

Cost-benefit optimization of sensor networks for SHM applications

Giovanni Capellari ^{1,†,*}, Eleni Chatzi ^{2,‡} and Stefano Mariani ^{1,‡}

¹ Politecnico di Milano, Dipartimento di Ingegneria Civile e Ambientale, Piazza Leonardo da Vinci 32, 20133 Milano, Italy: giovanni.capellari@polimi.it (G.C.); stefano.mariani@polimi.it (S.M.)

² ETH Zürich, Institut für Baustatik und Konstruktion Stefano-Franscini-Platz 5, 8093 Zürich, Switzerland; chatzi@ibk.baug.ethz.ch

* Correspondence: giovanni.capellari@polimi.it; Tel.: +39-02-2399-4267

† Presented at the 4th International Electronic Conference on Sensors and Applications, 15-30 November 2017; Available online: <https://sciforum.net/conference/ecsa-4>.

Published: 14 November 2017

Abstract: Structural health monitoring (SHM) is aimed to obtain information about the structural integrity of a system, e.g. via the estimation of its mechanical properties through observations collected with a network of sensors. In the present work, we provide a method to optimally design sensor networks in terms of spatial configuration, number and accuracy of sensors. The utility of the sensor network is quantified through the expected Shannon information gain of the measurements with respect to the parameters to be estimated. At assigned number of sensors to be deployed over the structure, the optimal sensor placement problem is ruled by the objective function computed and maximized by combining surrogate models and stochastic optimization algorithms. For a general case, two formulations are introduced and compared: (i) the maximization of the information obtained through the measurements, given the appropriate constraints (i.e., identifiability, technological and budgetary ones); (ii) the maximization of the utility efficiency, defined as the ratio between the information provided by the sensor network and its cost. The method is applied to a large-scale structural problem, and the outcomes of the two different approaches are discussed.

Keywords: Structural health monitoring; Bayesian inference; Cost-benefit analysis; stochastic optimization; information theory

1. Introduction

Structural Health Monitoring (SHM) allows to detect and estimate variations in the mechanical properties of structural and engineered systems [1,2] and, hence, make decisions about the actions needed to maintain or recover the overall structural safety [3]. Within a stochastic framework, the higher the SHM system effectiveness, the lower the uncertainties of the estimated quantities. The aim of the present work is the optimization of the SHM system, in terms of accuracy, number and spatial configuration of the sensors. Two main advantages can be therefore foreseen: first, the amount of information obtained from sensor measurements can be increased; second, the number of sensors can be reduced without sacrificing the information content. In this way, on one hand the overall cost of the SHM system can be reduced, on the other hand, as the amount of data to be analysed is cut down, the computational cost of the SHM method can be decreased, allowing for real-time applications (e.g. see [4]).

In the remainder of the paper, a theoretical introduction of the optimization formulation is given in Section 2. Then, its application to a structural problem is discussed in Section 3. Finally, in Section 4 some concluding remarks are gathered.

2. Theoretical framework

Let us assume that the goal of the SHM system is the estimation of a set of parameters (e.g. mechanical properties, geometrical properties, damage indices) defined within an appropriate mathematical numerical model, used to predict the response of the structure to given loads. Within a Bayesian framework, the prior probability density function (pdf) $p(\boldsymbol{\theta})$ (where $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is the random vector of the n_θ parameters to be estimated) can be updated into the posterior pdf $p(\boldsymbol{\theta}|\mathbf{y})$, when the measurements $\mathbf{y} \in \mathbb{R}^{n_y}$ are collected through the sensor network. Following a Bayesian experimental design approach, the overall information provided by the measurements can be quantified using the information theory, as introduced in [5]. Applying these concepts to the problem of optimal sensor placement, a strategy based on the combination of surrogate models (see [6]) and stochastic optimization (see [7]), was introduced in [4,8]. Let us call n_y the number of measurements, p_ϵ the pdf of the prediction error ϵ , and $\mathbf{d} \in \mathbb{R}^{n_d}$ the design variables which provide the spatial configuration of the network on the structure. The optimal sensor placement configuration \mathbf{d}^* can be obtained by maximizing the expected Shannon information gain $U(\mathbf{d})$, see [4]; in doing this, parameters n_y and p_ϵ are supposed to be fixed for a specific optimization problem.

