Article

On the quantum origin of a dark universe

Saurya Das [1]*, **Mohit Kumar Sharma** [2] **and Sourav Sur** [2]

1. Theoretical Physics Group and Quantum Alberta, Department of Physics and Astronomy, University of Lethbridge, 4401 University Drive, Lethbridge, Alberta T1K 3M4, Canada
2. Department of Physics and Astrophysics University of Delhi, New Delhi 110007, India; mrmohit254@gmail.com; sourav.sur@gmail.com
* Correspondence: saurya.das@uleth.ca

Abstract: It has been shown beyond reasonable doubt that the majority (about 95%) of the total energy budget of the universe is given by the dark components, namely Dark Matter and Dark Energy. What constitutes these components remains to be satisfactorily understood however, despite a number of promising candidates. An associated conundrum is that of the coincidence, i.e. the question as to why the Dark Matter and Dark Energy densities are of the same order of magnitude at the present epoch, after evolving over the entire expansion history of the universe. In an attempt to address these, we consider a quantum potential resulting from a quantum corrected Raychaudhuri/Friedmann equation in presence of a cosmic fluid, which is presumed to be a Bose-Einstein condensate (BEC) of ultralight bosons. For a suitable and physically motivated macroscopic ground state wavefunction of the BEC, we show that a unified picture of the cosmic dark sector can indeed emerge, thus resolving the issue of the coincidence. The effective Dark energy component turns out to be a cosmological constant, by virtue of a residual homogeneous term in the quantum potential. Furthermore, comparison with the observational data gives an estimate of the mass of the constituent bosons in the BEC, which is well within the bounds predicted from other considerations.

Keywords: Bose-Einstein condensate, dark energy, cold dark matter, cosmology of theories beyond the SM, quantum Raychaudhuri equation.

1. Introduction

A unified picture of the cosmic Dark Matter (DM) and Dark Energy (DE) has been one of the key aspirations of modern researches within the standard paradigm of spatially flat Friedmann-Robertson-Walker (FRW) cosmology[1–5]. While there have been innumerable attempts to emulate the nature and properties of DM and DE, a general consensus on the specific structure of the evolving dark sector remains elusive till date. Although candidates abound, e.g. WIMPs and axions for the DM [6–10], and a plethora of others for the DE (such as scalar fields, aerodynamic fluids and modified gravitational artefacts) [11–30], the emergent models have had their own pros and cons. One conundrum, of course, is the '*coincidence*', i.e. the vast amount of fine-tuning required to set up the initial conditions for the same order-of-magnitude contributions of DM and DE at the present epoch, if they purportedly stem out of different fundamental sources. This is indeed perplexing, given the strong observational support for the DE density to be closely akin to a cosmological constant Λ, whereas almost the entire bulk of the DM to be 'cold' and non-relativistic, with energy density $\rho^{(c)}(t) \sim a^{-3}(t)$ (where $a(t)$ is the cosmological scale factor, and t denotes the cosmic time coordinate).

One way to resolve the problem of coincidence (or, at least ameliorate it to a certain degree) is to make allowance for interaction(s) between the DE and DM components [31–38]. However, these components being so different in character, the physical realization of such interactions is often very difficult, unless they appear naturally, for e.g. via conformal transformations in the scalar-tensor formulations of gravitational theories [39–46]. A more

robust approach therefore is to bring a *unified* dark sector in the reckoning, i.e. to consider a scenario in which the DE, the DM, and possibly certain interaction(s) thereof, all having the same fundamental origin [47–51]. Of course, in that way the DE and the DM would lose their individuality, and would merely be the structural artefacts of the source, which thereby would become the all-important entity that shapes up the evolution profile of the cosmological dark sector.

In this paper, we resort to such a unified dark universe picture, by adopting an earlier proposal [52–55], albeit in a much broader sense. Specifically, we endeavor to deal with the plausible perception of the dark sector being described in entirety by a *Bose-Einstein condensate* (BEC) of light bosons extending across cosmological length scales. The density of the BEC, together with its *quantum potential*, manifest as the effective DE and DM densities. It is worth noting that in principle, every wavefunction comes associated with it a quantum potential, which effectively gets added to any starting classical potential, in determining the dynamics of the system — a fact that is often overlooked. In the scenario we focus on, the macroscopic BEC provides the requisite wavefunction.

