

A Statistical Method for Area Coverage Estimation and Loss Probability Analysis on Mobile Sensor Networks [†]

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Abstract: Sensor networks are formed by fixed or mobile sensor nodes and their functions are to capture the events that occur within a certain area and then relay to a central node. Normally sensor nodes are not able to transmit or receive information over long distances due to the need to use less energy and thus extend their useful life. Therefore, the number of sensor nodes in a given area directly influences the coverage of this area and the ability of information to be relayed by several sensors to the central node. If there are many missed messages the application will have its performance compromised. In this paper we use a statistical method-based on Monte Carlo approach to estimate the probability of message loss and area coverage. The position and proper motion of the sensors are randomly chosen and from that we estimate how many nodes can communicate with the central node directly or through another sensor working as relay. The free variables in our analysis are node density, node displacement velocity and sensor quantity. The results obtained are compared analytically with simple cases in order to validate the results obtained by the simulations performed.

Keywords: Sensor Networks; Mobile Ad-Hoc Network; Wireless Sensor Networks Coverage

1. Introduction

A sensor networks are characterized by the distribution of sensor nodes able to capture information, such as temperature, positioning, depth, wind speed or even capturing an image and transmitting the information to a system able to interpret and process the collected data. The sensors can be distributed in previously established positions or randomly in a given area, known as sensing area. They can also be fixed or mobile, depending on the application.

In certain cases, these sensors are best installed in remote terrains, such as forests, for example, or agricultural fields, or even in battle fields. For civilian applications, these sensors can be installed in more urbanized places, such as universities or industries, for example.

Regardless of the characteristics of the application, the sensor network must be able to meet the minimum service requirements, such as: capacity of delivery, transmission delay, power consumption, fault tolerance and sensing capacity. Other two requirements that must be considered part of quality of service in a sensor network, and that have a direct impact on the consumption of power are: connectivity and the coverage provided by the distribution of the sensor nodes [1]. According to [2], the attention of researchers has been increasingly drawn to this last parameter, since many times the sensor nodes are distributed randomly in the sensing area, and therefore the correct calculation of the ratio between coverage and number of nodes is essential in order to reach the objectives of the application.

In further consideration of coverage, in environmental monitoring e.g., there are fixed sensors, which simplifies the analysis of sensor distribution and, consequently, of coverage capacity: For a broad range of applications, it is impossible to divide the network into disconnected sets of nodes, making the problem of connectivity a crucial issue. For the purpose of this paper, the coverage and the connectivity of the sensor nodes are considered conjointly.

Other papers related to similar problems are present in [3],[4],[5],[6],[7],[8], [9],[10],[11],[12],[13], [14].

The scope of this paper is to present a mathematical model based on the calculation of the probability of a given sensor in movement to transmit or receive captured information or control messages, such as synchronization messages, using the sensing area, the sensor transmission range and the number of active nodes distributed in the target area as input parameters. To this end, the mathematical model that was developed is presented, considering sensor nodes moving at low speeds (e.g. a flock in a field), and at high speeds (e.g. drones over a forested or natural disaster area) and the deployment of the software model.

While leaving many sensor nodes in a state of dormancy increases the power efficiency of the network as a whole, it also reduces both the sensing area coverage as well as its connectivity [15].

An area is considered as having coverage when each point in the area to be monitored is under the surveillance of a sensor node, while a wireless sensor network is considered connected if each pair of sensor nodes is able to communicate directly or indirectly with other sensor nodes, with the purpose of discovering a minimum subset of active sensor nodes for the captured data to be sent to the processing system or the sink node [16].

2. Description the Problem and Methodology

The capacity of a message being delivered on the network depends on the existence of a path defined by the routing protocol. This dependence is related to the distance between the sensor nodes and the speed that these nodes are moving at. The implementation of this model is based on the assumption that the mobility of the sensor nodes always takes place within a defined sensing region and a known area.

Considering the t_0 and t_1 steps of a discrete event simulator, there is a probability that the sensor node will be connected at t_1 , assuming that it was connected at t_0 . Figure 1 presents two distributions of sensor nodes in a sensing area. At t_0 , the sensor nodes were distributed randomly. Therefore, there is a probability that a sensor node will be connected at t_0 , defined as P_{t_0} . At the next t_1 instant, due to the mobility of the sensor nodes, the distribution of the sensor nodes in the sensing area will not be the same. Therefore, there is a new probability that a sensor node will be connected, defined as P_{t_1} .

