# Sensitive Versus Rough Dependence on Initial Conditions in Atmospheric Flows

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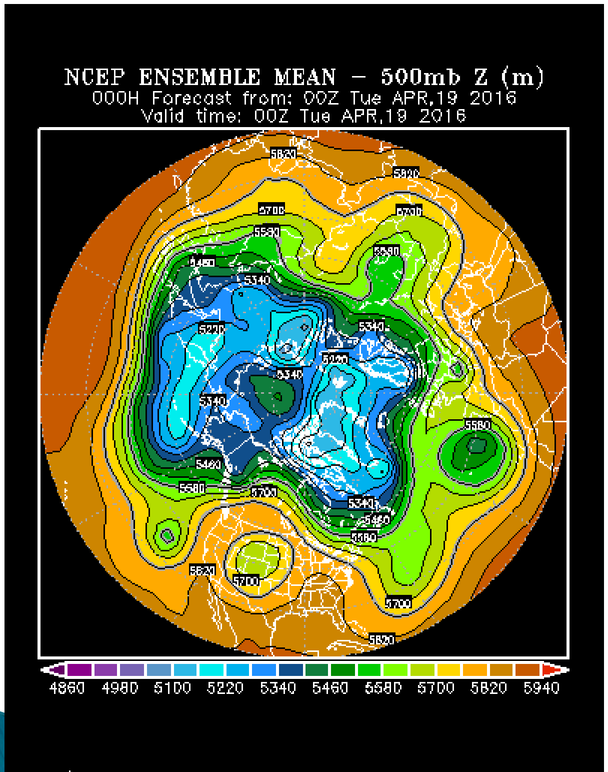
# Introduction

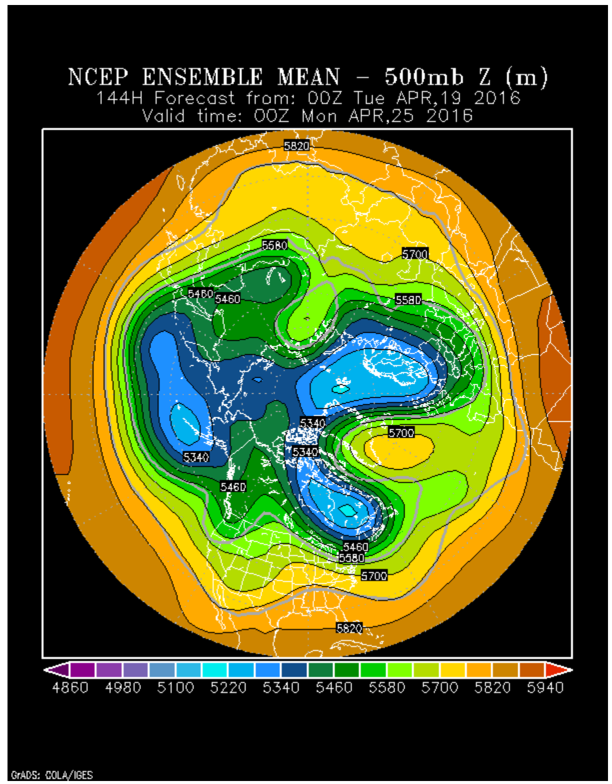
- Sensitive Dependence on Initial Conditions (SDOIC) is the idea that similar initial states can evolve very differently over time (slowly).

