Integrated Regional Enstrophy as a Measure of Kolmogorov Entropy

Anthony R. Lupo[1] and Andrew Jensen[2]

[1]Department of Soil, Environmental, and Atmospheric Sciences
University of Missouri
Columbia, MO

[2]Department of Math and Meteorology
Northland College
Ashland, WI

Introduction

* Predictability for blocking is still a problem for the onset and termination of blocking events.

* Integrated Regional Enstrophy (IRE) was related to the positive Lyapunov Exponent for atmospheric flows by Dymnikov et al. (1992).

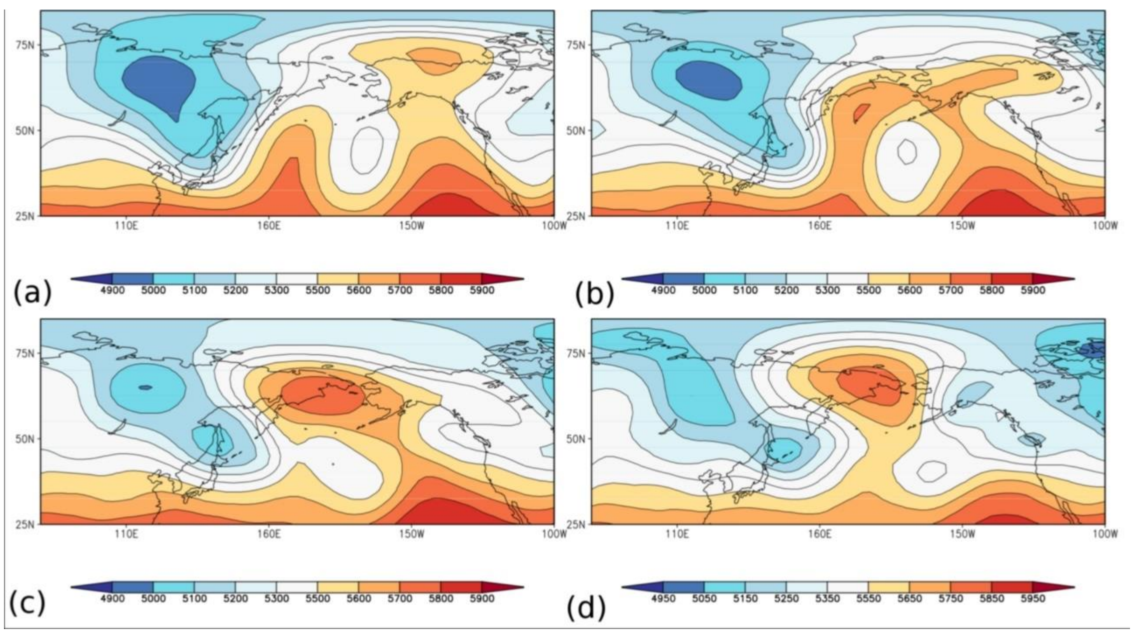
Introduction

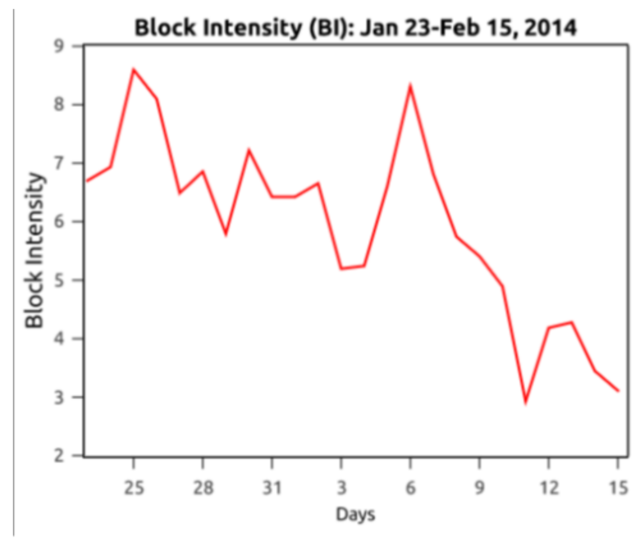
* IRE has been shown to be an effective diagnostic for identifying large-scale flow regime changes, and the onset and decay of blocking (Lupo et al. 2007; Hussain and Lupo, 2010, Jensen and Lupo 2013).

* Jensen (2015) analyzed a strong Pacific Region blocking event that persisted from late January through mid February 2014.

Case: Jan – Feb. 2014

* 4 – 7 Feb, 2014





Block Intensity

- Lupo and Smith (1995) - intensity index for blocking (BI) and is proportional to the strength of mid-latitude height gradients. This index was refined by Wiedenmann et al. (2002)
- BI = 100*(Zm/RC -1.0)
  - RC = (Zu + 2*Zm +Zd) / 4 (large-scale)
  - NH: 3.04 Units SH : 2.80 Units (~1970 – present)

Data

- The data used was the:
  - Potential Vorticity ERA-Interim re-analyses
  - NCAR/NCEP re-analyses

Integrated Regional Enstrophy

* Lyapunov Exponent – measures the rate of separation for the state of two systems that are initially close together.

