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Inheritance is a Surjection: Description and Consequences

Supplementary Notes and Basic Definitions

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Results and Discussion

In genetics, the biological descendants are the actual biological descendants of the identified parental pair.

In human systems, the descendants may be the actual biological descendants of the parental pair, but may also reflect other cultural assignments, such as by (non-inherited) devices such as adoption.

Conclusions

The well known mathematical name for this relation is surjection.

Theorems inferences for surjections include [1 page 178], [2], [3] [4].

Results and Discussion

[1, 2, 3, 4] show a surjection that also means that:

- The newly created offspring individuals are each "unique"
- The entities to which offspring are assigned as their parental organisms are identical or isomorphic in how the parents act.

Neither of those notions was in our concepts of the subjects here.

Thus we must first demonstrate if "uniqueness" and "identical or isomorphic forms of reproduction" exist for our result to be a surjection.

Conclusions: Uniqueness of Individuals

Radzevilavicius and Blackstone's [5] review of individuality of offspring populations shows that all individuals, of all species, are in some form unique.

Descriptions of standard genetic statistics include Mendel [7], Bernstein [8, 9]; Fischer [10]; Haldane [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]; Wright [21]; Nei [22]; Crow and Kimura [23] and useful comparisons including [24, 25, 26, 27, 28, 29, 30]. These statistics are of biological inheritance only.

Conclusions: Identical Means of Reproduction

The basic mechanics for construction of DNA and RNA molecules are structured by the group-theoretical results of physical mechanics.

For DNA, see Petoukhov [31, 32, 33, 34, 35], Petoukhov and Castro-Chaves [36], Petoukhov and Hu [37], Petoukhov, Petoukhova and Svirin [38], Hainen [39]’ Rakocevic [40] and others

For RNA, see similar work by Khrennikov and Kozyrev, Rakocevic, and by A. Dragovich and B. Dragovich [41, 42, 43, 44, 45, 46].

Since all must satisfy specific group-theoretical forms, so the isomorphisms of those group theoretical forms (required by how alleles may form under the allowed physical mechanics) means that all organisms are formed using the specific sets required by mathematical groups, thus acting under isomorphic rules of combination.

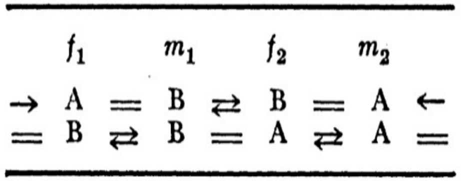
Results and Discussion: Cultural Kinship

In 1945 ethnographer Ruheman [50] created the first description of kinship which identified objects that are clearly mathematical groups.

The paper references many articles since then, which show that the descriptions of human cultural kinship are now considered to be mathematical groups. Thus, for each kinship system, when the culture applies its marriage rules in terms of kinship, it is similar to the results on genetics: the rules of the groups mean than all members of that culture apply isomorphic rules in how they select marriage partners.

Results and Discussion: Cultural Kinship

FIGURE 1: EXAMPLE FROM RUHEMAN'S STUDY OF AUSTRALIAN KINSHIP



Ruheman denoted each cultural system by placing names of kinship relations onto finite sets of offspring using the culture's language, and used diagrams to show how a terminology relates those sets of labels onto representative minimal set of parents. She assumed each system had distinct generations. She used labels A and B for moiety names. In this diagram, each generations minimal structure has two marriages. That minimal group thus has an order = s. The *structural number* of a rule is equal to the size of the minimal group representing that rule. Thus for Figure 1, the structural number of that rule is s = 2.

A marriage rule is also described as the *history* of that system, as it generates possible future relations.

Results and Discussion: Genetics vs. Cultural Kinship

In genetics, the results of particular matings record the actual number of surviving offspring from each parental pair. The results are summarized by genetic statistics noted in our Part 3. Statistics are biological.

For cultural kinship, knowing the order of a culture's minimal group, and using the fact that inheritance is a surjection, allows us to predict present and future average cultural populations statistics for that culture; such statistics may not be reflective of direct inheritance from parents.