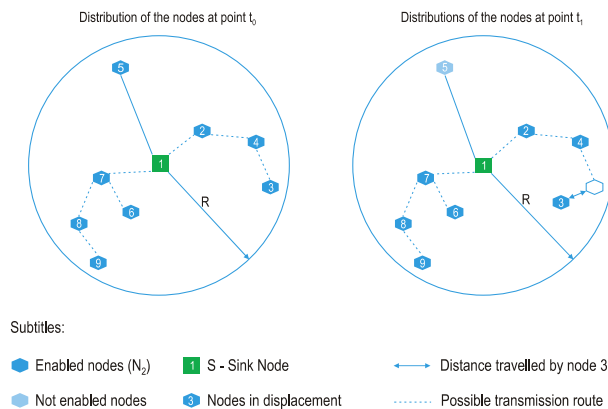


Figure 1. Displacement of Sensor Node 3 at Times t_0 and t_1 .

In the case of high speeds, this probability is a random variable that depends on the coverage area, while for low dislocation speeds, the probability of the sensor node being connected depends on the coverage area and the state of connectivity in the previous step.

The distance traveled by the node (D_p) is calculated by the following equation (1):

$$D_p = V_p / \Delta t, \text{ where } \Delta t = (t_1 - t_0) \tag{1}$$

If D_p is greater than r ($D_p > r$), then the dislocation speed is considered high and the probability of a given x_i sensor node receiving a synchronization message at t_0 is the same as the probability of this same sensor node receiving a message at t_1 . This is called the P_{random} Probability, as shown in equation (2) below:

$$P_{t_0}(x_i) = P_{t_1}(x_i) = P_{\text{random}} = P_{t_k}(x_i) \tag{2}$$

For the sensor node dislocation speed to be considered low, the distance traveled by the sensor node at Δt ($t_1 - t_0$) must be less than the distance of the radio transmitter's range ($D_p < r$) of the same sensor node, considering that at t_0 , there is a valid communication route between the sensor node and the sink node.

The probability of sensor node x_i receiving a message (e.g. synchronism message) at t_k , assuming that this message was received at t_0 , is calculated by equation (3):

$$P_{t_k}(x_i) = 1 - \frac{1 - P_{\text{random}}}{n \Delta t} \cdot k \cdot \Delta t \quad 0 < k \leq n + 1 \tag{3}$$

The probabilities of an x_i sensor node establishing communication with the sink node when moving at high or low speeds is given by P_{random} which is calculated estimating the coverage area.

For a simple case with two sensor nodes the probability of communication using an intermediate node as relay is presented in equation (4):

$$P \{ \rho \leq D_s(x_1) \leq \rho + \delta\rho \mid x_1 \text{ communicates with } S \} \cdot P \{ x_2 \text{ communicates with } x_1 \} = \frac{2 \pi \rho \delta\rho}{\pi r^2} \cdot \frac{r^2}{R^2} = \frac{2 \rho \delta\rho}{R^2} \tag{4}$$

One can thus remove the condition of the probability presented in equation (4) and eliminate parameter ρ , and obtain from this probability, based on parameters R (radius of the sensing area) and r (transmission range of the sensor node), resulting in equation (5), which shall be used to validate the model.

$$P\{X_2 \rightarrow X_1 \rightarrow X_S\} = \left(\frac{\pi r^4}{R^2} + \frac{4 r^4}{3 R^2} (\sqrt{3} - \pi) + \frac{r^4}{R^2} \left[\frac{16}{3} - 2\sqrt{3} \right] \right) \frac{1}{\pi R^2} \tag{5}$$

For the generalization of the P_{random} , one must calculate the total area of coverage by up to four sensors and discount the intersections. This result in the expression used in our simulator.

Based on the analysis of the several possible cases presented, one can determine that the area added by a given X_{li} sensor node, positioned within the range of the S central sensor node, can be generalized by equation (6), displayed below, as long as all X_{li} sensor nodes are ordered ascending from angle α , as shown in Figure 2.

$$A_{\text{added by } X_{li}} = A_{X_{li}} - A_{\text{intersection}(X_{li}/S)} - A_{\text{intersection}(X_{li}/X_{l-1})} + A_{\text{intersection}(X_{li}/X_{l-1}/S)} \tag{6}$$

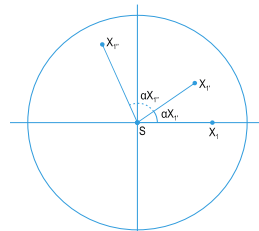


Figure 2. Angle α order.