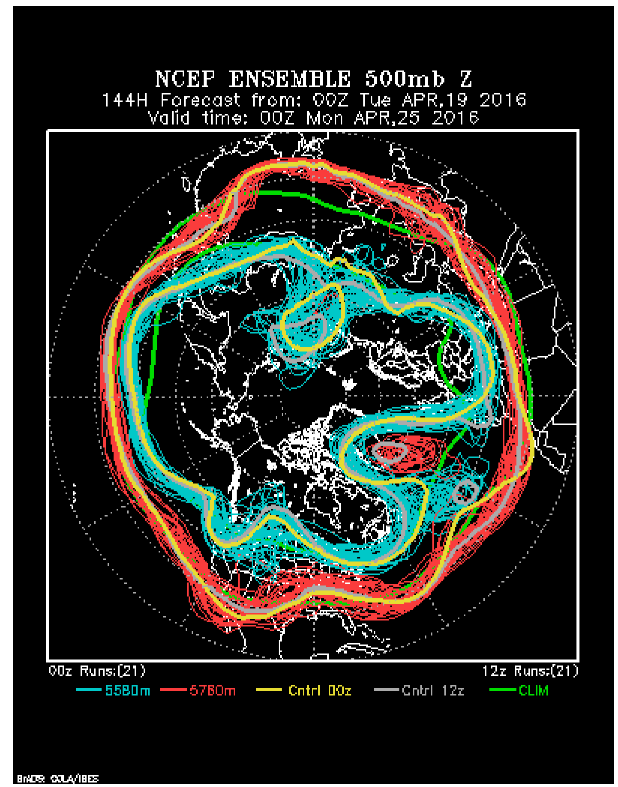
- The concept plagues weather forecasters, and techniques have been developed (e.g., Ensemble modeling) in order to mitigate the issue.

# Introduction

- Ensemble products:



# Motivation

- Occasionally, atmospheric phenomena may develop at an exponential rate, and initially similar states diverge rapidly with time.

- Thus, we introduce the concept of Rough Dependence on Initial Conditions (RDOIC) to explain this behavior and define it in terms of quantities we can measure.

# Background

- Reynolds number – ratio of inertial to viscous forces →
- Large Reynolds number → Atmosphere is three dimensional and dominated by inertial forces.
- Very large → atmosphere dominated by ‘violent’ turbulence. (Explosive development?)

# Derivation

- SDOIC → in a three dimensional system, at least one Lyapunov Exponent must be positive.

- It is a measure of 'stability' (rate of separation of trajectories) – characteristic exponent in the solution of a DIFFEQ:

$$\lambda_i = \lim_{t\to\infty}\left\{\lim_{\varepsilon(0)\to 0}\left[\frac{1}{t}\ln\left(\frac{\varepsilon_i(t)}{\varepsilon(0)}\right)\right]\right\}, i = 1, \ldots n$$

# Derivation

- In the atmosphere: Dymnikov (1992) showed that in a barotropic atmosphere, the positive LE can be expressed as:

$$\sum_{i>0} \lambda_i \approx \int_A \varsigma^2 dA$$

- Lupo et al. (2007) (then Hussain et al. 2010, Jensen and Lupo, 2013) call this quantity "Integrated Regional Enstropy" (IRE)

# Derivation

- Li (2014) – estimates the temporal growth of modes in the Navier–Stokes equations resulting in:

$$x(t) \leq e^{C\sqrt{t\,\mathrm{Re}}+Ct}\left(X(o)\right)$$

- This implies the exponential growth (as a function of ‘Re’ and time can be larger than that implied by the LE  (if equal – SDOIC).