For each cultural structural number s, are specific values for each s of *average family size* n_s and percentage of *reproducing females* p_s for each s. We then find cultural *average family size* n(t) and *percentage of reproducing married females* p(t) of each culture at time t, as the weighted average of the percentage of the society which uses structural number s, added over the entire society at each t. See our Part 6.

Results and Discussion: Kinship Population Examples

- Finding cultural rules of kinship and marriage are surjections (Part 2) allows to apply the statistics of the statistic SNSK.
- Most of the "standard statistics" in academic statistics courses are adaptations of the distributions of volumes of identical gas particles, distributed into specific (and distinct) parts of real space. But surjection means we have unique offspring, being assigned to parents who use identical or isomorphic means of process.
- The assumptions of the surjection are thus the opposite of what is required compared to use standard academic statistics.
- Statistics of SNSK are exactly what is required: unique objects (as individuals in the offspring population), and isomorphic means of treatment by the parents. This allows the computations of our Part 6.

Results and Discussion: Kinship Population Examples

SNSK make verifiable predictions of real ethnographies. [84] used the changes in the structural numbers to predict the changes in Western European population statistics from about AD 1000 to about AD 1950. It allowed to (approximately) forecast the changes in average family sizes as the system when from more exclusionary rules (higher s) to less exclusionary rules (lower s). In that period also was the beginning of more “Protestant” marriage rules with lower s, as local populations drew below the previous higher s required by the Catholic rules.

Results and Discussion: Kinship Population Examples

[48] includes that the possible changes among the available sets of histories at time t, might also limit what choices are available, and thus may limit what alternate forms of histories might be possible.

[87, 88] discuss not only the values of n_s and p_s but also how the distributions of choices (the mechanics of the application of the d^{-1} devices in our diagram) move parts of other cultural practices when a village might be subdividing.

There are thus many things that need to be better understood in ethnographies, which can be better explained when the use of proper statistics (here, using SNSK) is allowed.

Results and Discussion: Forecasting Evolution

Evolutionary biologist Wilson recently published *Genesis* [94], which intends to forecast the next stage in evolution of life. Even a sympathetic reviewer [95] said he reaches no such conclusion. But our Parts 4 and 6 also discuss forecasting evolution.

Schroeck [89, pages 223 – 253] used symmetrical configurations to describe biological evolution in bats. Schroeck is demonstrating the results of evolution. As the mechanisms of the other flying animals became fast enough, the bats internal locational devices “ … shows that bats perform at the quantum limit of uncertainty relations … ”, and also that “… we observe that joint time/frequency spectrograms are being used as a standard analysis procedure in studying biological and social systems. We list the ‘pictures’ of bird songs in many popular bird identification manuals, as well as the identification of vocal expressions of stump-tailed macaques … as examples.”

Results and Discussion: Forcasting Evolution

- The mechanics allowed by biological statistics of Part 3 citations, allowed biological evolution determined by physical mechanics, as allowed by mathematical groups (see articles in our Part 4).
- Schroeck thus also claimed (correctly) that birds began many millions of years ago to use verbal communication with each other. If humans did not have verbal communication, the sections of this paper describing human ability to describe society using kinship terminologies would not exist – thus their role in using mathematical groups to understand the demography of human societies would not be found, if indeed we could even recognize it. Human (biologically created) cognitive ability, some parts of which include birds [104 and other citations], verify Schroeck's thoughts; other species may have at least parts of these abilities.
- Thus biologically created devices in existing populations may allow creatures to create their own future biological evolution.

Results and Discussion

- Human logic (a product of biological evolution) can already forecast results of much of mathematics of physics.
- The human brain (a product of biological evolution) can forecast future versions of many domesticated crops and animals; indeed so have other species that have “tended” crops.
- Then why can’t humans forecast (indeed create) their own cultural evolution and natural biological evolution?
- Humans themselves of course are part of both forms of evolution; see also [108].

Conclusions

- Physical mechanics (Part 4) allows us to describe the objects on which evolution can take place.
- Genetic statistics (Part 3) describes “only” the means of demonstrating empirical evolutionary effects in actual use.
- We need to understand both forms of prediction, and to understand how they relate to each other.
- But evolution may only occur only in specific directions implied by what Part 4 devices can create and allow.

