Non-perturbative QED on the Hopf bundle

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Physical motivations

Quantum electrodynamics (QED) and the electroweak theory are very successful in explaining quantum phenomena for electromagnetic and weak interactions. Their predictions agree with experimental data to great precision. This progress was achieved despite the fact that the calculations are perturbative and one needs to involve, for example, a renormalization procedure. R. Feynman called this procedure "sweeping the garbage under the rug." L. Landau et al. wrote on this subject: "Although at present there exist methods to remove these singularities (regularization), which clearly lead to correct results, such method of action has the artificial nature. The singularities arise in the theory due to the pointlike interaction described by delta functions (operators of interacting fields are taken at one point)."

Physical motivations

Attempts to use this technique for strong interactions do not lead to a full success, and in gravity they are unsuccessful. This suggests that at present we do not clearly understand the nature of quantization. It is reasonable to assume that there should exist some well-defined mathematical procedure of quantization which can be applied to *any* field theory. It should be pointed out here that the nonperturbative quantization technique was perhaps first suggested by W. Heisenberg in his book “Introduction to the unified field theory of elementary particles”.

Physical motivations

Taking all this into account, it is of great interest to construct nonperturbative QED for some simple case in order to compare perturbative and nonperturbative QED. It would allow one to understand the physical essence of such phenomena like the renormalization, convergence of the Feynman integral, etc. Simultaneously, it would be possible to construct perturbative QED according to conventional methods. After that, there would appear the possibility of comparison of perturbative and nonperturbative quantum theories.
Consistent with this, here, we find a discrete spectrum of classical solutions describing noninteracting Dirac and Maxwell fields in a spacetime with a spatial cross-section in the form of the Hopf bundle. Then we use the spectra obtained to quantize the Dirac equation and Maxwell's electrodynamics. Finally, we suggest a scheme of nonperturbative quantization of coupled Dirac and Maxwell fields.

Classical electrodynamics plus the Dirac equation

Consider first classical electrodynamics coupled to spinors obeying the Dirac equation in $\mathbb{R} \times S^3$ spacetime with a spatial cross-section in the form of the Hopf bundle $S^3 \to S^2$. Such a theory can be regarded as a relativistic quantum theory of an electron interacting with an electromagnetic field and living on the the Hopf bundle. The source of electromagnetic field is taken in the form of a massless Dirac field. The corresponding Lagrangian can be chosen in the form

$$L = \frac{i}{2}\left(\bar{\psi}\gamma^\mu\psi_{;\mu} - \bar{\psi}_{;\mu}\gamma^\mu\psi\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \qquad (1)$$

Classical electrodynamics plus the Dirac equation

Corresponding fundamental equations are

$$i\gamma^{\mu}\psi_{;\mu} = 0, \tag{2}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\nu}}\left(\sqrt{-g}F^{\mu\nu}\right) = -4\pi j^{\mu}. \tag{3}$$

They will be solved in $\mathbb{R} \times S^3$ spacetime with the Hopf coordinates χ, θ, φ on a sphere S^3 with the metric

$$ds^2 = dt^2 - \frac{r^2}{4}\left[\left(d\chi - \cos\theta d\varphi\right)^2 + d\theta^2 + \sin^2\theta d\varphi^2\right] = dt^2 - r^2 dS_3^2.$$

To solve Eqs. (2) and (3), we employ the following $\mathfrak{Ansatze}$ for the spinor and electromagnetic fields:

$$\psi_{nm} = e^{-i\Omega t} e^{in\chi} e^{im\varphi} \begin{pmatrix} \Theta_1(\theta) \\ \Theta_2(\theta) \\ 0 \\ 0 \end{pmatrix} , \tag{4}$$

$$A_\mu = \left\{ \phi(\theta), r\rho(\theta), 0, r\lambda(\theta) \right\} , \tag{5}$$

where m and n are integers.