2. BEC Cosmological Formulation in the standard setup

Let us begin with the general form of the cosmological equations in the standard spatially flat FRW framework:

$$H^2(t) \equiv \left[\frac{\dot{a}(t)}{a(t)}\right]^2 = \frac{\kappa^2}{3}\rho(t) , \qquad (2.1)$$

$$\dot{H}(t) + H^2(t) \equiv \frac{\ddot{a}(t)}{a(t)} = -\frac{\kappa^2}{6}\left[\rho(t) + 3p(t)\right] , \qquad (2.2)$$

where $\kappa^2 = 8\pi G$ is the gravitational coupling constant, and $a(t)$ is the FRW scale factor, normalized to unity at the present epoch, $t = t_0$, i.e. $a(t_0) = 1$. The overhead dot $\{\cdot\}$ denotes $\partial/\partial t$, and $\rho(t)$ and $p(t)$ are respectively the conserved total energy density and pressure of the system. The latter is presumably a collection of fluids and(or) fields, the individual energy densities and pressures of which may not necessarily be conserved.

Eqs. (2.1) and (2.2) are of course classical equations. However, if a constituent entity is inherently quantum mechanical, and described by a wave-function of the form

$$\Psi(\vec{x}, t) = \mathcal{R}(\vec{x}, t)\, e^{i\, S(\vec{x}, t)} , \qquad \text{with} \quad \mathcal{R}, S \in \mathbb{R} , \qquad (2.3)$$

then as shown in [52–54], an additional term arises due to the quantum correction of the field equations, even at the background cosmological level. Specifically, the geodesic flow equation, or the Raychaudhuri equation, (2.2) gets modified by a *quantum potential* given by [52]

$$U_Q = \frac{\hbar^2}{m^2}\, h^{\mu\nu}\, \nabla_\mu \nabla_\nu \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right) , \qquad (2.4)$$

with m being the total mass parameter of the quantum constituent, and $h_{\mu\nu}$ denoting the (induced) metric of the three-dimensional spatial hypersurfaces of constant cosmic time. One can, as usual, express $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$, where $u_\mu = (\hbar/m)\,\partial_\mu S$ is the world velocity of the evolving (quantum) cosmic 'fluid', which can appropriately be taken to be a BEC of rest mass m and described by a macroscopic wave-function of the form (2.3), provided the value of m remains within certain limit [53,54].

Indeed, for a suitable choice of the amplitude $\mathcal{R}$, the BEC probability density $\rho^{(B)} \sim |\mathcal{R}|^2$ is shown to mimic the energy density of a classical dust-like matter, presumably the cold dark matter (CDM), whereas the potential U_Q is positive (repulsive) and even a constant, that acts as a cosmological constant dark energy (DE) component Λ, albeit of a quantum origin [54,55]. However, from a rather general perspective, such a CDM mimicry of $\rho^{(B)}$ may only be partial, since the potential U_Q may evolve dynamically and contribute to the CDM as well. That is to say, $\rho^{(B)}$ or U_Q may not individually be responsible, in

general, for shaping up the course of evolution of only the CDM or only the DE. Instead, their combined effect may be looked upon as what leads to some specific evolution profile of the entire cosmological dark sector. Strikingly however, one may still reckon the constancy of the effective DE component, and that the CDM effectively results from a close interplay of not only the evolving $\rho^{(B)}$ and U_Q, but also the density of the visible (baryonic) matter content of the universe. While demonstrating all these in this paper, we shall endeavor to make it clear that the formalism developed herein differs substantially from the plethora of proposals encompassing the ways to treat the BEC as a plausible CDM candidate in the literature [56–83]. In particular, it is worth noting here that, to the best of our knowledge, neither the quantum effects of the BEC on the background level cosmic evolution, nor the possibility of a BEC accounting for the whole dark sector, has been explored with rigour.