# Derivation

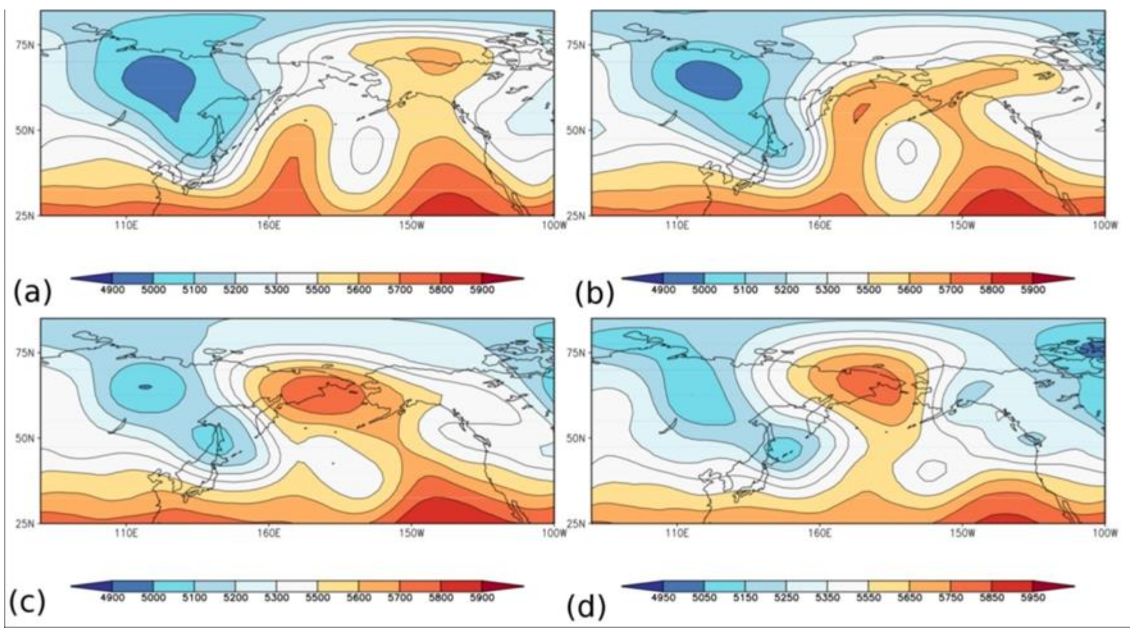
- If IRE is the LE – then using the previous equation produces the following inequality:

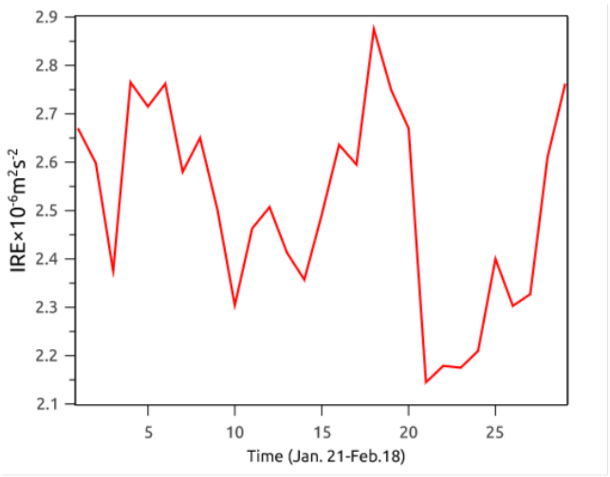
$$\int_A \zeta^2 dA \le C\sqrt{t\,\mathrm{Re}} + C_1 t$$

- Thus, if IRE is smaller than the LE implied by the Re, the we have RDOIC not SDOIC. This can be an issue in rapid development.

# Case – Blocking

- 23 Jan, 2014 – 16 Feb, 2014



# Case – Blocking

- Need to estimate RE:

$$\mathrm{Re} = \frac{\bar{V}L}{\upsilon}$$

- And constants from Li (2014):

$$C - \frac{8}{\sqrt{2e}} \max_{\tau\in[0,T]} \left\| u(\tau) \right\|_n$$

$$C_1 - 4 \max_{\tau\in[0,T]} \left\| u(\tau) \right\|_n - \frac{\sqrt{2e}}{2} C$$

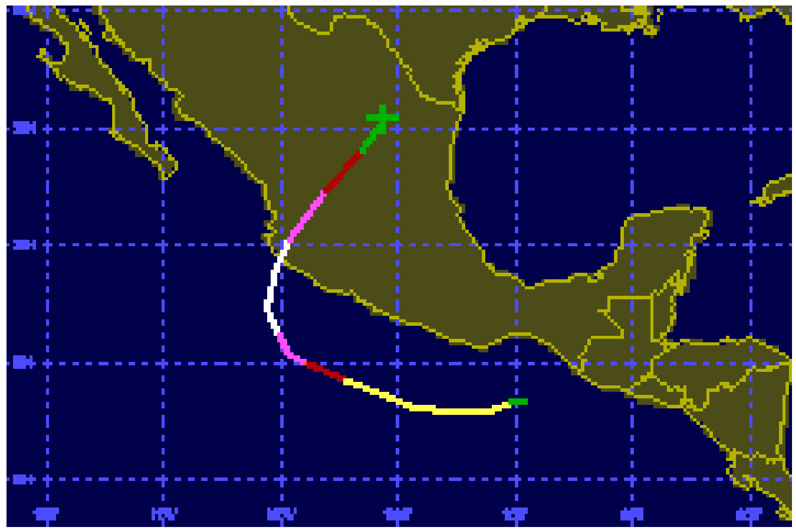
# Case – Blocking

- RDOIC – is going to be evident if the time-scale for development and evolution is smaller than that implied by the Reynolds number.