On substitution of the Ansatze (4) and (5) in Eqs. (2) and (3), we have the following set of equations:

$$\Theta_1' + \Theta_1 \left(\frac{\cot\theta}{2} + n + er\rho \right) + \Theta_2 \left(\frac{1}{4} - \frac{r\Omega}{2} - n\cot\theta - \frac{m}{\sin\theta} - er\rho\cot\theta - \frac{er\lambda}{\sin\theta} + \frac{er}{2}\phi \right) = 0, \tag{6}$$

$$\Theta_2' + \Theta_2 \left(\frac{\cot\theta}{2} - n - er\rho \right) + \Theta_1 \left(-\frac{1}{4} + \frac{r\Omega}{2} - n\cot\theta - \frac{m}{\sin\theta} - er\rho\cot\theta - \frac{er\lambda}{\sin\theta} - \frac{er}{2}\phi \right) = 0, \tag{7}$$

$$\frac{1}{\sin\theta} \left(\sin\theta\,\phi' \right)' = \phi'' + \cot\theta\,\phi' = -\frac{er^2}{4} \left(\Theta_1^2 + \Theta_2^2 \right), \tag{8}$$

$$\left(\frac{\rho'}{\sin\theta} \right)' + \left(\cot\theta\,\lambda' \right)' = \frac{\rho''}{\sin\theta} - \frac{\cot\theta}{\sin\theta}\rho' + \cot\theta\,\lambda'' - \frac{\lambda'}{\sin^2\theta} = \frac{er^2}{8} \left[\cos\theta \left(\Theta_1^2 - \Theta_2^2 \right) + 2\sin\theta\,\Theta_1\Theta_2 \right], \tag{9}$$

$$\left(\frac{\lambda'}{\sin\theta} \right)' + \left(\cot\theta\,\rho' \right)' = \frac{\lambda''}{\sin\theta} - \frac{\cot\theta}{\sin\theta}\lambda' + \cot\theta\,\rho'' - \frac{\rho'}{\sin^2\theta} = \frac{er^2}{8} \left(\Theta_1^2 - \Theta_2^2 \right), \tag{10}$$

where e is a charge in Maxwell theory and the prime denotes differentiation with respect to θ.

Consider first the simplified case of “frozen” electric and magnetic fields with zero scalar and vector potentials, $\phi = \rho = \lambda = 0$. In this case the parent Dirac equations (6) and (7) take the form

$$\Theta_1' + \Theta_1 \left(\frac{\cot\theta}{2} + n \right) + \Theta_2 \left(\tilde{\Omega} - n\cot\theta - \frac{m}{\sin\theta} \right) = 0,$$
$$\Theta_2' + \Theta_2 \left(\frac{\cot\theta}{2} - n \right) + \Theta_1 \left(-\tilde{\Omega} - n\cot\theta - \frac{m}{\sin\theta} \right) = 0.$$

Classical vacuum solutions to the Dirac equation

Their solution is

$$\psi_{nml} = e^{-i\Omega_{nl} t} e^{in\chi} e^{im\varphi} \begin{pmatrix} \Theta_{1,nml} \\ \Theta_{2,nml} \end{pmatrix} = \psi_{+1/2,nml} + \psi_{-1/2,nml},$$

where $\psi_{\pm 1/2,nml}$ describe the states of particles with the projection of the spin $\pm 1/2$:

$$\begin{aligned}
\psi_{+1/2,nml} =& \tilde{c}_1 e^{-i\Omega_{nl} t} e^{in\chi} e^{im\varphi} \left(1-\cos\theta\right)^{\alpha/2} \left(1+\cos\theta\right)^{\beta/2} P_l^{(\alpha,\beta)}\left(\cos\theta\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\
\psi_{-1/2,nml} =& \tilde{c}_1 \frac{n+\tilde{\Omega}_{nl}}{2n+2m+1} e^{-i\Omega_{nl} t} e^{in\chi} e^{im\varphi} \left(1-\cos\theta\right)^{(\alpha+1)/2} \left(1+\cos\theta\right)^{(\beta+1)/2} \\
& \times P_{l-1}^{(\alpha+1,\beta+1)}\left(\cos\theta\right) \begin{pmatrix} -1 \\ 1 \end{pmatrix}.
\end{aligned}$$