In general, the expected Shannon information gain is a function of \mathbf{d} , n_y and p_ϵ . The prediction error $\epsilon = \epsilon_m + \epsilon_n$ depends on both the model error ϵ_m and the measurement noise ϵ_n . In [9], it was proven that the spatial correlation among different measurements, which is embedded into p_{ϵ_m} , affects the optimal sensor configuration \mathbf{d}^* . On the other hand, if the environmental effects are neglected, the pdf of the measurement noise ϵ_n can be directly linked to the employed sensors, as the probability model p_{ϵ_n} depends on the sensors characteristics. The sensor network can be therefore optimized, in terms of spatial configuration, number and type of sensors, by maximizing the expected Shannon information gain according to:

$$(\mathbf{d}^*, n_y^*, p_{\epsilon_n}^*) = \arg \max [U(\mathbf{d}, n_y, p_{\epsilon_n})] \quad (1)$$

where p_{ϵ_m} is instead supposed to be constant.

Assuming that ϵ_n is sampled from a zero mean Gaussian pdf $p_{\epsilon_n} = \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_n)$, where $\mathbf{0} \in \mathbb{R}^{n_y}$ is the mean vector, and $\boldsymbol{\Sigma}_n \in \mathbb{R}^{n_y \times n_y}$ is the covariance matrix, then $U = U(\mathbf{d}, n_y, \boldsymbol{\Sigma}_n)$. For the sake of simplicity, we next assume that there is no correlation between measurements and so $\boldsymbol{\Sigma}_n = \sigma^2 \mathbf{I}$, where σ is the standard deviation of measurement noise and $\mathbf{I} \in \mathbb{R}^{n_y \times n_y}$ is the identity matrix. The optimization statement in Eq. (1) thus becomes:

$$(\mathbf{d}^*, n_y^*, \sigma^*) = \arg \max [U(\mathbf{d}, n_y, \sigma)] \quad (2)$$

The function $\bar{U} = U(\mathbf{d}^*, n_y, \sigma)$, corresponding to the maximum of the objective function for each value of n_y and σ , is computed for the corresponding optimal sensor configuration \mathbf{d}^* . As \mathbf{d}^* depends on the choice of $\{n_y, \sigma\}$, then $\bar{U} = \bar{U}(n_y, \sigma)$ is implicitly a function of n_y and σ only. It can be proven [10] and numerically shown [11] that \bar{U} increases as the number of sensors gets higher (more information is provided by the SHM system). Moreover, if σ increases then \bar{U} decreases, since the structure response gets hidden by the measurement noise [12]. Thus, it follows that \bar{U} is a monotonically increasing function of n_y and a monotonically decreasing function of σ : additional constraints need therefore to be handled in order to obtain the optimal solution of Eq. (2).

Three types of constraints are here taken into account:

- (a) technological constraint $\sigma > \sigma_{best}$, with σ_{best} designating the standard deviation of the measurement noise of the most accurate sensor available on the market, to provide measurements \mathbf{y} ;
- (b) identifiability constraint $n_y > n_{obs}$, with n_{obs} designating the minimum number of measurements required to guarantee identifiability and observability of the parameters $\boldsymbol{\theta}$ (see [2,13–15]);

- (c) cost constraint $C(n_y, \sigma) \leq B$, with $C(n_y, \sigma)$ designating the cost model of the SHM system and B the maximum available budget.

