^{2/3} x^{-5/3}, \quad \Phi_{TT}(x) \approx \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)^{-1/3} x^{-5/3}$$

2) $K\Gamma_T^2 >> 1, K\Gamma_U^2 >> 1$; ***Significant atmospheric stratification and shear***

Solving the parametric equation in the approximation $H_T \approx 4/\left(K\Gamma_T^2\right) - 1$ and $M \approx K^{-1}$

$$K(x) \approx \left(\frac{1}{2\Gamma_U^2}\right)^{1/4} x^{-1}, \quad \Phi(x) \approx 2\left(\frac{1}{2\Gamma_U^2}\right)^{1/2} x^{-1}, \quad \Phi_{TT}(x) \approx \left(\frac{1}{2\Gamma_U^2}\right)^{-1/2} \left(\frac{1}{\Gamma_T^2}\right) x^{-1}$$

Form a combined expression that convolves both instances above for the spectrum of temperature fluctuations

Combined spectrum containing 1) and 2) as limiting cases to get this form (See text) →

$$\phi_{TT}(k) \approx \left(\frac{B}{k}\right)\frac{k^{1/3} + k_U^{1/3}}{k + k_T}, \qquad k_U \equiv \left(\frac{C}{2B}\right)^3, \qquad k_T \equiv \frac{C}{2A}$$

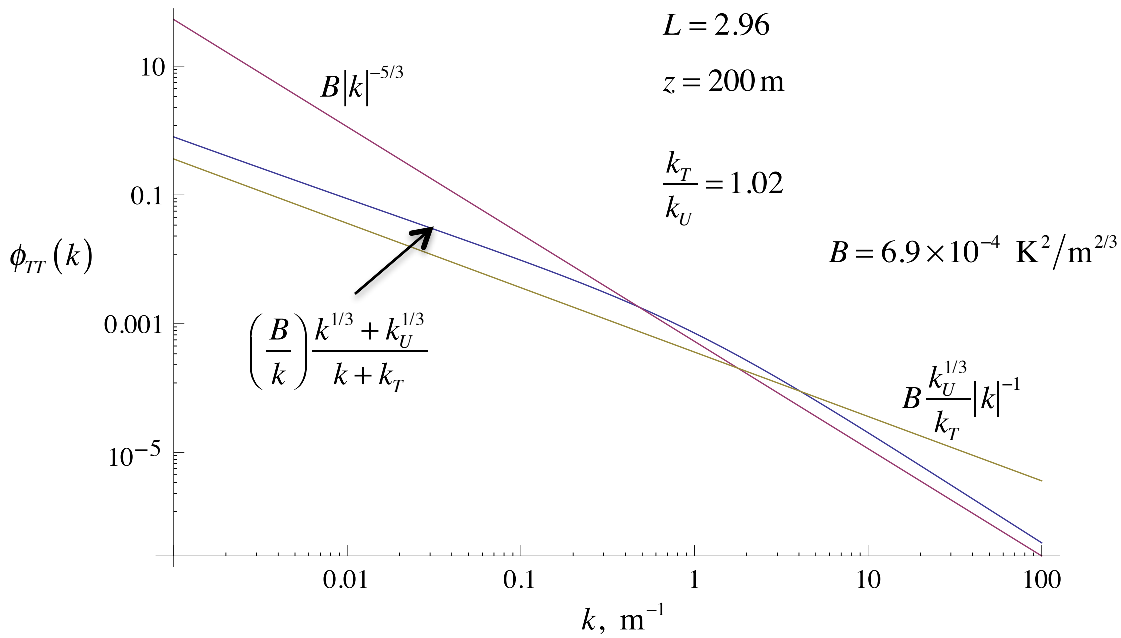
$$A \equiv 2^{1/2}\left|\frac{d\bar{U}}{dz}\right|\left|\frac{d\bar{T}}{dz}\right|^{-2} \gamma^{-1} b^{-2} N^2 \varepsilon^{-1}, \; B \equiv \left(\frac{2}{3}\right) 4^{1/3} \gamma^{-2/3} b^{-1} N \varepsilon^{-1/3}, \; C \equiv 2^{1/4}\left|\frac{d\bar{U}}{dz}\right|^{1/2} \gamma^{-1/2} b^{-1} N \varepsilon^{-1/2}$$

Turbulence Theory … (cont'd)

Pasquill Stability Class F Example of Composite Spectrum

(L=Stability Parameter)

$\partial \bar{U}/\partial z = 0.09$ (m/s)/m $\qquad \partial \bar{T}/\partial z = 0.04$ K/m



One Dimensional Spatial Spectrum $\phi_{TT}(k)$ Displaying Both Buoyancy and Inertial Sub-Ranges for an Unstable Atmosphere at a Height of z=200 m

Application to Profiling Radiometer Measurements

- An analysis performed in the text shows that there is no need for a 'frozen-flow' hypothesis correction to be made in the compilation of $D_T\left(\bar{U}\Delta t\right)$ using measurements of T profiles over a temporal interval of $\Delta t \sim 40\,\mathrm{sec}$ at these large-scale turbulence sizes.