With the energy given as

$$\tilde{\Omega}_{nl} = \pm(n+l), \text{ where } l = 0, 1, \ldots$$

Classical vacuum solutions to the Maxwell equations

The Maxwell equations on the Hopf bundle are

$$
\begin{aligned}
\phi'' + \cot\theta\phi' - \frac{p^2+q^2+2pq\cos\theta}{\sin^2\theta}\phi + \lambda\omega\frac{p\cos\theta+q}{\sin^2\theta} + \rho\omega\frac{p+q\cos\theta}{\sin^2\theta} &= 0,\\
\rho'' - \cot\theta\rho' + \cos\theta\lambda'' - \frac{\lambda'}{\sin\theta} + \rho\left(\frac{\omega^2}{4} - q^2\right) + \lambda\left(\frac{1}{4}\omega^2\cos\theta + pq\right) - \phi\frac{\omega\left(p+q\cos\theta\right)}{4} &= 0,\\
\frac{1}{4}\omega\sin^2\theta\;\phi' - \lambda'\left(p\cos\theta+q\right) - \rho'\left(p+q\cos\theta\right) &= 0,\\
\lambda'' - \cot\theta\lambda' + \cos\theta\rho'' - \frac{\rho'}{\sin\theta} + \lambda\left(\frac{\omega^2}{4} - p^2\right) + \rho\left(\frac{1}{4}\omega^2\cos\theta + pq\right) - \phi\frac{\omega\left(q+p\cos\theta\right)}{4} &= 0.
\end{aligned}
$$

Classical vacuum solutions to the Maxwell equations

Deriving an analytic solution to the set of Maxwell's equations runs into great difficulty. Therefore we can find numerical solutions and then show that there are analytic solutions for some particular values of the numbers p, q and of the functions ϕ, ρ, and λ.

Quantization of the Dirac field

Field operators $\hat{\psi}$ and $\hat{\psi}^\dagger$ can be written in the form

$$\hat{\psi} = \sum_n \sum_{|m|\leqslant n} \sum_l \Big[\hat{b}_{nml} e^{i(-\Omega_{nl} t + n\chi + m\varphi)} \Xi_{nml}\left(\cos\theta\right) + \left(\hat{c}_{nml}\right)^\dagger e^{i\left(-\Omega_{-n,l} t - n\chi - m\varphi\right)} \tilde{\Xi}_{nml}\left(\cos\theta\right) \Big],$$

$$\hat{\psi}^\dagger = \sum_n \sum_{|m|\leqslant n} \sum_l \Big[\left(\hat{b}_{nml}\right)^\dagger e^{-i(-\Omega_{nl} t + n\chi + m\varphi)} \Xi_{nml}\left(\cos\theta\right) + \hat{c}_{nml} e^{-i\left(-\Omega_{-n,l} t - n\chi - m\varphi\right)} \tilde{\Xi}_{nml}\left(\cos\theta\right) \Big],$$

where the spinors $\Xi_{nml}, \tilde{\Xi}_{nml}$ are

$$\Xi_{nml}(\cos\theta) = \begin{pmatrix} \Theta_{1,nml} \\ \Theta_{2,nml} \end{pmatrix}, \quad \tilde{\Xi}_{nml}(\cos\theta) = \begin{pmatrix} \Theta_{2,-n,-m,l} \\ \Theta_{1,-n,-m,l} \end{pmatrix}.$$

Quantization of the Dirac field

The energy is given by

$$\tilde{\Omega}_{nl} = \pm(n + l), \text{ where } l = 0, 1, \ldots$$

and

$$|n| \geqslant 1, |m| \leqslant |n|$$

are quantum numbers.

We impose the following anticommutation relations on these operators:

$$
\begin{aligned}
\left\{\hat{b}^s_{nml}, \left(\hat{b}^r_{pqt}\right)^\dagger\right\} &= \boxed{f_{nml}}\,\delta^{rs}\delta_{mp}\delta_{nq}\delta_{lt},\\
\left\{\hat{c}^s_{nml}, \left(\hat{c}^r_{pqt}\right)^\dagger\right\} &= \boxed{f_{nml}}\,\delta^{rs}\delta_{mp}\delta_{nq}\delta_{lt},\\
\left\{\hat{b}^s_{nml}, \hat{b}^r_{pqt}\right\} &= \left\{\hat{c}^s_{nm}, \hat{c}^r_{pqt}\right\} = \cdots = 0,
\end{aligned}
$$

where f_{nml} is a numerical factor possibly depending on n, m, and l, and this factor is chosen such that the infinite sum over n and l in the propagator (11) would be convergent.