Let us first recollect an important result derived in [53,54]: *for a copious supply of bosons of mass $\lesssim 6\,eV$, the corresponding critical temperature T_c exceeds the ambient temperature of the universe at all epochs.* To be more precise, the critical temperature of an ideal gas of ultralight bosons of mass m, below which they will drop to their lowest energy state and form a BEC is found in [53], following the procedure outlined in [84–86],

$$T_c(t) \simeq 4.9\, m^{-1/3}\, a^{-1}(t) \quad [\text{in } K, \text{ for } m \text{ in eV}]\,, \tag{2.5}$$

under the assumption that the BEC density $\rho^{(B)}$ accounts for the DM density in entirety. Consequently, for $m \lesssim 6\,eV$, one has $T(a) < T_c(a)\,\forall\, a$, where $T(a) = 2.7\,a^{-1}$ is the ambient temperature of the universe. This therefore points to the formation of a BEC of the bosons in the early phase of evolution of the universe, once the BEC density is taken to be of the order of the critical density of the universe at the present epoch [1].

Now, the observed isotropy of the universe (at large scales) makes the stringent demand that the wave-function of this embedded BEC has to be spherically symmetric after all. Henceforth, from an analogy commensurable with a limiting Newtonian cosmological approximation (see refs.[54,55]), we shall assume such a wave-function to be of the form:

$$\Psi(r,t) = R(t)\, e^{-r^2(t)/\sigma^2}\, e^{-iE_0 t/\hbar}\,, \tag{2.6}$$

where σ and E_0 are real and positive-valued constants, and

$$r(t) = \sqrt{h_{ij}(t)\, x^i x^j} = x\, a(t)\,, \tag{2.7}$$

with $x = \sqrt{\delta_{ij} x^i x^j}$ denoting the comoving radial coordinate. Eq. (2.6) essentially means taking the quantum amplitude $\mathcal{R}(r,t)$ in Eq. (2.3) to be a Gaussian (of spread σ), modulo a time-dependent factor $R(t)$. On the other hand, the phase is considered to be purely a function of time, viz. $S(t) = E_0 t/\hbar$, with the parameter E_0 having the usual interpretation of the ground state energy of the BEC. Of course, in an effective cosmic fluid description, one requires $E_0 = m$ — the rest mass energy of the fluid, in order that the world velocity u_μ satisfies the normalization condition $u_\mu u^\mu = -1$.

Note also that the above equation (2.7) is just the standard relationship between the radial coordinate distances in the proper frame and the comoving frame, however with the induced metric being identified with that of the three-dimensional spatial hypersurfaces of the FRW bulk space-time, i.e. $h_{ij} \equiv a^2(t)\delta_{ij}$. This is justified from the point of view that we are simply following the usual course one takes while studying the cosmic evolution driven by an intrinsically inhomogeneous source (in this present context, the BEC). To be more specific, the local inhomogeneities being usually treated as small perturbations over a homogeneous FRW background, it is natural for us to make such an identification, while

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[1] Note however the difference between the expression (2.5) and that derived in [87]. This is because the former is obtained upon considering the BEC of neutral bosons, which are their own anti-particles, with no charge or other hidden quantum numbers and a vanishing chemical potential (e.g. gravitons or axions). In contrast, the authors in [87] have considered charged bosons with a non-vanishing chemical potential, distinct particles and anti-particles and a net BEC charge.

limiting our attention to the background level BEC cosmology in this paper[2]. The foremost task that remains then is to solve explicitly the background cosmological equations for the scale factor $a(t)$ (or at least, the expansion rate $H(a)$), which would nonetheless require only the homogeneous (or the purely time-dependent) parts of the BEC density $\rho^{(B)}$ and the quantum potential U_Q. Henceforth, we shall make no allusion to the x-dependence of these quantities, or in other words, denote by $\rho^{(B)}$ and U_Q their homogeneous parts only. The former is of course given simply by

$$\rho^{(B)}(t) = |R(t)|^2 , \tag{2.8}$$

whereas the latter, derived from the above expression (2.4), turns out to be

$$U_Q(t) = \frac{3\hbar^2}{m^2}\left[H(t)\,\dot{Y}(t) + \frac{4}{\sigma^2}\left\{F(t) + \frac{2}{\sigma^2}\right\}\right], \tag{2.9}$$

with the functions $Y(t)$ and $F(t)$ defined as follows:

$$Y(t) := \frac{\ddot{R}(t)}{R(t)} + \frac{3H(t)\,\dot{R}(t)}{R(t)} , \tag{2.10}$$

$$F(t) := \dot{H}(t) + 5H^2(t) + \frac{2H(t)\,\dot{R}(t)}{R(t)} . \tag{2.11}$$

Clearly, the emphasis is therefore on the function $R(t)$ in the BEC wave amplitude, a suitable form of which leads to a viable cosmological scenario with a unified dark sector, as demonstrated in what follows.