- In the case of this blocking event, the time-scale for growth was 'typical' of the synoptic and planetary-scale (about three days)! Predictable….

# Case – Hurricane Patricia

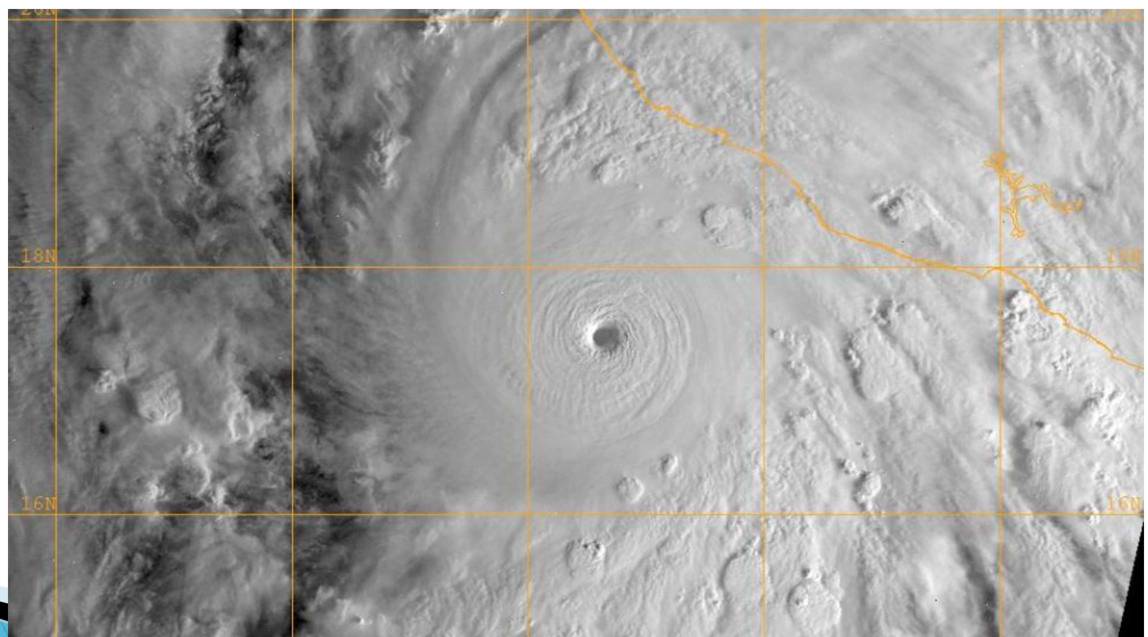
- 20–24 October – Cat 5 – 200 mph sustained winds!



- Deepened: 124 hPa in 36 hr (1004 to 880 hPa)

# Case – Hurricane Patricia

- Deepened at ten times (!) the rate of the definition for mid-latitude explosive cyclogenesis (24 hPa / 24 hr * sin (lat)/sin (60))



# Case – Patricia

- RDOIC – is going to be evident if the time-scale for development and evolution is smaller than that implied by the Reynolds number.

- In the case of Patricia, the time-scale for growth was characteristic of that of the Meso-$\gamma$ scale (convection)! RDOIC *probably* describes this case.

# Summary and conclusions

- SDOIC is a problem for weather forecasting, and the uncertainty that it implies has been accommodated using various ensemble products.

- We develop an expression to quantify RDOIC as a function of variables that are meteorologically relevant.

# Summary and Conclusions

- In the case of blocking from 23 Jan – 16 Feb 2014 – SDOIC characterized the time scale for development of this event. This is probably true for most blocking events – predictable.

