POLITECNICO MILANO 1863

Cost-benefit optimization of sensor networks for SHM applications

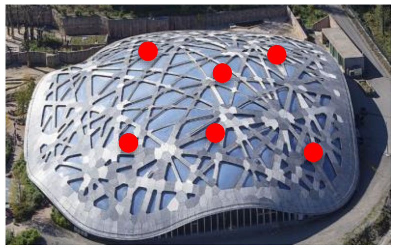
Giovanni Capellari
Eleni Chatzi
Stefano Mariani

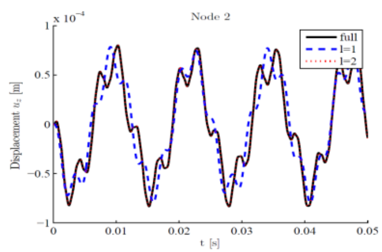
POLITECNICO MILANO 1863
ETH zürich

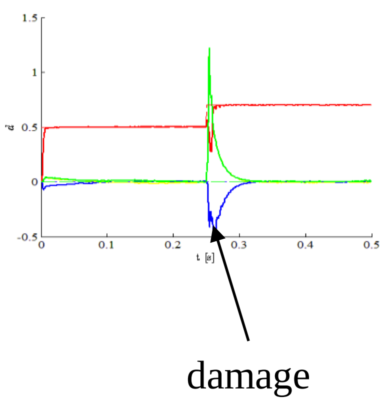
4th International Electronic Conference on Sensors and Aplications, 10-15 November 2017

Motivation

Structural Health Monitoring can be conceptually divided in three stages: in our work, we will focus on the design of the sensor network







SHM system design
$\boldsymbol{d}$

⇩

Data collection
$\boldsymbol{y}$

⇩

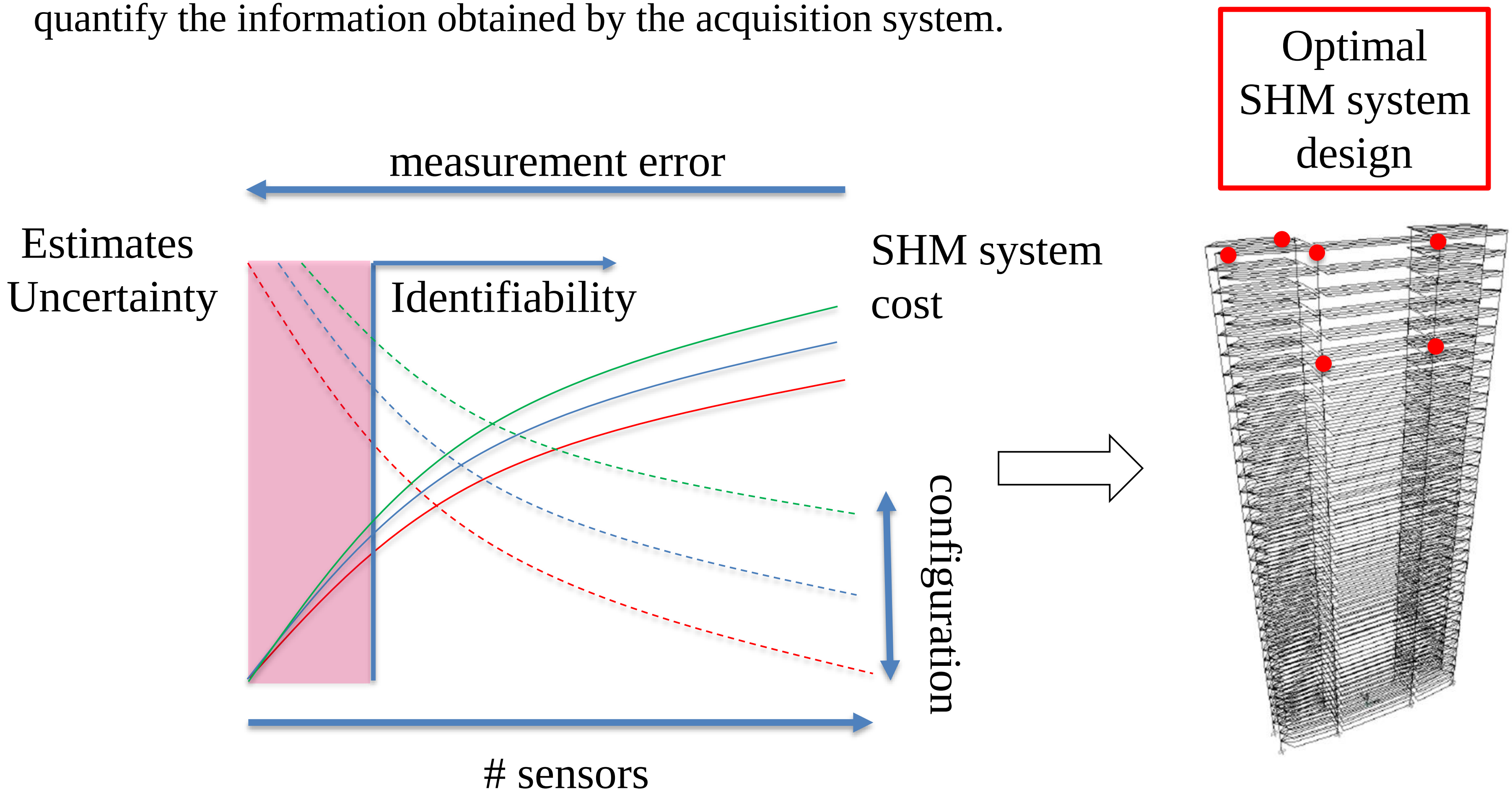
Parameters estimation
$\boldsymbol{\theta}$