3. BEC Cosmological Evolution and the Unified Dark Sector

For the BEC to emulate an effective CDM evolution (either totally or partially), the corresponding (homogeneous) density $\rho^{(B)}(t)$ requires to fall off with time, i.e. with the expansion of the universe. In particular, with the wave amplitude having the functional dependence

$$R(t) = R_0\,a^{-3/2}(t) , \quad \text{where} \quad R_0 \equiv R(t_0) : \text{constant} , \tag{3.12}$$

we have the BEC evolving precisely as a dust-like matter, viz.

$$\rho^{(B)}(t) = \rho_0^{(B)}\,a^{-3}(t) , \quad \text{with} \quad \rho_0^{(B)} \equiv \rho^{(B)}(t_0) : \text{constant} , \tag{3.13}$$

at the background cosmological level. The (homogeneous) quantum potential $U_Q(t)$ is then given by Eq. (2.9), albeit with the constituent functions reducing to

$$Y(t) = -\frac{3}{2}\left[\dot{H}(t) + \frac{3H^2(t)}{2}\right], \tag{3.14}$$

$$F(t) = \dot{H}(t) + 2H^2(t) . \tag{3.15}$$

Consequently, the Friedmann equations (2.1) and (2.2) yield

$$\rho(t) = \frac{2}{\kappa^2}\left[2Y(t) + 3F(t)\right], \tag{3.16}$$

$$p(t) = \frac{4}{3\kappa^2}\,Y(t) , \tag{3.17}$$

so that we can re-write Eq. (2.9) as

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[2]   A rather rigorous study of the consequences of the BEC and the quantum potential at the level of the linear perturbations is being carried out in a subsequent work [88].

$$U_Q(t) = \frac{24\hbar^2}{m^2\sigma^4}\left[1 + \frac{\kappa^2\sigma^2}{12}\left\{\left[1 + \frac{3\kappa^2\sigma^2}{8}a(t)\,p'(t)\right]\rho(t) - 3p(t)\right\}\right], \tag{3.18}$$

where the prime $\{'\}$ denotes derivative with respect to the scale factor $a(t)$. Eq. (3.18) is a convenient form of the quantum potential, which explicitly shows how it depends on the physical variables, viz. the total (or the 'critical') energy density $\rho(t)$ and the total pressure $p(t)$.

Now, since we are primarily interested in the late-time expansion history of the universe, it suffices to consider the visible matter to be baryonic dust, having an energy density

$$\rho^{(b)}(t) = \rho_0^{(b)}\,a^{-3}(t), \quad \text{with} \quad \rho_0^{(b)} \equiv \rho^{(b)}(t_0) : \text{constant}. \tag{3.19}$$

More specifically, it is reasonable to resort to the following decomposition of the total energy density:

$$\rho(t) = \rho^{(b)}(t) + \rho^{(c)}(t) + \rho_X(t), \tag{3.20}$$

where we denote the total effective CDM density by

$$\rho^{(c)}(t) = \rho_0^{(c)}\,a^{-3}(t), \quad \text{with} \quad \rho_0^{(c)} \equiv \rho^{(c)}(t_0) : \text{constant}, \tag{3.21}$$

and the total effective (and possibly dynamical) DE density by $\rho_X(t)$. Note that $\rho^{(c)}(t)$ may not necessarily be equal to $\rho^{(B)}(t)$, i.e. the BEC density may not be the total CDM density, as some contribution may come from the the quantum potential U_Q as well. Nevertheless, our main interest is in the evolution of the total dark sector density, $\rho^{(c)}(t) + \rho_X(t)$, while the corresponding pressure is that due to the DE component only. In fact, this pressure is the lone contribution to the total pressure $p(t)$, as the visible matter is dust-like.