Thus, the generalization of P_{random} to establish communication between sensor node X_2 , positioned outside the range of central sensor node S , and the sink sensor node, by means of sensor node X_1 , positioned within the range of the sink sensor node, is calculated by equation (7)

$$P_{\text{random}} = \frac{\sum \text{Added Areas} + \text{Fundamental Area}}{A_{\text{SR}}} \tag{7}$$

where A_{SR} is the sensing area and is calculated by: $A_{\text{SR}} = \pi R^2$

3. Simulation

To analyze the probability of a given node to establish communication with the sink sensor node, a software was developed in MATLAB, based on the Monte Carlo Method. This statistical simulation method uses a random sequence of numbers to develop simulations. In other words, it is considered a universal numerical method to solve problems by means of random sampling (approximating the solution).

There is no need to write down the differential equations describing the behavior of complex systems for this method. The only requirement is that the physical or mathematical system be described (modeled) in terms of functions of density of probability distribution (FDP). Once these distributions have been established, the Monte Carlo Simulation can begin random sampling, based on them. This process is repeated innumerable times, and the desired result is obtained by means to statistical techniques on a given number of executions (samples) that can vary from dozens to millions of times [17].

3.1. Validation

Considering two sensor nodes, X_1 and X_2 with a transmission range of r , randomly distributed in a circular sensing area A_{SR} , with a radius of R and an S sink node, positioned in the center of the A_{SR} area, there is a Probability $P(E_i)$ of 4 possible events occurring, as described in table 1

Table 1. Possible Events.

Event	Sensor X_1	Sensor X_2	Description
E_1	$X_1 \times S$	$X_2 \times S$	X_1 and X_2 do not communicate directly with S node
E_2	$X_1 \rightarrow S$	$X_2 \times S$ $X_2 \times X_1$	X_1 communicates directly with S node X_2 does not communicate with X_1
E_3	$X_1 \rightarrow S$	$X_2 \rightarrow X_1$	X_1 communicates directly with S node; X_2 communicate directly with X_1
E_4	$X_1 \rightarrow S$	$X_2 \rightarrow S$	X_1 and X_2 communicates directly with S node

For each event in table 1, simulations were carried out using the developed software. The results were compared with the results returned from the mathematical solution developed and presented in section 3.

Table 2. Validation Parameters.

Parameters	Values
radius_area	$R = 4r$
radius_range	r
qtd_sensors	2 (X_1 and X_2 sensors plus sink node)
A	1 (all nodes are actives)

Event E_3 is mathematically modeled in section 3, and its extrapolation was implemented in software., so that fort a given X_i sensor node to establish communication with the sink node through n hops. Thus, to validate the software, 22 simulations were executed, with a fixed number of repetitions defined for each simulation. The first simulation was repeated 8,000 times and the last 60,000 times. The calculated and simulated results for the other events, as well as the mean of the simulated results achieved for event 3 are presented in Table 3

Table 3. Probability per event.

Event	Calculated Results	Simulated Results
E ₁	87,89 %	87,74 %
E ₂	11,39 %	11,56 %
E ₃	0,33 %	0,33 %
E ₄	0,39 %	0,37 %

The main event analyzed is E₃, since this event represents the probability of a sensor node communicating with the sink node through another sensor node ($X_2 \rightarrow X_1 \rightarrow S$). As observed in the Table 3, the average probability obtained with the simulation of the developed software was 0.33%, while the mathematically calculated probability is 0.33% (using equation 5). This shows that it is possible to use software to achieve simulation results for extrapolated scenarios, in other words, with quantities higher than 2 sensor nodes, without the need of developing complex mathematical models.

3.2. Experiments and Measurements

In order to simulate a scenario as similar as possible to a real-life situation, we used a circular sensing area equivalent to a 4000-hectare farm [18], sensors with a 100-meter transmission range and the sink node placed in the center of the Area. Table 4 shows all the input parameters used in the simulation. In the simulation scenario, the delay in communication between the sensor and sink sensor node is not taken into consideration.

Table 4. Simulation Parameters.

Parameter	Value
radius_area	3570 m
radius_range	100 m
qtd_sensors	Variation between 4000 and 10000
A	1 (All nodes are actives)
qtd_executions	200

The achieved results, shown in figure 3, demonstrate that if 4000 sensors are distributed over an area to be monitored, the probability of a new sensor randomly distributed in this same area being able to establish contact with the sink node is 0.96%. However, if in this same area, there are 7000 sensors, the probability of communication soars to 92%. This means that for an increase of 75% in the number of sensors, the probability of there being communication is 96.8% higher. On the other hand, to achieve a 99.77% probability of communication, the number of sensors must be increased 2.5 times.