⇩

Decision making

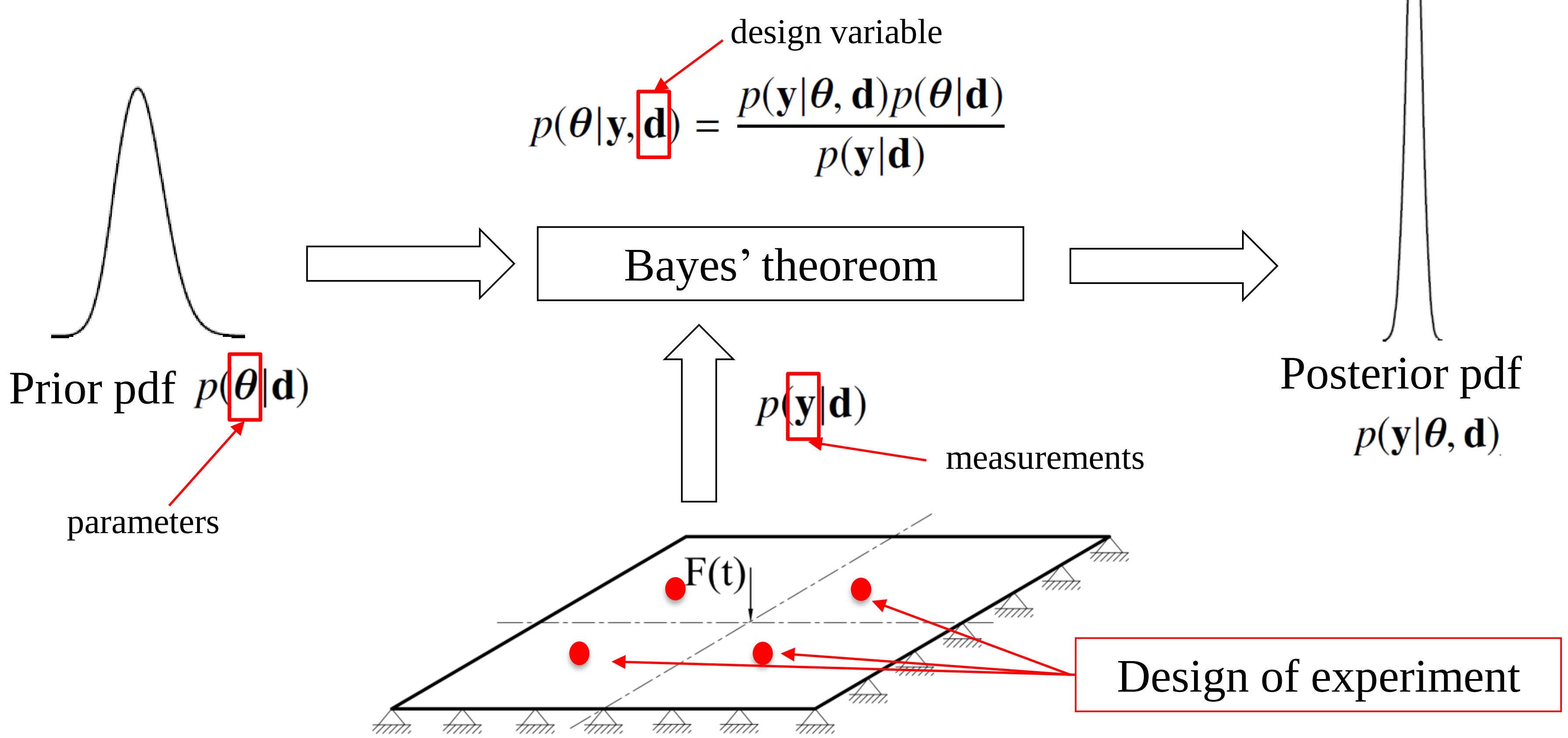
Motivation

The usefulness of the sensor network depends on the number, type and location of the sensors. Therefore, we need a method to quantify the information obtained by the acquisition system.



Stochastic approach

The Bayesian framework allows to take into account all the inherent uncertainties in the measurment process. The goal of the optimal SHM design is to find the experimental settings such that the uncertainties of the estimated paramaters are minimized.



Bayesian experimental design

We first compute the optimal sensor placement by maximizing the expected Shannon information gain. The objective function is numerically approximated through a Monte Carlo sampling approach.

$$\mathbf{d}^* = \arg\max_{\mathbf{d}\in\mathcal{D}} \underbrace{\left[\int_{\mathcal{Y}} \int_{\Theta} u(\mathbf{d}, \mathbf{y}, \boldsymbol{\theta}) p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) p(\mathbf{y}|\mathbf{d}) d\boldsymbol{\theta} d\mathbf{y} \right]}$$

Utility function:
Kullback-Leibler divergence
between prior and posterior

$$D_{KL}\left[p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) || p(\boldsymbol{\theta}|\mathbf{d})\right]$$

Expected Shannon information gain

Monte Carlo approximation

$$\frac{1}{N}\sum_{i=1}^{N}\left\{\ln\left[p(\mathbf{y}^i|\boldsymbol{\theta}^i, \mathbf{d})\right] - \ln\left[\frac{1}{N}\sum_{j=1}^{N} p(\mathbf{y}^i|\boldsymbol{\theta}^j, \mathbf{d})\right]\right\}$$

Alternative estimators: Kraskov, KDE, etc.