On the other hand, the quantum corrected Raychaudhuri equation for the system constituted by only the (visible) baryonic dust and the BEC (which also emulates a dust-like fluid) is given by

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{\kappa^2}{6}\left[\rho^{(b)}(t) + \rho^{(B)}(t)\right] + \frac{1}{3}U_Q(t). \tag{3.22}$$

Comparing this with the Friedmann equation (2.2), and using Eqs. (3.13) and (3.19), we find

$$\rho(t) = \frac{\rho_0^{(b)} + \rho_0^{(B)}}{a^3(t)} - \frac{2}{\kappa^2}U_Q(t) - 3p(t). \tag{3.23}$$

Hence, the above equation (3.18) can be reduced to that of the quantum potential expressed (for convenience) as a function of the scale factor $a(t)$:

$$U_Q(a) = \frac{6\,\alpha^2}{1 + \alpha^2\sigma^2\,f(a)}\left[1 + \frac{\kappa^2\sigma^2}{12}\left\{\frac{[\rho_0^{(b)} + \rho_0^{(B)}]f(a)}{a^3} - 3[1 + f(a)]p(a)\right\}\right], \tag{3.24}$$

where we have used the following notation and definition:

$$\alpha \equiv \frac{2\hbar}{m\sigma^2}, \qquad f(a) := 1 + \frac{3\kappa^2\sigma^2\,a\,p'(a)}{8}. \tag{3.25}$$

Again, from Eq. (3.23) and the Friedmann equations (2.1) and (2.2) (or more specifically, the conservation relation $\rho'(a) = -(3/a)[\rho(a) + p(a)]$) it follows that the quantum potential can also be expressed in an integral form as

$$U_Q(a) = \frac{\kappa^2}{2a^3}\left[\rho_0^{(B)} - 3\int da \cdot a\left\{a^2 p(a)\right\}'\right]. \tag{3.26}$$

So, for a given form of $p(a)$ we can in principle determine $U_Q(a)$, and hence the the total density $\rho(a)$, or equivalently the Hubble rate $H(a)$ that describes the cosmic expansion history (and its future extrapolations).

Let us now consider a rather simplified BEC cosmological setup with the total pressure

$$p = -\Lambda = \text{constant}. \tag{3.27}$$

That this is indeed tenable and the corresponding system of equations, given explicitly by the above set (3.23) – (3.26), do admit an exact solution describing an effective ΛCDM evolution had already been demonstrated in a few earlier works involving one of us (SD) [53–55]. However, certain inherently intriguing aspects of such a solution remain to be explored, particularly from the point of view of asserting stringent parametric limits pertaining to the physical realization of a unified cosmological dark sector. We endeavor to do so in this paper, by first noting that Eq. (3.27) immediately implies Eq. (3.26) reducing to

$$U_Q(a) = \kappa^2 \Lambda \left(1 + \frac{\beta}{a^3}\right), \tag{3.28}$$

where β is a (dimensionless) integration constant. Comparing Eqs. (3.24) and (3.28) we identify

$$\Lambda = \frac{6\,\alpha^2}{\kappa^2(1 - 2\alpha^2\sigma^2)}, \qquad \beta = \frac{\alpha^2\sigma^2\left[\rho_0^{(b)} + \rho_0^{(B)}\right]}{2(1 + \alpha^2\sigma^2)\Lambda}, \tag{3.29}$$

since, by Eq. (3.25), we have $f = 1$ for constant p. It then follows from Eq. (3.23) that the total energy density of the universe is given by

$$\rho(a) = \frac{\rho_0^{(m)}}{a^3} + \Lambda, \tag{3.30}$$

i.e. apart from a cosmological constant Λ, the universe only has a dust-like matter content of energy density $\rho^{(m)}(a) = \rho_0^{(m)} a^{-3}$, whose value at the present epoch ($t = t_0$ or, $a = 1$) can easily be identified as

$$\rho_0^{(m)} = (1 - \epsilon)\left(\rho_0^{(b)} + \rho_0^{(B)}\right), \tag{3.31}$$

where the dimensionless quantity

$$\epsilon = \frac{\kappa^2\sigma^2\Lambda}{3(2 + \kappa^2\sigma^2\Lambda)} = \frac{\alpha^2\sigma^2}{1 + \alpha^2\sigma^2}, \tag{3.32}$$

is positive definite and of particular importance in the context of our entire analysis in this paper. Specifically, the above solution (3.30) is the same as the one that describes the standard ΛCDM evolution of the universe composed of a dust-like baryonic matter plus the CDM, and dark energy in the form of the constant Λ. However, the key point to note here is that the effective CDM density at the present epoch, viz.