Quantization of the Dirac field

The fermion propagator is defined as usual by the expression

$$
\begin{aligned}
& iS_{\alpha\beta}\left(t-t', \chi-\chi', \theta, \theta', \varphi-\varphi'\right) = \left\{\hat{\psi}_\alpha\left(t, \chi, \theta, \varphi\right), \hat{\bar{\psi}}_\beta\left(t', \chi', \theta', \varphi'\right)\right\} \\
& = 2\tilde{S}_{\alpha\beta} \sum_n \sum_{|m|\leqslant n} \sum_l f_{nml} \Big\{ \cos\Big[-\Omega_n(t-t') + n(\chi-\chi') + m(\varphi-\varphi')\Big] \\
& \times \sum_s F_{nml}^{(s,\alpha,\beta)}\left(\cos\theta\right) F_{nml}^{(s,\alpha,\beta)}\left(\cos\theta'\right) \Big\}.
\end{aligned} \tag{11}
$$

It turns out that for $\boxed{f_{nml}} = 1$ this sum diverges as $N, L \to \infty$. On the other hand, if one takes, for example,

$$\boxed{f_{nml}} = \frac{1}{n^a} \quad \text{with} \quad a \geqslant 8,$$

this sum converges.

Quantization of Maxwell's electrodynamics

The operator of the electromagnetic field four-potential can be written in the form

$$\hat{A}_\mu = \sum_{p,q,n} \left[\hat{a}_{pqn} e^{i\left(\frac{\omega_{pqn}}{r}t + p\chi + q\varphi\right)} \left(\tilde{A}_\mu\right)_{pqn}(\theta) + \left(\hat{a}_{pqn}\right)^\dagger e^{-i\left(\frac{\omega_{pqn}}{r}t + p\chi + q\varphi\right)} \left(\tilde{A}_\mu\right)_{pqn}(\theta) \right]$$

with the following standard commutation relations for the creation, $\left(\hat{a}_{pqn}\right)^\dagger$, and annihilation, $\hat{a}_{pqn}$, operators of the quantum state $\left|pqn\right\rangle$:

$$\left[\hat{a}_{pqn}, \left(\hat{a}_{rsm}\right)^\dagger\right] = f_{pqn}\delta_{pr}\delta_{qs}\delta_{nm} \quad \text{(all other commutators are zero).}$$

NP quantization of Dirac and Maxwell's electrodynamics

The results obtained above concerning the quantization of free Dirac and Maxwell fields need to be compared with the usual quantization in a box in Minkowski spacetime. The main difference is that in the box eigenfunctions are plain waves (because the spacetime is locally flat). On a sphere, plain waves cannot be eigenfunctions of the Dirac equation, since the spacetime has a nonzero curvature. For this reason, the propagators are not translationally invariant; they are functions of θ, θ' and not functions of their difference, $(\theta - \theta')$.

NP quantization of Dirac and Maxwell's electrodynamics

Now we suggest a method of nonperturbative quantization of Maxwell-Dirac theory on the Hopf bundle. The fact of compactness of a three-dimensional sphere S^3 very much simplifies the procedure of quantization of free fields: when quantizing, no Dirac delta functions appear, since they are replaced by the Kronecker symbols. This would lead us to expect that the quantization of interacting fields on a sphere S^3 will be considerably simplified as well. Notice also that there appears a considerable difference in the behavior of quantum fields on a sphere S^3 and in Minkowski space: the propagators of free fields on the compact space are not translationally invariant.

NP quantization of Dirac and Maxwell's electrodynamics

According to Heisenberg, the process of nonperturbative quantization consists in that a set of equations describing interacting fields is written in the operator form

$$
\begin{aligned}
i\gamma^\mu \hat{\psi}_{;\mu} &= 0, && (12)\\
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left(\sqrt{-g} \hat{F}^{\mu\nu} \right) &= -4\pi \hat{j}^\mu. && (13)
\end{aligned}
$$

