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On the Isobaric Surface Shape in the Geostrophic State of the Atmosphere

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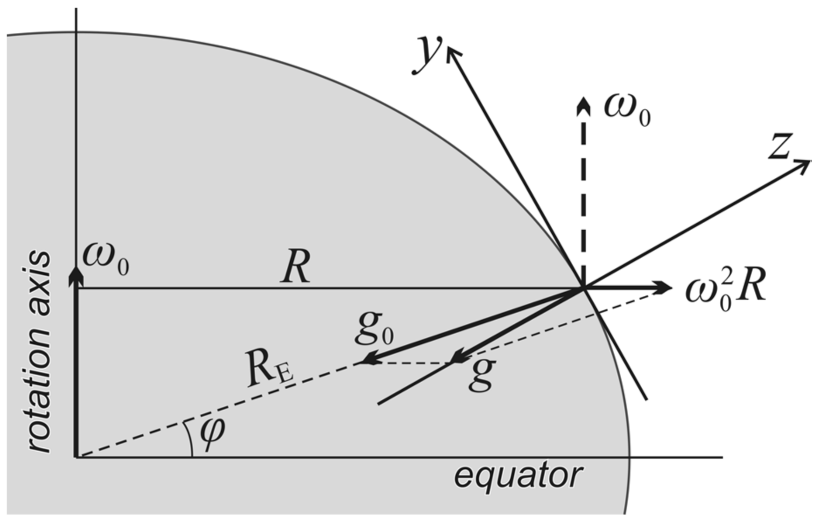
Introduction

The question of the disturbed isobaric surface shape in the geostrophic state of the atmosphere is still an open. The knowledge of the disturbed isobaric surface shape can improve our fundamental understanding of a number of significant atmospheric phenomena. In particular, in this paper we place special emphasis on the polar vortex phenomenon. This phenomenon represents a persistent, large-scale low-pressure area located near either of the Earth's poles.
Distinct polar vortices have also been observed on other planetary bodies of the solar system.
The nature of the polar vortex is still not clear. The question is raised here as to whether the polar vortex results from the atmosphere geostrophic state disturbance owing to the non-uniform heating of the Earth from equator to the pole or it is inherent in the geostrophic state itself.
It can be expected that the analysis of the isobaric surface geometry should demonstrate correlation with the fact of existence of a pronounced low-pressure area near the poles.

The goal of the present work is to determine (even if qualitatively) the disturbed isobaric surface shape in the geostrophic state of the atmosphere. This will allow solving the above-mentioned problems.

Main equations

A system of coordinates is related to the Earth surface; abscissa axis (*x*-axis) is directed along parallel; ordinate axis (*y*-axis) is directed along meridian; applicate axis (*z*-axis) is perpendicular to the Earth's surface:



The projections of the stationary state motion equation in the coordinates system where the *x*-*y*-plane is tangent to geoid are:

$$0 = -\frac{1}{\rho_s}\frac{\partial p_g}{\partial x} + 2v\omega_0 \sin\varphi - 2w\omega_0 \cos\varphi \qquad (1)$$

$$0 = -\frac{1}{\rho_s}\frac{\partial p_g}{\partial y} - 2u\omega_0 \sin\varphi \qquad (2)$$

$$0 = -\frac{1}{\rho_s}\frac{\partial p_g}{\partial z} + \alpha\Delta Tg \ + 2u\omega_0 \cos\varphi \qquad (3)$$

The free fall acceleration in the last equation equals to $g = g_0 - R_E \omega_0^2 \cos^2 \varphi$

Here, contrary to the usual definition of the geostrophic state, we have the additional Equation (3) for the description of the geostrophic state. As a matter of fact, the goal of the present study is to reveal the novel aspects of the geostrophic state description resulting due to the accounting of Equation (3).