Conclusions

The foundation question thus are:

- Given the things that Part 4 allows, do those necessarily cause the results of evolution?
- Can the mechanical foundations of biological evolution (Part 4) can lead us (humans!) to forecast how future biological, and cultural, evolution might occur?
- Can we (humans) predict them?

The answer to all is likely "yes".

Supplementary Materials

See **References** to the full paper.

Definitions

generation – a defined limited but continuous duration of time, called t

members – a non-empty set of “individuals” of some species alive at time t

parents -- a pair of members of the same generation who are assigned as creating new individuals at t

offspring – a set of individuals who are assigned in generation t + 1 to be “descendants” from a set of parents in t

evolutionary process – a process that creates, or changes what may be created in evolution, by means of defined interactions between two individuals of the same generation

genetics – an evolutionary process in which the parents are creating new individuals by means of transmission of genes:

- the position in the genome where there are or can be variants is a **locus**.
- the alternative forms of DNA at the position of a locus are **alleles**.

mathematical anthropology – a mathematical representation of a cultural evolutionary process in which the parents are identified by cultural rules and to which offspring are assigned.

surjection – a collection of two sets of non-empty objects P and O, where $P \cap O = 0$, and where each member of P has at least one member of O linked to it. (Note therefore that the members of P can be called “parents” and the members of O can be called “offspring”.)

(mathematical) group – a non-empty set G with an operation + such that for each g in G:

- an object o in G such that for every g in G, $g + o = o + g = g$;
- an object in G called identity or I;
- for each g in G is a member g^{-1} in G such that for each g,
  $g + g^{-1} = g^{-1} + g = I$;
- for any pair of objects g, h in G, $g + h$ and $h + g$ are members of G, and $g + h = h + g$.

describing or transmitting cultural information:

- α, β are used to represent a particular cultural rule, also called a **history.**
- the proportion of each generation population G_t using history α is $\mathbf{v}_{\alpha t}$.
- each cultural rule of marriage has a **structural number** *s*, which is set by the size of the minimal group representing that rule (page 4).
- let s be the **structural number** of history α.
- let n_s = **average family size** (average number of offspring to a set of parents in time period t) of a system with structural number s.
- Define the proportion of (socially ascribed) **reproducing females** as p_s. Let $p_s = 2/n_s$.
- The specific values of p_s, and n_s are presented in [3 appendix 2]

Part 6, **Equations 1 through 4 show how to predict for each t**:

- the value of the expected surviving **average family size** n(t);
- average percent of assumed **reproducing females** p(t);
- average expected **change in population growth** r(t);

The demographic papers [80, 81, 82] also use a notation r(t); since we use that here for a different concept, we thus need a distinct representation for that demographic result. We thus use R(t) for the demographic concept described in [80, 81, 82] as r(t). The relation between r(t) and R(t) are given by Equation 4.

mathematical diagram:

Let:	Mean:
D	biological population operators of the population at the indexed time t,
G_t	is the population size at time t
M_t	the sets of married or biologically mating couples at time t, with $M_t \subseteq G_t$
B_{t+1}	a partition of G_{t+1} into sets of offspring with the same parents, with $B_{t+1} \subseteq G_{t+1}$
μ	a surjection corresponding to assignments of M_t,
π	a partition of G_{t+1} showing kin groups of a population within a generation t+1, as assigned by the genetic or cultural rules of marriage
d^{-1}	a surjection corresponding to descent
d	an injection corresponding to ancestor

so that the d^{-1} surjection maps the progress of population change showing sets of descendants in generation t + 1 onto the sets of their parents in generation t.

This gives us a diagram summarizing our mathematical representations:

$$\begin{tikzcd}
G_t \arrow[r, "D"] \arrow[d, "\mu"] & G_{t+1} \\
M_t \arrow[r, "d^{-1}"'] & B_{t+1} \arrow[u, "\pi"']
\end{tikzcd}$$

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