Numerical approaximation of the objective function: meta-modeling

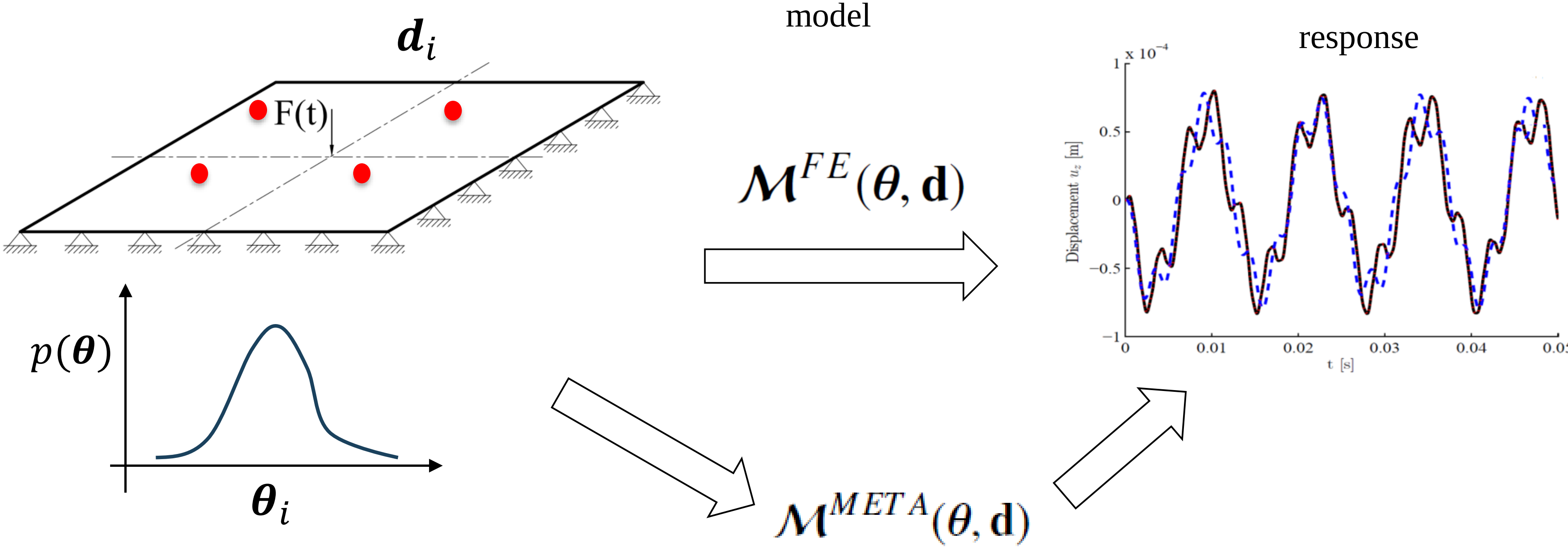
In order to reduce the computational cost of the estimator, the model response is computed through a meta-model, which is built combining a model order reduction method (Principal Component Analysis) and a surrogate modeling technique (Polynomial Chaos Expansion) (see Capellari et al. 2017)

$$p\left(\boldsymbol{y}^i \middle| \boldsymbol{\theta}^j, \boldsymbol{d}\right) = p_\epsilon\left(\boldsymbol{y}^i - \mathcal{M}\left(\boldsymbol{\theta}^j, \boldsymbol{d}\right)\right)$$

$$\boldsymbol{y} = \mathcal{M}(\boldsymbol{\theta}, \boldsymbol{d}) + \boldsymbol{\epsilon}$$

model

prediction error



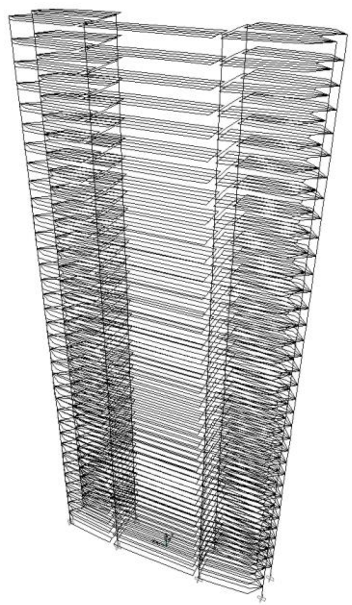
Optimal SHM system design: information maximization

The SHM system design can be optimized in terms of number, type and spatial configuration of the sensors by maximizing the expected Shannon information gain.

