Modified Commutators vs Modified Operators in a Quantum Gravity minimal length scale*

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arXiv:2009.12348

Quantum Gravity Models

- Generic quantum gravity models predict minimal length.
- Minimal length implies photon dispersion.
- Modified energy-momentum relation for photons.*

$$p^2c^2 = E^2[1 + f(E/E_{QG})] \tag{1}$$

- E_{QG} is often set to the Planck scale ($\sqrt{\hbar c^5/G} \approx 10^{19}$ GeV).

*G. Amelino-Camelia *et al.* Nature, **393**, 763 (1998).

Modified Energy-Momentum and Photon Dispersion.

- Taylor expand the modified energy-momentum relationship (1) and ignore the higher order terms.

$$p^2c^2 = E^2[1 + \xi(E/E_{QG}) + \mathcal{O}(E/E_{QG})^2] \tag{2}$$

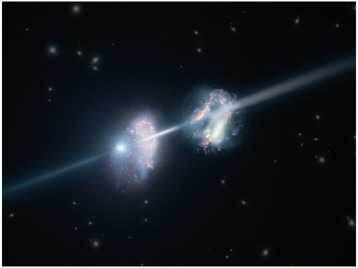
- Photons with different energies are then predicted to have different velocities.

$$v = \frac{\partial E}{\partial p} \approx c\left(1 - \xi \frac{E}{E_{QG}}\right) \tag{3}$$

Testing for Photon Dispersion

- Gamma Ray Bursts (highly energetic photon emissions)
- Short GRB are preferred for testing (less intrinsic lag)
- Different energies lead to different arrival times.

$$\delta t = \xi \frac{L}{c} \frac{\delta E}{E_{QG}} \tag{4}$$



GRB 090510

- GRB detected by Fermi Telescope 2009.*
- Searched for LIV-induced time delay using gamma rays from
  1. Energy: 35 MeV to 31 GeV ($E < 28\ \mathrm{GeV}$ for 99%)
  2. Burst interval: 0.5 s to 1.45 s
- $\left|\frac{\delta t}{\delta E}\right| = \frac{L}{c}\frac{1}{E_{QG}} < 30\mathrm{ms}\ \mathrm{GeV}^{-1}$ for 99% of gamma rays

  $\Rightarrow 1.6\ E_{Pl} < E_{QG}$ but expected result: $E_{QG} \leq E_{Pl}$
- For other data: $1.19 E_{Pl} < E_{QG}$ to $102 E_{Pl} < E_{QG}$

*Doi: 10.1038/nature08574 (2009).

A problem for Minimal Length

- Observational data currently show no such dispersion.
- Implies spacetime is smooth? No minimum length scale?
- A new approach is required.

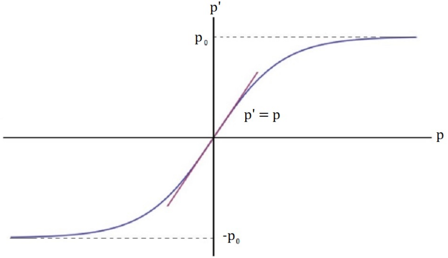
Quantum Gravity via Modified Operators

- We now introduce new forms of the modified momentum operators.

$$p' = p_0 \tanh\left(\frac{p}{p_0}\right) \tag{5}$$

$$p' = p_0 \arctan\left(\frac{p}{p_0}\right) \tag{6}$$

- These have bounds of $\pm \mathrm{p}_0$ as $p \to \infty$, while at low p they reduced to the standard momentum operators.



Energy-Momentum Relation

- Modified momentum leads to modified energies in the energy-momentum relationship.

$$E'^2 = p'^2 c^2 + m^2 c^4 \tag{7}$$

- For photons, the energy expressions then become:

$$E' = p_0 c \;\tanh\left(\frac{p}{p_0}\right) = p' c \tag{8}$$

$$E' = p_0 c \;\arctan\left(\frac{p}{p_0}\right) = p' c \tag{9}$$

No Change in Photon Velocities

- We see now that there is no change to the regular dispersion relationship (all photons should arrive at the same time).

$$c = \frac{\partial E'}{\partial p'} \tag{10}$$

- No conflict with observational data of GRB's.

- Is there still a minimum length scale?

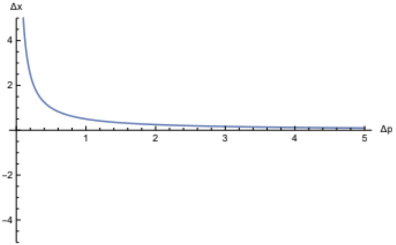
Minimum Length in Heisenberg Uncertainty Principle

- Heisenberg uncertainty principle (HUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad \rightarrow \quad \Delta x \propto \frac{1}{\Delta p}$$

- $\Delta x \rightarrow 0$ as $\Delta p \rightarrow \infty$.

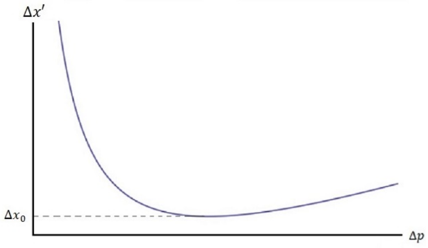
- No minimum length scale from HUP. $\Rightarrow$ Need to modify HUP.



Modification of HUP to GUP

- Generalized Uncertainty Principle (GUP)*

$$\Delta x' \Delta p \geq \frac{\hbar}{2}\left(1 + \beta \Delta p^2\right) \quad \rightarrow \quad \Delta x' \propto \frac{1}{\Delta p} + \beta \Delta p$$



- Minimum length at local minima. ($\Delta x_0 = \hbar\sqrt{\beta}$ *at* $\Delta p = \frac{1}{\sqrt{\beta}}$)

A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D **52**, 1108 (1995).*

Another Modification of the HUP

- Minimal length from modified operators and standard HUP.

$$\Delta x' \Delta p' \geq \frac{\hbar}{2} \tag{11}$$

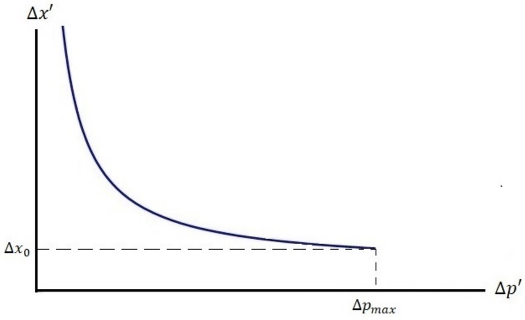
- Modify position operators to maintain the form of (11).

$$\hat{x}' = i\hbar \cosh^2\left(\frac{p}{p_0}\right)\partial_p \quad \text{for} \quad \hat{p}' = p_0 \tanh\left(\frac{p}{p_0}\right) \tag{12}$$

$$\hat{x}' = i\hbar \left[1 + \left(\frac{p}{p_0}\right)^2\right]\partial_p \quad \text{for} \quad \hat{p}' = p_0 \arctan\left(\frac{p}{p_0}\right) \tag{13}$$

Minimum Length Scale in modified HUP

- When the momentum reaches it's maximum value, we see a minimum length value for $\Delta x'$: ($\Delta x' \geq \frac{\hbar}{2p_0}$)
- This model maintains a dispersion relationship consistent with observational data.



Summaries and Conclusions

- Modified operators can lead to no dispersion.
- This still allows us to have a minimum length.
- This doesn't conflict with current observational data.
- Modified operators are a preferable approach to Quantum Gravity for proposing a minimal length scale.

End of Presentation

- Thank you for your time.