- Thus, one can use the well-known expression relating to the temperature fluctuation spectra

$$D_T\left(\bar{U}\Delta t\right) = 2\int_{-\infty}^{\infty}\left(1-\exp\left[-ik\bar{U}\Delta t\right]\right)\phi_{TT}(k)\,dk$$

or what is the same thing now that no corrections for Δt are required,

$$D_T(d) = 2\int_{-\infty}^{\infty}\left(1-\exp\left[-ikd\right]\right)\phi_{TT}(k)\,dk, \quad d = \bar{U}\Delta t$$

- Using the composite large-scale turbulence spectrum derived earlier, this gives

$$D_T\left(\bar{U}\Delta t\right) = C_T^2\,F\left(\bar{U}\Delta t\right)$$

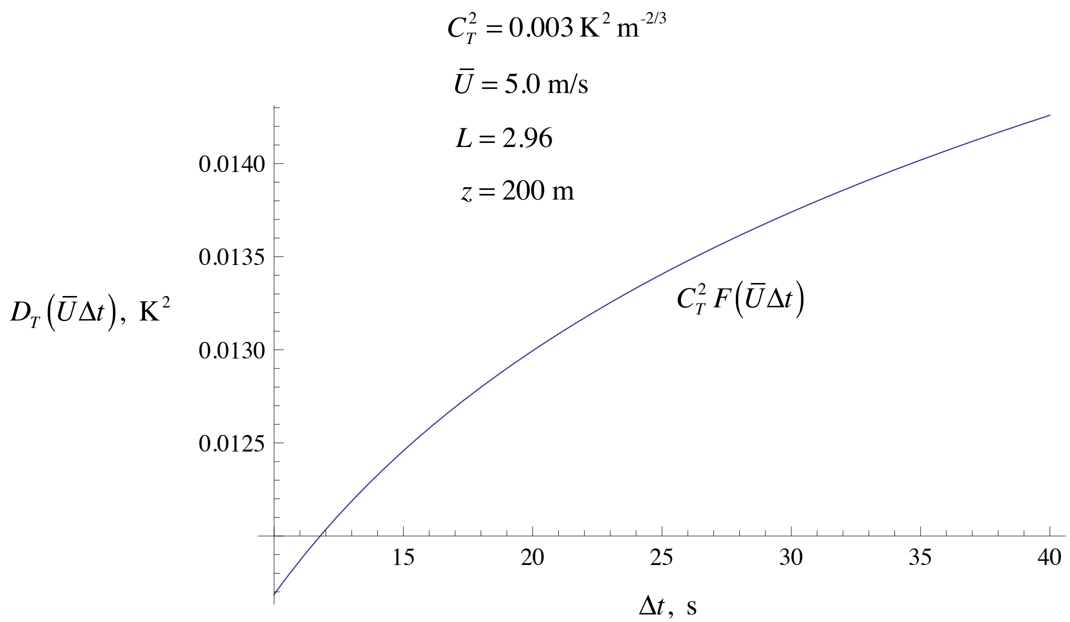
where

A confluent hypergeometric function

$$F\left(\bar{U}\Delta t\right) = \frac{1}{2}\left(\frac{1}{k_t^{2/3}}\right)\left\{\frac{2}{3}\sqrt{3}\pi\left(1-\frac{\mathrm{Re}\left\{\Psi\left(\frac{1}{3};\frac{1}{3};-ik_t\bar{U}\Delta t\right)\right\}}{\Gamma\left(\frac{2}{3}\right)}\right)+\left(\frac{k_U}{k_t}\right)^{1/3}\left(\mathrm{Re}\left\{\Psi\left(1;1;-ik_t\bar{U}\Delta t\right)\right\}+0.577+\log\left(k_t\bar{U}\Delta t\right)\right)\right\}$$

<u>Application to Profiling Radiometer Measurements</u> (cont'd)

An idea of what the temporal evolution look like of the temperature structure function for large-scale turbulence



Temperature Structure Function as a Function of Integration Time for a Stable Atmosphere with Minimum Expected Value of C_T^2 at a Height of z=200 m.

- From this example of minimum expected C_T^2, resolution requirements can be obtained: $\Delta T_{\min} \sim \sqrt{D_{T_{\min}}\left(\bar{U}\Delta t\right)}$ @ $\Delta t = 40\,\mathrm{sec}$ gives $\Delta T_{\min} \sim 0.11\,\mathrm{K}$. Similarly, for the water-vapor case, $C_{Q_{\min}}^2 = 0.1\left(\mathrm{g/m}^3\right)^2/\mathrm{m}^{2/3}$ giving
$$\Delta Q_{\min} \sim \sqrt{D_{Q_{\min}}\left(\bar{U}\Delta t\right)} \sim 0.68\ \mathrm{g/m}^3.$$

These values are within the resolution range of currently available profiling radiometers