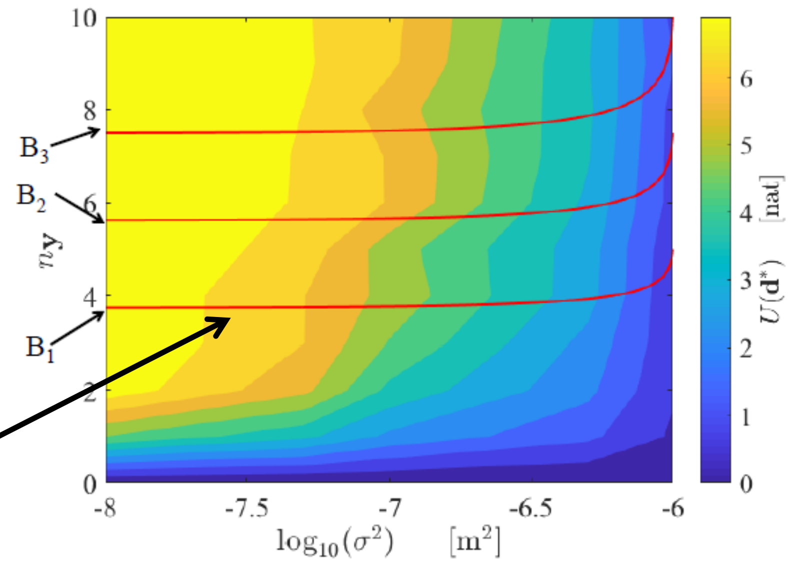
Information maximization

$$(\mathbf{d}^*, n_{sens}^*, \sigma^*) = \arg\max \left[U(\mathbf{d}, n_{sens}, \sigma) \right]$$

$$\text{subject to} \begin{cases} n_{sens} > n_{obs} & \text{(identifiability)} \\ \sigma > \sigma_{best} & \text{(technology)} \\ C(n_{sens}, \sigma) \leq B & \text{(budget)} \end{cases}$$