The resulting constrained optimization problem is formulated as follows:

$$\begin{aligned} (\mathbf{d}^*, n_y^*, \sigma^*) &= \arg \max [U(\mathbf{d}, n_y, \sigma)] \\ \text{subject to } &\begin{cases} n_y > n_{obs} \\ \sigma > \sigma_{best} \\ C(n_y, \sigma) \leq B \end{cases} \end{aligned} \quad (3)$$

As far as the cost model is concerned, the simplest formulation, includes a constant overall contribution C_0 , which takes into account the cost of data acquisition hardware, database, assemblage, etc., and a variable contribution, which takes instead into account the cost of the sensors. The associated expression is:

$$C(n_y, \sigma) = C_0 + c(\sigma) n_y \quad (4)$$

where $c(\sigma)$ is the cost per unit sensor.

One possible approach for solving the optimization problem would consist in defining a new design variable to account for n_y and σ ; next, the optimal solution is obtained by applying an optimization algorithm for stochastic problems, like the Covariance Matrix Adaptation-Evolution Strategy [7]. When only a limited number of sensor types is available, an alternative approach is based on the computation of the function $\bar{U} = U(\mathbf{d}^*, n_y, \sigma) = U(n_y, \sigma)$ on a set of points $\{n_y, \sigma\}$, as shown in Section 3.

The optimization problem introduced in Eq. (3) allows to design sensor network such that the provided information is maximized, given a certain budget B . Following a usual method in decision making strategies (see [16]), an alternative optimization rationale would be to maximize the ratio between the expected Shannon information gain and the cost of the SHM system, in a sort of cost-benefit analysis. Thus, the following utility-cost index (UCI) is defined:

$$\text{UCI}(\mathbf{d}, n_y, \sigma) = \frac{U(\mathbf{d}, n_y, \sigma)}{C(n_y, \sigma)} \quad (5)$$

where the associated measurement unit is [nat/€], [nat] being natural unit of information. The resulting optimization problem then becomes:

$$\begin{aligned} (\mathbf{d}^*, n_y^*, \sigma^*) &= \arg \max [\text{UCI}(\mathbf{d}, n_y, \sigma)] \\ \text{subject to } &\begin{cases} n_y > n_{obs} \\ \sigma > \sigma_{best} \\ C(n_y, \sigma) \leq B \end{cases} \end{aligned} \quad (6)$$

The above formulation allows to obtain the most efficient SHM design, i.e., to maximize the information per unitary cost.

As regards the solution of the optimization problem, the objective function to be maximized turns out to be now $\bar{\text{UCI}} = \text{UCI}(\mathbf{d}^*, n_y, \sigma) = \text{UCI}(n_y, \sigma)$, where \mathbf{d}^* .

The application of the two formulations defined in Eqs. (3) and (6) to the optimal design of a SHM system is presented in Section 3.

3. Results

The method presented in Section 2 is applied to the Pirelli tower, a 130 m tall building in Milan. The associated finite element model features a total number of 4106 nodes, each one with 6 degrees of freedom, i.e. the displacement components u_{x_1} , u_{x_2} and u_{x_3} along the three axes x_1 , x_2 and x_3 of a orthonormal reference frame and the 3 rotation components φ_{x_1} , φ_{x_2} and φ_{x_3} about the same axes. For

further details on the model, the reader may refer to [17]. We herein assume that measurements can be either displacements or rotations; $n_\theta = 6$ parameters, including both geometrical and mechanical properties, ought to be inferred (see [8] for further details on the choice of the parameters).