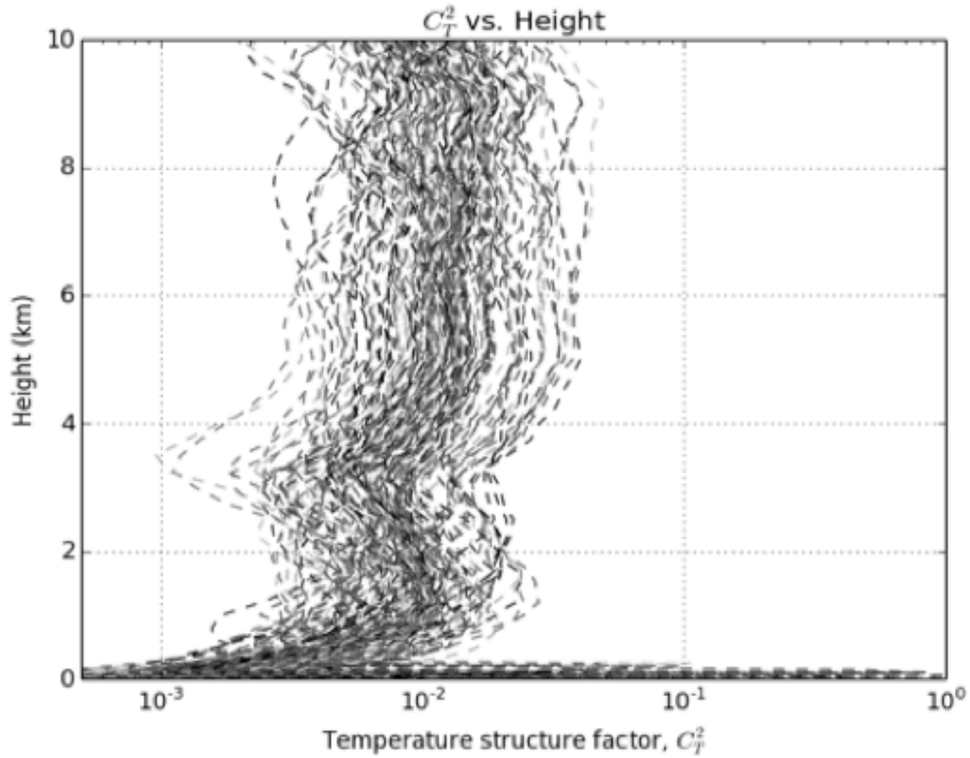
Application to Profiling Radiometer Measurements (cont'd)

Two additional requirements:

1) Need to get a good description of vertical profile of horizontal wind components to capture values for $\bar{U}$. This can be done with a wind profiling radar but this represents too much overhead for a remote-sensing scenario. Use instead a statistical wind profile model that makes use of historical archive of wind data that or near the particular site. A very good modeling approach is noted and used in the text.

2) Integrating time interval of the radiometer Δt essentially 'averages-out' initial profiles existing from Earth surface to a threshold height $h_{\min}$ below which C_T^2 profiles cannot be resolved. Assuming isotropic behavior of the turbulent inhomogenieties, one can simply place this minimum height at the value $h_{\min} = \bar{U}\Delta t$. Behavior of C_T^2 below $h_{\min}$ can be obtained by simple gradient measurements of T (and water-vapor Q) taken at two vertical locations near the Earth's surface, forming the prevailing gradient Richardson number, which will help secure C_T^2 near the surface as well as augment estimates to help bound values of the characteristic spatial frequencies k_U and k_T needed in the large-scale turbulence spectrum discussed earlier.

Application to Profiling Radiometer Measurements (cont'd)



Calculated Vertical Profile of C_T^2 from Temperature Measurements Using a Radiometrics Corp. MP-3000A Microwave Profiling Radiometer.

- Figure confirms the potential of the remote sensing method. Description of experiment is described in the text.
- No concurrent atmospheric measurements were made.
- Stable atmosphere values of $k_t = 0.9$ m^{-1} and $k_U / k_t = 1.0$ m^{-1} were used.
- Statistical wind profile model was applied as described in text.

Application to Profiling Radiometer Measurements (cont'd)

Procedure To Use a Profiling Radiometer With the Model Just Presented

1) Configure temperature and water-vapor sensors along a short (~5 m) vertical direction to assess gradient Richardson number for temperature and humidity.

2) Apply Similarity Theory to find prevailing stability parameter as well as profile estimates for the characteristic spatial frequencies k_U and k_T as well as estimate for structure parameter values below threshold height $h_{\min}$.

3) Form structure function $D_T\left(\bar{U}\Delta t\right)=\frac{1}{N}\sum_{i=0}^{N}\left(T\left(t_i+\Delta t\right)-T\left(t_i\right)\right)^2$ for temperature (as well as humidity).

4) Apply statistical wind profile model appropriate for location.

5) Evaluate $F\left(\bar{U}\Delta t\right)$ function.

6) Determine corresponding profile $C_T^2=D_T\left(\bar{U}\Delta t\right)/F\left(\bar{U}\Delta t\right)$.

The next order of business is to apply this procedure concurrent with independent assessments of C_T^2 and C_Q^2 profiles for quantitative verification.