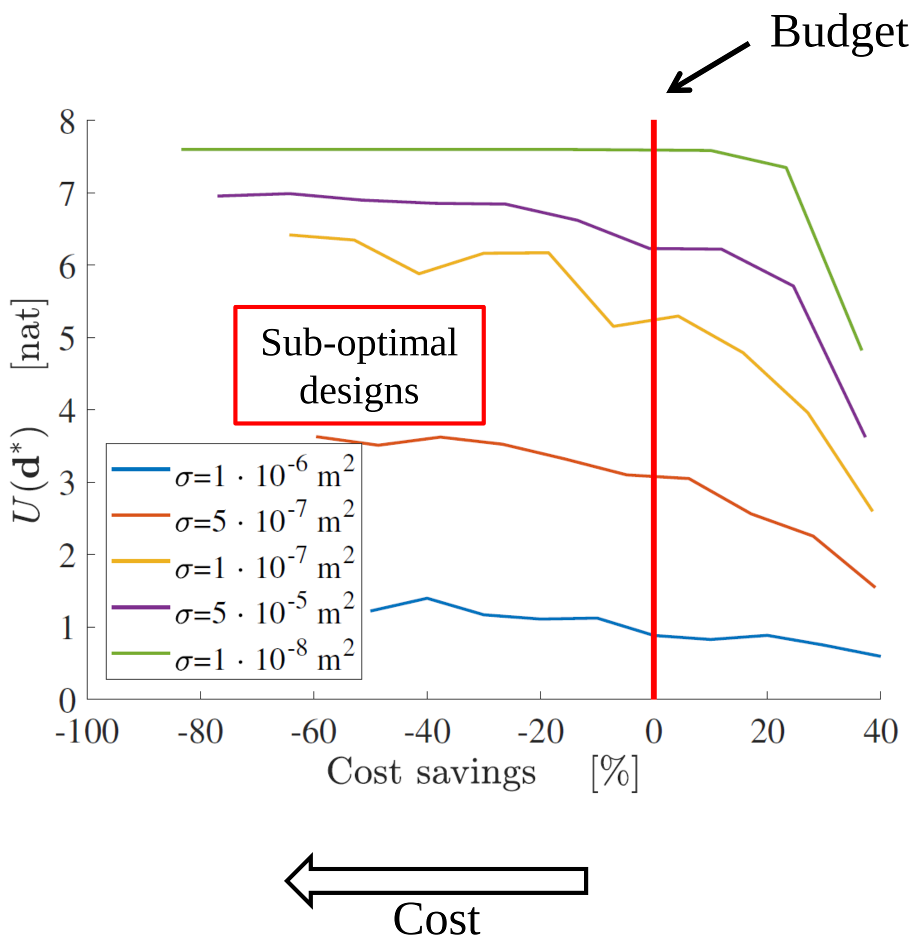
Pirelli tower



SHM cost model: $C(n_{sens}, \sigma) = B$

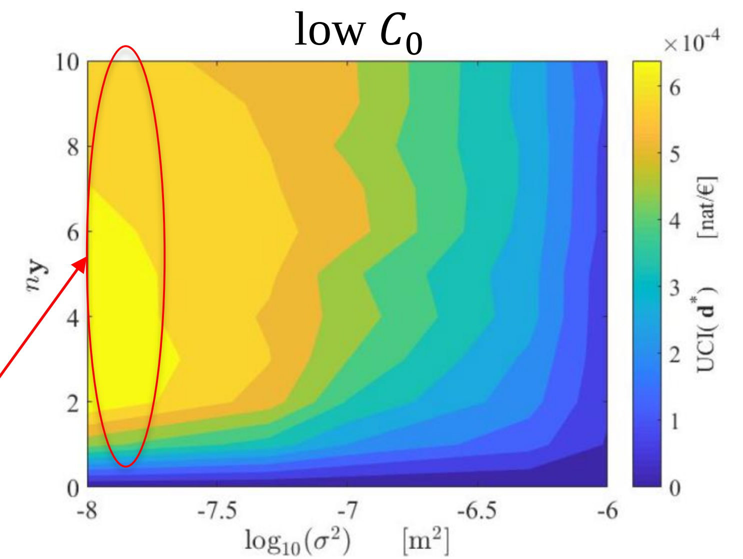
Optimal SHM system design: Pareto front

The Pareto fronts for different standard deviations (i.e. different types of sensors) are derived: these represent a prompt tool which can be use to design the SHM system.

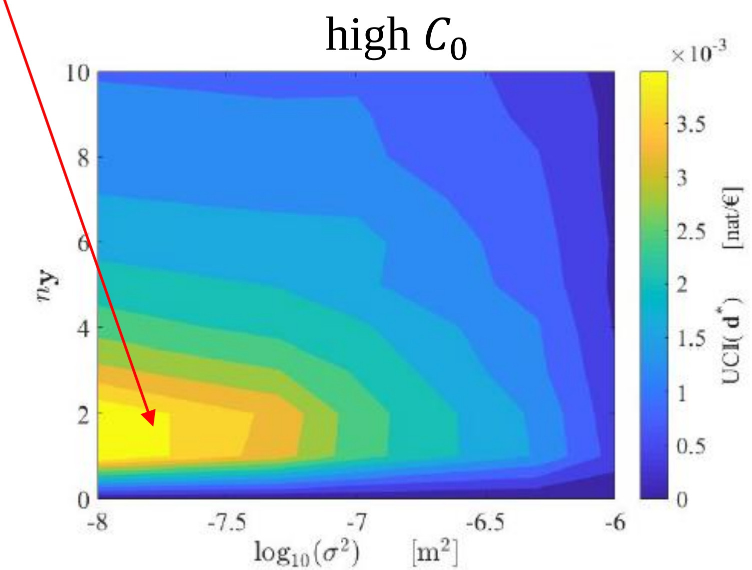


Optimal SHM system design: efficiency maximization

If the designer needs to choose both the experimental settings and the budget to be spent, a different approach should be followed: the amount of information per monetary unit is maximized.



‘law of diminishing marginal utility’



Efficiency maximization

$$(\boldsymbol{d}^*, n_{\mathbf{y}}^*, \sigma^*) = \arg\max \left[\frac{U(\boldsymbol{d}, n_{\mathbf{y}}, \sigma)}{C(n_{\mathbf{y}}, \sigma)} \right]$$

$$\text{subject to} \begin{cases} n_{\mathbf{y}} > n_{obs} \\ \sigma > \sigma_{best} \\ C(n_{\mathbf{y}}, \sigma) \leq B \end{cases}$$

Cost model: $C(n_{\mathbf{y}}, \sigma) = C_0 + c(\sigma)\, n_{\mathbf{y}}$

Conclusions

- SHM system design ⟸ **Bayesian optimal experimental design**

- Take into account:
  - Measurements uncertainties
  - Model uncertainties
  - Type of measured data with respect to quantities to be inferred

- **Maximization of expected information gain** between prior and posterior

- Use of **surrogate model** (PCE) for MC approximation and **stochastic optimization** (CMA-ES) methods for computational speed-up

- Methods for designing the SHM network in terms of $\boldsymbol{n_{sens}}, \boldsymbol{\sigma}$ and $\boldsymbol{d}$:
  - Information maximization
  - Pareto frontiers
  - Efficiency maximization

- **Quantitative comparison** between different design solutions