$$\rho_0^{(c)} = \rho_0^{(m)} - \rho_0^{(b)} = (1 - \epsilon)\rho_0^{(B)} - \epsilon\rho_0^{(b)}, \tag{3.33}$$

is not entirely given by the present-day value $\rho_0^{(B)}$ of the BEC density, but is actually *smaller* than the latter. Interestingly, the reduction is not only given by an amount $\epsilon\rho_0^{(B)}$, but also by an amount $\epsilon\rho_0^{(b)}$, where $\rho_0^{(b)}$ is the present-day value of the baryonic energy density. Now, recalling that the baryons have primarily been supposed to constitute only the visible matter content of the universe, their role in inflicting a reduction in the effective CDM density (from that of the BEC) is nonetheless quite striking. Such a reduction can in fact be looked upon as the consequence of the baryons backreacting on the geometrical structure

of the space-time, because of the quantum correction to the Raychaudhuri equation which leads to the quantum potential U_Q. Therefore, notwithstanding the classical dust-like characterization of the baryonic matter, one can infer that the effect of the baryons on the CDM is inherently quantum mechanical. The extent of such an effect is crucially determined by the parameter ϵ, given by Eq. (3.32), which we shall henceforth refer to as the *quantum backreaction* (QB) parameter. Let us express this parameter in a rather convenient form as

$$\epsilon = \frac{H_0^2 \, \sigma^2 \, \Omega_0^{(\Lambda)}}{2 + 3H_0^2 \, \sigma^2 \, \Omega_0^{(\Lambda)}} \, , \tag{3.34}$$

where $\Omega_0^{(\Lambda)}$ denotes the value of the Λ-density parameter $\Omega^{(\Lambda)}(t) = \Lambda/\rho(t)$ at the present epoch ($t = t_0$), and $H_0 \equiv H(t_0)$ is the Hubble's constant. It is then straightforward to obtain the following stringent bounds being imposed on the extent of the QB from entirely physical considerations:

- First of all, the total matter density of the universe has to be positive definite. Therefore, apart from being positive-valued by definition, the parameter $\epsilon < 1$, so that by Eq. (3.31), $\rho_0^{(m)} > 0$ (of course, under the presumption that $\rho_0^{(b)} > 0$ and $\rho_0^{(B)} > 0$, which in turn imply $\rho_0^{(c)} > 0$ as well as $\rho_0^{(b)} < \rho_0^{(m)}$).

- Now, from Eq. (3.34), the second equality in Eq. (3.32) and the definition of the constant α given by the first equation in (3.25), we have the BEC mass parameter expressed as

$$m \simeq \frac{\hbar \, H_0}{\epsilon} \sqrt{2(1 - \epsilon)(1 - 3\epsilon)\Omega_0^{(\Lambda)}} \, , \tag{3.35}$$

whose real-valuedness (i.e. $m^2 > 0$) makes evident the more restrictive condition $\epsilon < \frac{1}{3}$.

- Moreover, the overall credibility of a BEC cosmological formulation demands that the energy density due to the BEC should not exceed the total matter density. Referring therefore to the present epoch, we should have $\rho_0^{(B)} < \rho_0^{(m)}$, so that by Eqs. (3.30) and (3.31),

$$\epsilon < \frac{\Omega_0^{(b)}}{1 - \Omega_0^{(\Lambda)} + \Omega_0^{(b)}} \, , \tag{3.36}$$

where $\Omega_0^{(b)}$ is the present-day value of the baryon density parameter $\Omega^{(b)}(t) = \rho^{(b)}(t)/\rho(t)$. Statistical estimations using recent observational data from various probes fairly conform to the best fit parametric values for ΛCDM to be $\Omega_0^{(\Lambda)} \simeq 0.7$ and $\Omega_0^{(b)} \simeq 0.05$, whence by Eq. (3.36), $\epsilon \lesssim \frac{1}{7}$. For instance, the widely accepted Planck 2018 results for the Cosmic Microwave Background (CMB) anisotropy observations (power spectrum TT,TE,EE+lowE), in combination with Lensing and Baryon Accoustic Oscillations (BAO), show the estimates (upto the 68% confidence limits):

$$\Omega_0^{(\Lambda)} = 0.6889 \pm 0.0056 \, , \qquad \text{and}$$
$$\Omega_0^{(b)}h^2 = 0.02242 \pm 0.00014 \, , \quad \Omega_0^{(c)}h^2 = 0.11933 \pm 0.00091 \, , \tag{3.37}$$

with $100\, h \equiv H_0$ [Km s^{-1} Mpc^{-1}] $= 67.66 \pm 0.42$. Therefore, plugging in the best fits of $\Omega_0^{(\Lambda)}$ and $\Omega_0^{(b)}$ in Eq. (3.36) we get $\epsilon < 0.136$, which is an even tighter constraint (compared to $\epsilon < \frac{1}{3}$ found earlier).