NP quantization of Dirac and Maxwell's electrodynamics

The main idea of nonperturbative quantization consists in that the operator field equations are replaced by an infinite system for all Green's functions. As a result, one arrives at the following infinite set of equations:

$$
\begin{aligned}
i\gamma^\mu \left\langle \hat{\psi}_{;\mu} \right\rangle =&0,\\
\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}\left\langle \hat{F}^{\mu\nu}\right\rangle\right) =&-4\pi\left\langle \hat{j}^\mu\right\rangle,\\
i\gamma^\mu \left\langle \hat{\bar{\psi}}\hat{\psi}_{;\mu} \right\rangle =&0,\\
i\gamma^\mu \left\langle \hat{A}_\nu\hat{\psi}_{;\mu} \right\rangle =&0,\\
\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}\left\langle \hat{F}^{\mu\nu}\hat{\psi}\right\rangle\right) =&-4\pi\left\langle \hat{j}^\mu\hat{\psi}\right\rangle,\\
&\ldots
\end{aligned}
$$

NP quantization of Dirac and Maxwell's electrodynamics

It is evident that this infinite set of equations cannot be solved explicitly and analytically. Therefore, the question arises as to whether it is possible to find its approximate solution. The problem of how to cut off an infinite set of equations is called *the closure problem*, and it is well known in turbulence modeling. In that case, the Navier-Stokes equation is averaged, and it is known as the Reynolds-averaged Navier-Stokes equation. But this equation contains an unknown quantity – the Reynolds-stress tensor, for which one has to have an extra equation called the Reynolds-stress equation, which in turn contains more unknown functions, and so on.

NP quantization of Dirac and Maxwell's electrodynamics

For better understanding of the situation, it is useful to consider some simple example illustrating this process.Replacing the functions by operators, we then have

$$
\hat{\psi}_{nm0} = e^{-i\Omega t} e^{in\chi} e^{im\varphi} \hat{\Theta}_{nm0}(\theta) \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix},
$$

$$
\hat{A}_\mu = \left\{ \hat{\phi}(\theta), r\hat{\rho}(\theta), 0, r\hat{\lambda}(\theta) \right\}.
$$

NP quantization of Dirac and Maxwell's electrodynamics

In this case we obtain the following equations coming from the quantum equations:

$$\hat{\Theta}' \mp \hat{\Theta}\left[\cot\theta\left(\mp\frac{1}{2} + n + er\hat{\rho}\right) + \frac{1}{\sin\theta}\left(m + er\hat{\lambda}\right)\right] = 0,$$

$$\frac{1}{\sin\theta}\left(\sin\theta\hat{\phi}'\right)' = -er^{3/2}\frac{\hat{\Theta}^2}{2},$$

$$\frac{1}{\sin\theta}\left(\sin\theta\hat{\rho}'\right)' = \pm er^{3/2}\frac{\hat{\Theta}^2}{4},$$

$$\hat{\lambda}' = -\hat{\rho}'\cos\theta.$$

NP quantization of Dirac and Maxwell's electrodynamics

After quantum averaging, the first equation will be the equation for $\left\langle \hat{\Theta} \right\rangle$, the second one – for $\left\langle \hat{\phi} \right\rangle$, the third one – for $\langle \hat{\rho} \rangle$, and the fourth one – for $\left\langle \hat{\lambda} \right\rangle$. But these equations contain the following new Green's functions:

$$G_{\Theta\rho}(\theta_1, \theta_2) = \left\langle \hat{\Theta}(\theta_1)\hat{\rho}(\theta_2) \right\rangle , \quad G_{\Theta\lambda}(\theta_1, \theta_2) = \left\langle \hat{\Theta}(\theta_1)\hat{\lambda}(\theta_2) \right\rangle ,$$

$$G_{\Theta\Theta}(\theta_1, \theta_2) = \left\langle \hat{\Theta}(\theta_1)\hat{\Theta}(\theta_2) \right\rangle ,$$

for which one must have their own equations.