The system (1)–(3) yields the geostrophic wind velocity horizontal projections:

$$u_{\mathrm{g}} = -\frac{1}{2\omega_0 \rho_{\mathrm{s}} \sin\varphi} \frac{\partial p_{\mathrm{g}}}{\partial y} \quad (4)$$

$$v_{\mathrm{g}} = \frac{1}{2\omega_0 \rho_{\mathrm{s}} \sin\varphi} \frac{\partial p_{\mathrm{g}}}{\partial x} \quad (5)$$

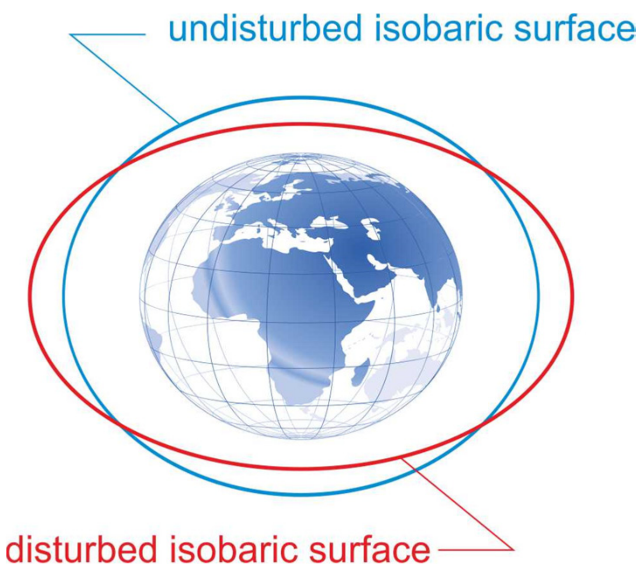
$$u_{\mathrm{g}} = \frac{1}{2\omega_0 \rho_{\mathrm{s}} \cos\varphi} \frac{\partial p_{\mathrm{g}}}{\partial z} - \frac{\alpha g}{2\omega_0 \cos\varphi} \Delta T \quad (6)$$

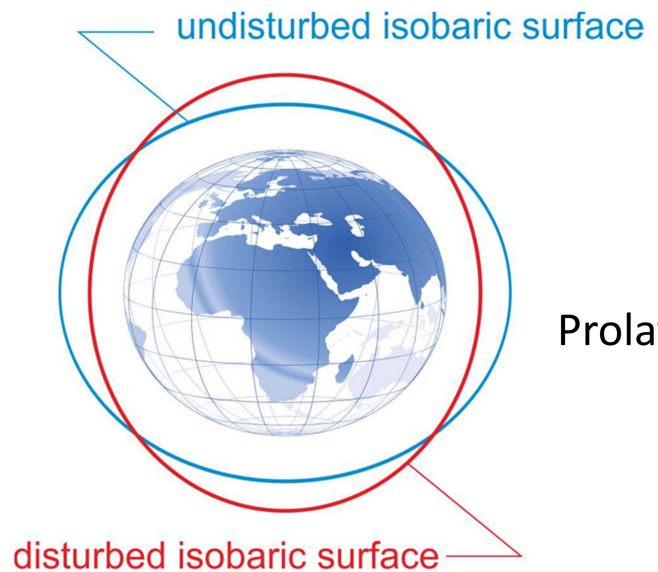
Analysis of Equations (4)–(6) demonstrates that, if we assume that the geostrophic state of the atmosphere corresponds to the isobaric surface of certain shape, which still needs to be determined, then at a given isobaric surface shape the warm air will move westward (on condition that $\partial p_{\mathrm{g}} / \partial z \leq 0$) and the cold air will move eastward (on condition that $\Delta T < \frac{1}{\alpha g \rho_{\mathrm{s}}} \partial p_{\mathrm{g}} / \partial z$)

Shape of the disturbed isobaric surface

In the general case, it follows from Equations (4)–(6) that the pressure disturbance gradient has three nonzero components. So, if in the statics state the pressure gradient is directed along **g**, then in the general case of the geostrophic state at the arbitrary point of isobaric surface the total pressure gradient will be deflected from the **g** direction. In this case, contrary to the statics state, the isobaric surface will not be perpendicular to **g** because it is perpendicular to the pressure gradient. Depending on the signs of components of the pressure disturbance gradient, the total pressure gradient can be deflected from **g** both towards the rotation axis and away from the rotation axis. In the first case, the isobaric surface will be extended along the Earth's rotation axis in comparison with the isobaric surface in the statics state. Such surface we will call the prolate geoid for short. In such a case, the pressure at the poles will be greater than the pressure in the statics state. In the second case, the isobaric surface will be flattened at the poles along the Earth's rotation axis in comparison with the isobaric surface in the statics state. Such surface we will call the oblate geoid for short. In such a case, the pressure at the poles will be lower than the pressure in the statics state.