* Example: a simple ODE

$$\dot{x} = -\lambda x$$

$$x(t) = x(0)e^{rt} = x(0)e^{-\lambda t} = c_o e^{-\lambda t}$$

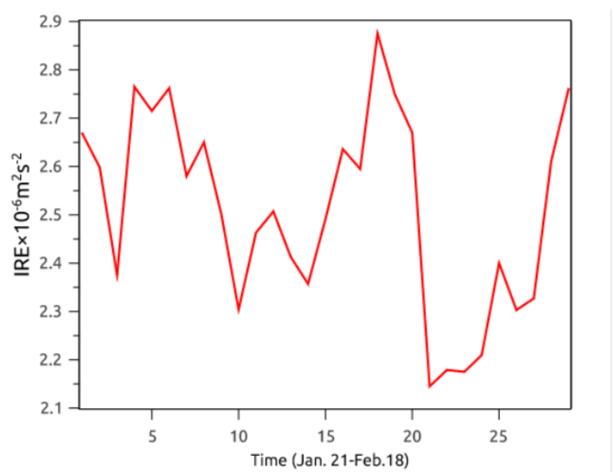
$$-or-$$

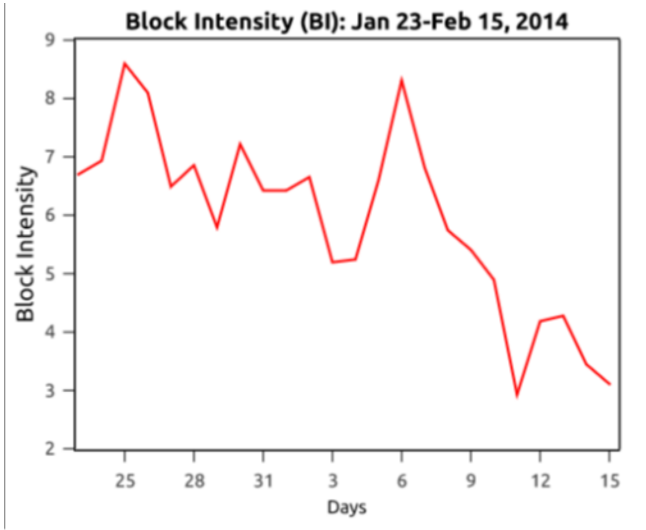
$$\dot{x} + \lambda x = 0$$

$$\lambda = \frac{1}{t}\ln\left[\frac{\varepsilon(t)}{\varepsilon(0)}\right]$$

Integrated Regional Enstrophy

- Dymnikov used the barotropic vorticity equation to show: $$\sum_i \lambda_i^+ = \int_A \zeta^2 dA = IRE$$
- And for the Jensen Block, IRE (left) and BI (right):





Kolmogorov Entropy

- Entropy: 2nd law of thermodynamics – entropy of a system cannot decrease.

$$dS_i \geq 0 \qquad\qquad dS_{ext} = \frac{dQ}{T}$$

- Entropy is extensive property – if the process is irreversible, this quantity increases.

Kolmogorov Entropy

* Kolmogorov Entropy is also called "metric Entropy" and is a quantity inversely proportional to predictability.

$$KolE = \sum_{\sigma_i > 0} \sigma_i$$

* Where (σ_i) is the positive Lyapunov Exponents. Other studies (e.g., Zeng et al. 1992 used this quantity to study atmospheric flows.

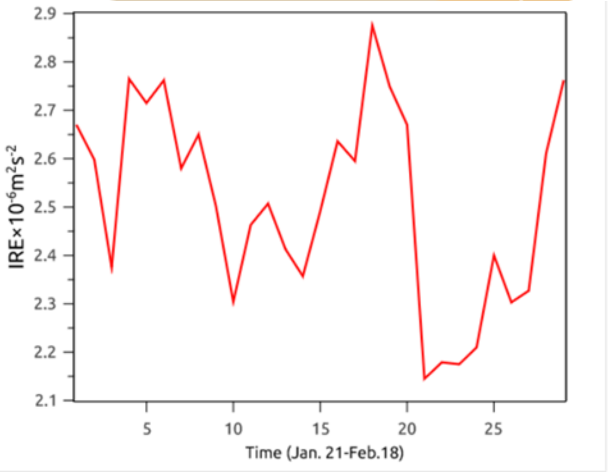
IRE = Kolmogorov Entropy

- Then: $K = \iint \delta^2 \, dA$

- Thus, Kolmogorov Entropy is IRE. We calculated using an R program for "correlation entropy" which is a lower bound of Kolmogorov (Eckmann and Ruelle, 1985 - Reviews of Modern Physics)

Results

* IRE for Pacific winter 2014 block:



* Kolmogorov Entropy for the entire event was 0.75 units (23 Jan – 16 Feb). During onset 0.86 units (14-24 Jan), and during the mid-point intensification 1.39 units (30 Jan – Feb 9).

So What?

* The results of the IRE and Kolmogorov Entropy demonstrate that during intensification, both quantities were high.

* Thus, predictability is low during block onset, but also during intensification. No wonder models have difficulty with them once they onset.

Summary and Conclusions

* This study shows that IRE and Block Intensity are related to Kolmogorov Entropy which is a measure of predictability.

* The Kolmogorov Entropy during the Jensen (2014) blocking event qualitatively matched the results of the IRE diagnostic and BI. Thus, these quantities have implications for predictability of blocking.

The End

* Questions?
* Comments?
* Criticism?
* LupoA@missouri.edu