NP quantization of Dirac and Maxwell's electrodynamics

The equation for the Green function $G_{\Theta\rho}(x,y)$ can be obtained by multiplying the operator equation for $\hat{\Theta}$ on the right by $\hat{\rho}$ and by performing the quantum averaging,

$$G'_{\Theta\rho} \mp G_{\Theta\rho}\left[\cot\theta\left(\mp\frac{1}{2}+n\right)+\frac{m}{\sin\theta}\right] \mp er\left\langle\hat{\Theta}\hat{\rho}^2\right\rangle \mp er\left\langle\hat{\Theta}\hat{\lambda}\hat{\rho}\right\rangle = 0.$$

NP quantization of Dirac and Maxwell's electrodynamics

The Green function $G_{\Theta\rho}(\theta_1, \theta_2)$ is a function of two variables θ_1 and θ_2; hence, one has to have one more differential equation for the variable θ_2. This equation can be obtained by multiplying the equation for ρ on the left by $\hat{\Theta}$ and by performing the quantum averaging,

$$\frac{1}{\sin\theta}\left(\sin\theta\, G'_{\Theta\bar{\rho}}\right)' = \pm e r^{3/2} \frac{\left\langle \hat{\Theta}^3 \right\rangle}{4}.$$

NP quantization of Dirac and Maxwell's electrodynamics

As a result, we arrive at an infinite set of equations for an infinite number of Green's functions

$$
\begin{aligned}
\left\langle \hat{\Theta} \right\rangle' \mp \left\langle \hat{\Theta} \right\rangle \left[\cot\theta \left(\mp\frac{1}{2} + n \right) + \frac{m}{\sin\theta} \right] \mp erG_{\Theta\rho} \mp erG_{\Theta\lambda} &= 0, \\
\frac{1}{\sin\theta} \left(\sin\theta \left\langle \hat{\phi} \right\rangle' \right)' &= -er^{3/2} \frac{G_{\Theta\Theta}}{2}, \\
\frac{1}{\sin\theta} \left(\sin\theta \left\langle \hat{\rho} \right\rangle' \right)' &= \pm er^{3/2} \frac{G_{\Theta\Theta}}{4}, \\
\left\langle \hat{\lambda} \right\rangle' &= -\left\langle \hat{\rho} \right\rangle' \cos\theta, \\
G'_{\bar{\Theta}\rho} \mp G_{\Theta\rho} \left[\cot\theta \left(\mp\frac{1}{2} + n \right) + \frac{m}{\sin\theta} \right] \mp er \left\langle \hat{\Theta}\hat{\rho}^2 \right\rangle \mp er \left\langle \hat{\Theta}\hat{\lambda}\hat{\rho} \right\rangle &= 0, \\
\frac{1}{\sin\theta} \left(\sin\theta \, G'_{\Theta\bar{\rho}} \right)' &= \pm er^{3/2} \frac{\left\langle \hat{\Theta}^3 \right\rangle}{4}, \\
G'_{\bar{\Theta}\lambda} \mp G_{\Theta\lambda} \left[\cot\theta \left(\mp\frac{1}{2} + n \right) + \frac{m}{\sin\theta} \right] \mp er \left\langle \hat{\Theta}\hat{\rho}\hat{\lambda} \right\rangle \mp er \left\langle \hat{\Theta}\hat{\lambda}^2 \right\rangle &= 0, \\
G'_{\Theta\bar{\lambda}} &= -G'_{\Theta\bar{\rho}} \cos\theta, \\
G'_{\bar{\Theta}\Theta} \mp G_{\Theta\Theta} \left[\cot\theta \left(\mp\frac{1}{2} + n \right) + \frac{m}{\sin\theta} \right] \mp er \left\langle \hat{\Theta}\hat{\rho}\hat{\Theta} \right\rangle \mp er \left\langle \hat{\Theta}\hat{\lambda}\hat{\Theta} \right\rangle &= 0, \\
\ldots &= \ldots
\end{aligned}
$$

Conclusions

We have considered the Dirac equation and Maxwell's electrodynamics in $\mathbb{R} \times S^3$ spacetime. The distinctive feature of these theories on the Hopf bundle is that they have discrete spectra of solutions both for the Dirac equation and for Maxwell's electrodynamics.

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- The algebra of these fields is determined by the infinite set of the Schwinger-Dyson equations as a whole.
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- The properties of the operators of interacting fields can differ drastically from those of free fields.
- The algebra of these fields is determined by the infinite set of the Schwinger-Dyson equations as a whole.
- For strongly interacting quantum fields, it is impossible to introduce creation and annihilation operators.
- Separate consideration of the notion of quantum state is required.
- It is possible that the properties of the operators of interacting fields and of quantum states are related to the properties of the complete set of Green's functions defined by the Schwinger-Dyson equations.

Thanks for your attention !