Oblate





Prolate

The above reasoning is evidently inapplicable to the points at the equator and at the poles. Consider the points at the equator. We have:

$$\partial p_{\mathrm{g}} / \partial x = 0 \qquad \partial p_{\mathrm{g}} / \partial y = 0 \tag{7}$$

$$u_g = \frac{1}{2\omega_{0y}} \left(\frac{1}{\rho_{\mathrm{s}}} \frac{\partial p_{\mathrm{g}}}{\partial z} - g\alpha\Delta T \right) \tag{8}$$

Equations (7) and (8) should be used at the equator instead of Equations (4) – (6).

From Equations (7) and (8) it follows that the total pressure gradient is co-directional with the statics state pressure gradient. Thus, the isobaric surface is perpendicular to **g** at the equator, i.e., the isobaric surface is parallel to the geoidal isobaric surface of the statics state.

It follows from Equations (7) and (8) that the geostrophic wind velocity has only one component at the equator. This velocity component is directed along the equator. If we assume that $\partial p_{\mathrm{g}} / \partial z = 0$, then Equation (8) dictates that the warm air will move to the negative direction (eastward), and cold air will move to the positive direction.

At any point infinitely near the equator the meridional component of the velocity will also takes place. From the continuity of motion it can be concluded that the warm air starting from the equator goes along the spiral trajectory in the negative direction towards the pole. Analogously, the cold air will move from the equator in the positive direction along the spiral trajectory towards the pole. It should be noted that within the geostrophic model the curved trajectory cannot be realized. But we can consider the local direction of the geostrophic wind at each point.

Let us next analyze the disturbed isobaric surface shape at the poles. The projections of the atmosphere motion equation onto the coordinate axes have the form:

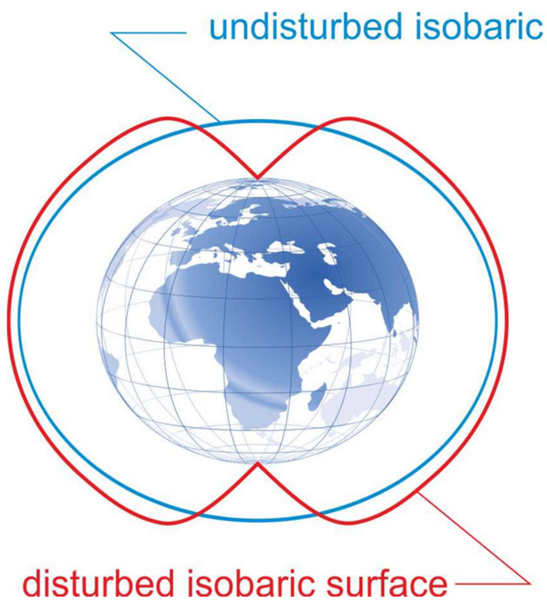
$$-\frac{1}{\rho_{\mathrm{s}}}\frac{\partial p_{\mathrm{g}}}{\partial x}\bigg|_0 + 2v_{\mathrm{g}}\omega_0 = 0 \qquad (9)$$

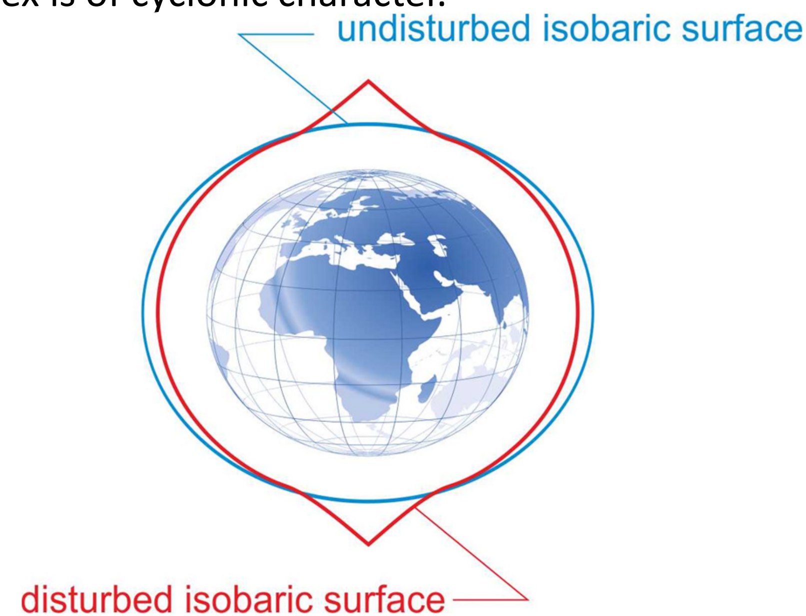
$$-\frac{1}{\rho_{\mathrm{s}}}\frac{\partial p_{\mathrm{g}}}{\partial y}\bigg|_0 - 2u_{\mathrm{g}}\omega_0 = 0 \qquad (10)$$

$$\alpha \Delta T g_0 - \frac{1}{\rho_{\mathrm{s}}}\frac{\partial p_{\mathrm{g}}}{\partial z} = 0 \qquad (11)$$