Based on the data achieved with 8000 sensors, the probability of communication is 97.59%. However, to achieve a probability of communication 1.0223 times higher, the number of sensors would have to be increased by 25%. This shows that, for a given number of distributed sensor nodes, there is a saturation of the probability of communication, which leads to the conclusion that any gains from increased sensor node distribution, without evaluating the coverage, is minimal.

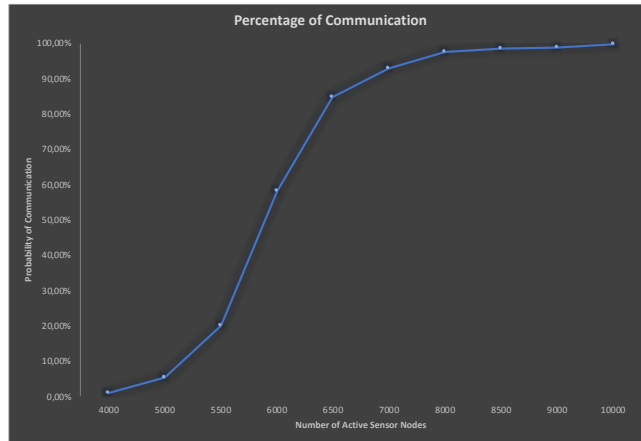


Figure 3. Number of Nodes x Communication Percentage.

In order to evaluate the behavior of the probability of communication in relation to the variation of the coverage radius of the sensor (r/R), another simulation was carried out. For this study, the number of sensor nodes was fixed, and the variation of the sensor node transmission range (r) was executed in steps of 50 meters, starting at 100m (base scenario).

To establish the number of sensor nodes to be used in the simulation, a new simulation was made, with the transmission range of the sensor node fixed at 0.1 of the Radius of the Sensing area, that is, $r = 0.1R$, and the number of active sensor nodes was varied from 100 sensors up, with additions of 100 units.

Based on the results achieved with this simulation, it was established that with 450 active sensors, the probability of communication is 49.48% (approximately 50%), and, therefore, this was the amount of sensor nodes chosen to verify the behavior of the sensor nodes based on the relation r/R . The behavior of the probability of communication based on the relation (r/R) is presented in figure 4.

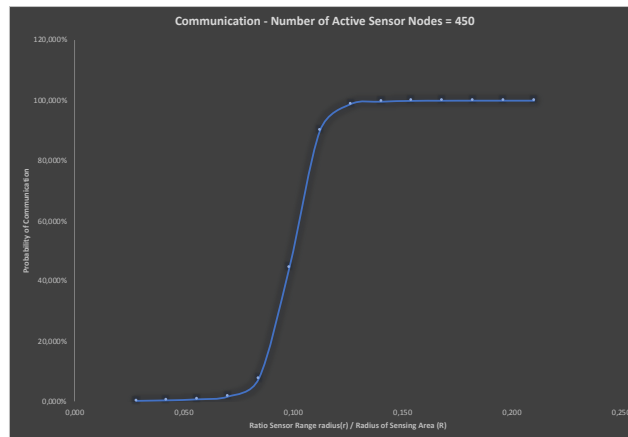


Figure 4. Relation r/R x Probability of Communication.

An analysis of the results shows that the relation r/R is a determining factor only in one segment of the relation r/R . Based on the approximate value of $r/R = 0.12$, the probability of communication remains practically the same. This is relevant from the viewpoint of sensor network designers who need specific equipment that have both optimal coverage, which means higher transmission ranges, and sensors with long lasting batteries.

4. Conclusions

This paper establishes a way to evaluate the probability of communication between a given sensor node and the sink node. The achieved results demonstrate an increasing relationship between number of sensor nodes and the probability of communication, by which it is possible to evaluate the gains in communication by increasing the number of nodes in a sensing area.

This interpretation can be extrapolated, bringing one to the conclusion that the probability of communication demonstrates the coverage percentage of the monitored area.

With the software developed to execute the simulation, it is possible to reduce the complexity of mathematical models to analyze the coverage and communication in sensor networks that require large amounts of sensor nodes.

Another relevant contribution lies in the analysis of the relation between the range of the sensor, the sensing area and the number of nodes, since these factors bear a significant impact on the life of the sensor batteries. This them has been studies over the years, and the analysis of the r/R relation from this perspective contributes to increase the efficiency and minimize the deployment costs of sensor networks.

The proposed model can also be used for different applications, such as FANETs, military applications, the monitoring of vehicles on highways, border surveillance, to name a few.

It is the author's intention, based on the proposed model, to evaluate the impact of the level of synchronization on a sensor node, considering the delay in communication variable and the mobility variable when nodes are engaged in low-speed dislocations.

5. Patents

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