In Fig. 1a, the contour plot of the objective function $\bar{U}(n_y, \sigma) = U(\mathbf{d}^*, n_y, \sigma)$ is shown at varying number n_y and accuracy of the sensors (measured through σ). As previously discussed, the plot shows that the maximum value of the expected Shannon information gain increases as n_y gets higher and as the standard deviations decreases. It can be also observed that the increase in the expected Shannon information gain due to each additional measurement gets lower as more measurements are considered. In other words, the derivative of the expected Shannon information gain with respect to the number of sensors $\frac{\partial \bar{U}}{\partial n_y}$ is a decreasing function of n_y . Interpreting the optimization problem within a decision-making view, it is interesting to underline that this behaviour corresponds to the so-called "law of diminishing marginal utility" (also known as Gossen's First Law [18]), which was proposed for problems of resources allocation optimization. This law states that the marginal utility of a certain system, due to an additional unit, decreases as the supply of units increases. In the current problem of optimal SHM system design, the utility, i.e., the benefit of the sensor network is quantified by the expected Shannon information gain (see Section 2), and the unit is represented by each measurement.

If the cost model defined in Eq. (4) is employed, the red lines in Fig. 1a represent different budget constraints, i.e., the solutions $\{\sigma, n_y\}$ of the equation $B = C_0 + c(\sigma) n_y$, with B being the available budget (in the example $B_1 = 2000$ €, $B_2 = 2500$ €, $B_3 = 3000$ €). This graph allows the designer to choose the optimal SHM sensor network characteristics σ^* and n_y^* and the associated optimal configuration \mathbf{d}^* , which corresponds to the maximum $U(\mathbf{d}, n_y, \sigma)$: it is worth noting that in this case the solution is basically ruled by the budget constraint. A discussion about the optimal configurations \mathbf{d}^* obtained through the optimization procedure can be found in [8].

A different approach for decision making is to define a Pareto front for $U(\mathbf{d})$ versus cost savings, as shown in Fig. 1b: each line here represents the optimal design for a certain standard deviation σ , i.e. a certain type of sensors. Along the x-axis quantity the cost saving is represented, which is defined as the cost function normalized with respect to the chosen budget; the vertical straight line represents the budget B . Any design point located on the left of each line represents a non-optimal solution, i.e. the associated cost does not correspond to the best choice of $\{\mathbf{d}, n_y, \sigma\}$.

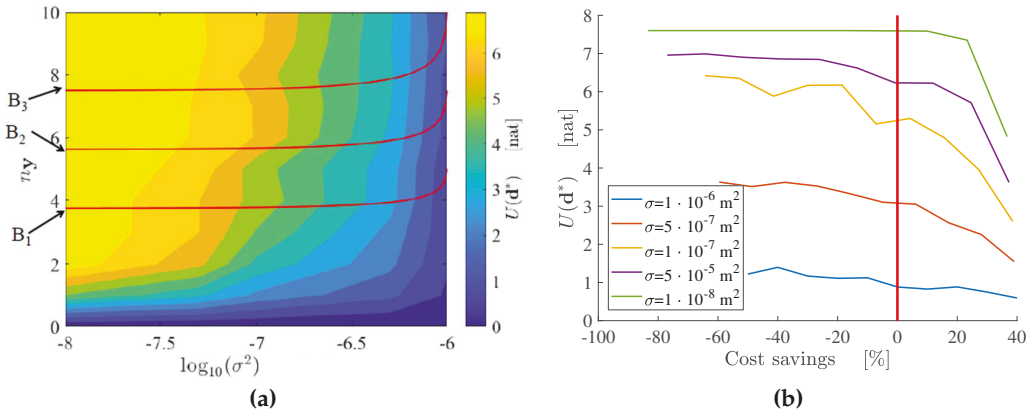


Figure 1. (a) Contour plot of $\bar{U}(n_y, \sigma)$, where lines represents the budget constraints $B = C(\sigma, n_y)$, with $B_1 = 2000$ €, $B_2 = 2500$ €, $B_3 = 3000$ €. (b) Pareto front of the expected Shannon information gain with respect to the SHM system cost.