It can be seen that all three components of the pressure disturbance gradient are nonzero, and the geostrophic wind velocity is also nonzero. Thus, in the geostrophic state the total pressure gradient will be deflected from the $\mathbf{g}_0$ direction. In this case, contrary to the statics state, the isobaric surface will not be perpendicular to the rotation axis.

Consider the isobaric surface shape at the poles when overheat is equal to zero. As is seen from Equations (9), (10), two cases are possible. In the first case the geostrophic wind velocity is equal to zero at the pole. Then the *x*- and *y*-components of the pressure disturbance gradient are equal to zero. In this case we have the isobaric surface shape in the form of prolate or oblate geoids. In the second case the geostrophic wind velocity is nonzero at the pole. Then the *x*- and *y*-components of the pressure disturbance gradient are also nonzero. In this case the pressure disturbance gradient vector makes a right angle ($\pi/2$) with the statics state pressure gradient vector. Hence, the resulting pressure gradient vector will be deflected from vertical. This will lead to the disturbance of the isobaric surface at the pole. The isobaric surface will not be perpendicular to the rotation axis at the pole. It is clear that the pole is the exceptional point (in the sense of smoothness) of the function describing the isobaric surface. When the overheat at the equator is positive, the isobaric surface will have the prolate geoidal shape and the warm air will move along the spiral towards the pole where the local maximum is taking place and the wind velocity has a finite value. In this case the polar vortex is of anticyclonic character. When the overheat at the equator is negative, the isobaric surface will have the oblate geoidal shape and the cold air will move along the spiral towards the pole where the local minimum is taking place and the wind velocity has a finite value. In this case the polar vortex is of cyclonic character.





Equations (9) – (11) demonstrate that the negative overheat diminishes the pressure minimum depth at the pole and the positive overheat elevates the pressure maximum at the pole.

The presented analysis does not give any preference to the appearance of either polar pressure maximum or polar pressure minimum, it is only demonstrates the possibility of extremums of the pressure field at the poles in the geostrophic state.

Thus, the presented discussion of the disturbed isobaric surface geometry demonstrates the existence of a pronounced persistent low- or high-pressure area near the poles. As follows from the analysis above, the polar vortices may be an inherent attribute of the geostrophic state of the atmosphere.

Conclusions

Thus, in the present work it has been found that for the wind velocity projection onto the parallel two equivalent expressions exist: the first demonstrates the velocity dependence on the pressure gradient along the parallel; and the second demonstrates the velocity dependence on the vertical pressure gradient. When $\partial p_s / \partial y < 0$ for the existence of the zonal eastward transport of warm air the vertical pressure disturbance gradient must be positive and greater than certain value. Otherwise for the warm air mass only east wind can be observed and the cold air mass will move in the eastward direction.

It has been demonstrated that the following alternative isobaric surface geometries are possible in the geostrophic state. The isobaric surface has a shape of oblate or prolate geoid and the pressure at the pole is correspondingly lower or higher than the pressure in the statics state. The geostrophic wind velocity, divergence and vorticity are equal to zero in these two cases. The prolate geoidal shape of the isobaric surface corresponds to the positive value of the overheat at the equator; the oblate geoidal shape corresponds to the negative value of the overheat at the equator. The pressure minimum and maximum can occur at the poles in the mentioned cases of oblate and prolate geoid. In such instances, the geostrophic wind velocity is nonzero at the poles. It follows from here that the polar vortexes can be a special feature of the geostrophic state of the atmosphere. But the problem of mathematical definition of the exact shape of disturbed isobaric surface in the geostrophic state remains open. There is no doubt that in addition to the geostrophic nature, a various geophysical factors can influence on the polar vortex formation process and complicate the resulting picture of the phenomenon.