- In the case of Hurricane Patricia – developed at a ‘hyper’ explosive rate. Thus, RDOIC *likely* characterizes this event (still need to determine), and the probability that models could have captured it are small.

# The End!

- Questions?
- Comments?
- Criticisms?
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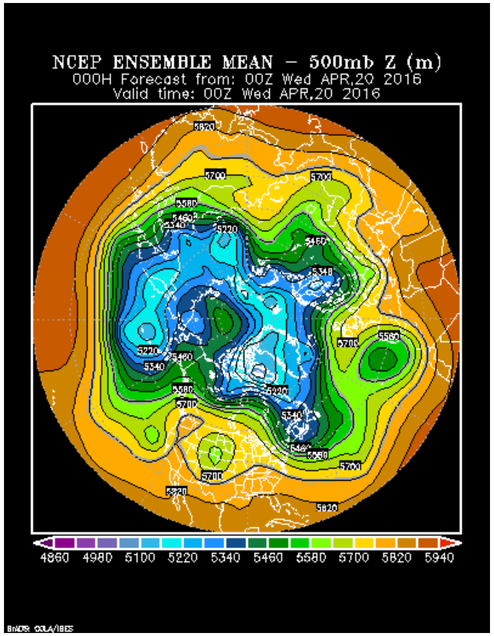
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Valid time: 00Z Wed APR,20 2016
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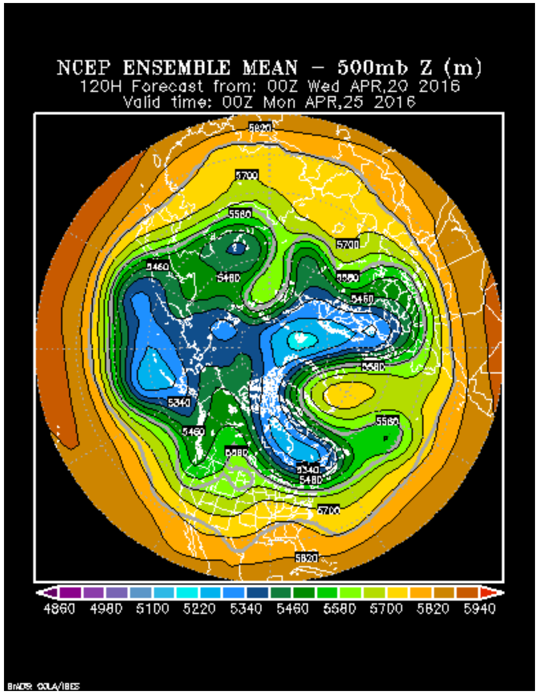
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120H Forecast from: 00Z Wed APR,20 2016
Valid time: 00Z Mon APR,25 2016
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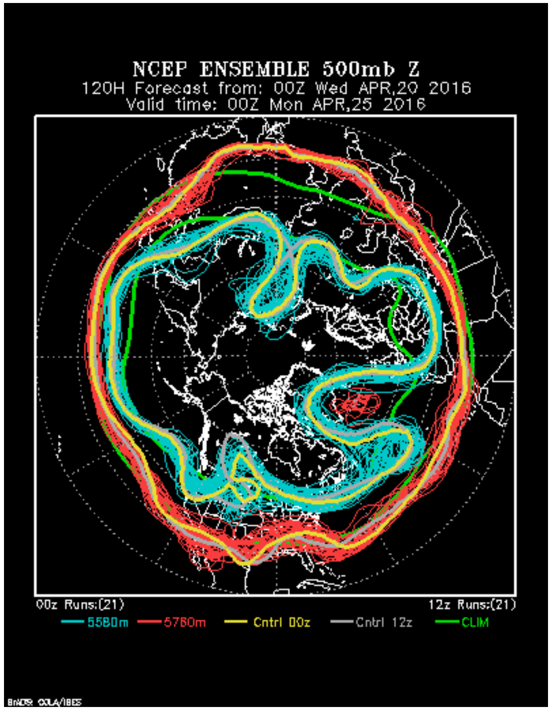
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