The alternative design optimization approach defined in Eq. (6) is based on the maximization of the ratio $UCI(n_y, \sigma) = \frac{\bar{U}(n_y, \sigma)}{C(n_y, \sigma)}$. The resulting optimal solution obviously depends on the cost model: in Fig. 2a the SHM system is supposed to have a lower cost $C_0 = 500$ €; in Fig. 2b the SHM system is supposed to have a higher cost $C_0 = 1000$ €. In both cases, the most efficient allocation resources is

obtained if the best sensors, in terms of measurement noise, are chosen; the optimal number of sensors depends instead on the cost model: the higher C_0 , the higher n_y^* .

It is worth noting that, while the function $\bar{U}(n_y, \sigma)$ always increases with n_y and σ , the function $\overline{UCI}(n_y, \sigma)$ presents a maximum for a finite value of n_y . As previously discussed, the increase in information associated with each additional sensor decreases as more sensors are considered: from a cost-benefit point of view, it is therefore worthless to add sensors, i.e., to increase the SHM cost, if the associated additional benefit (the additional expected Shannon information gain) becomes too low.

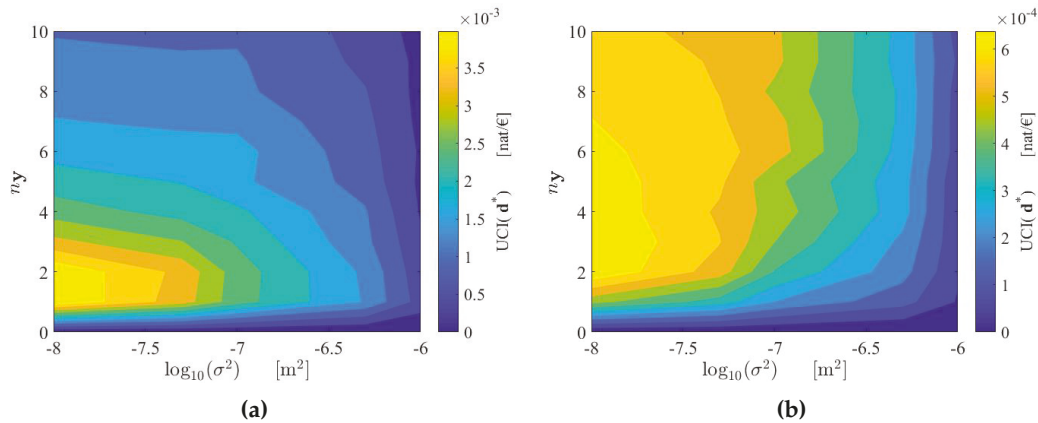


Figure 2. Contour plot of $UCI(\mathbf{d}^*, n_y, \sigma) = \frac{\bar{U}(\mathbf{d}^*, n_y, \sigma)}{C(n_y, \sigma)}$, with (a) $C_0 = 500$ € and (b) $C_0 = 1000$ €.

4. Conclusions

In the present work, a strategy to optimally design sensor networks for SHM applications, in terms of spatial configuration, accuracy and number of sensors, has been introduced. Two optimization formulations have been proposed, related either to the maximization of the expected Shannon information gain or to the maximization of the ratio between the information and the cost of the SHM system. The application of the two strategies to a large-scale structural problem has been discussed, highlighting the effect of the cost budget on the optimal solution. A Pareto front approach has been also discussed, allowing to obtain a prompt tool for the SHM system design.

Acknowledgments: G. C. wishes to acknowledge the partial financial support by IDEA League through a Student Grant. The authors also acknowledge the Chair of Risk, Safety and Uncertainty Quantification and the Computational Science and Engineering Laboratory at ETH Zürich for having provided the MATLAB-based softwares UQLab and CMA-ES, used in the implementation of the method. Authors are also indebted to Gianluca Barbella and Federico Perotti, who formerly provided a numerical model of the Pirelli tower.

Author Contributions: The authors contributed equally to this work.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

| | |
|-----|------------------------------|
| SHM | Structural Health Monitoring |
| pdf | Probability Density function |
| UCI | Utility-cost index |

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