The most interesting outcome of this smallness of ϵ is of course the enhancement of the BEC mass m from its 'Hubble value' ($m_H \simeq 10^{-32}$ eV)[3], since by Eq. (3.35), m varies approximately as ϵ^{-1}. Hence, the smaller the value of ϵ, the greater would be the order-

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[3] The Hubble value is nothing but the one obtained by assuming the BEC constituents to be the gravitons, whence taking the Gaussian spread $\sigma \simeq H_0^{-1}$ one gets $m \equiv m_H \simeq 2\sqrt{2}H_0 \simeq 10^{-32}$ eV, as $H_0 \simeq 10^{-42}$ GeV in the units $c = 1 = \hbar$ (see ref. [54]).
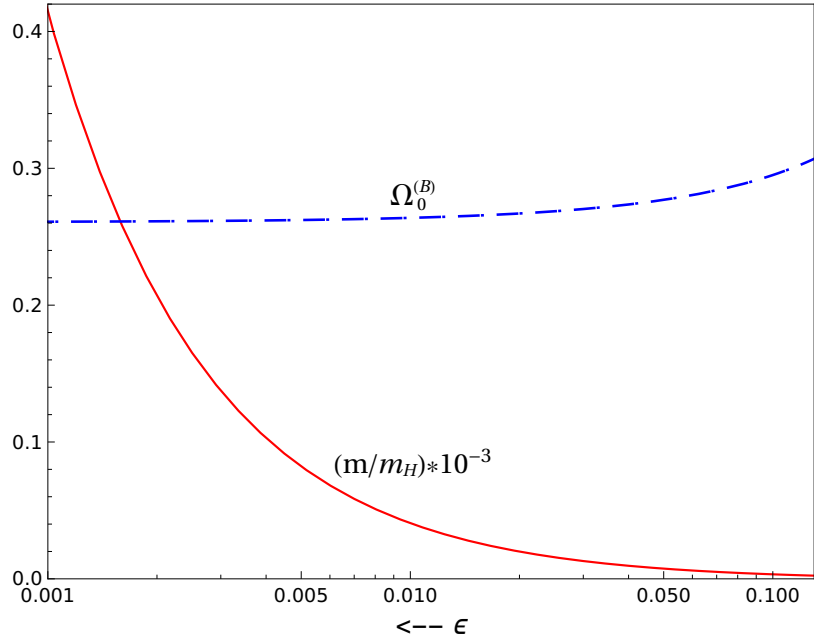
Figure 1. Enhancement of the BEC mass parameter m, relative to its Hubble value m_H (scaled by a factor of 10^3), and the variation of the BEC density parameter at the present epoch, $\Omega_0^{(B)}$, with the decrease of the quantum backreaction parameter ϵ from about 0.1 to 0.001. The ϵ-axes ticks are in the log scale.

of-magnitude enhancement of m, which is in fact desirable from the point of view of preventing small-scale structure formation of the CDM because of the uncertainty principle [57]. Such a relative mass enhancement with decreasing ϵ (from its limiting value of about 0.1 found above) is shown in Fig. 1, alongwith the ϵ-variation of the BEC density parameter at the present epoch, $\Omega_0^{(B)} \equiv \left. [\rho^{(B)}/\rho] \right|_{t=t_0}$, obtained from Eq. (3.33). In all the requisite calculations for these plots, we have used the best fit values of the Planck 2018 TT,TE,EE+lowE+Lensing+BAO parametric estimations for ΛCDM quoted in the Eq. (3.37) above.

4. Conclusion and Outlook

To summarize, in this paper we have studied a macroscopic BEC stretching across cosmological length scales as a potential CDM candidate. While there have been a number of attempts in this direction earlier, what is new in our work is the observation that the quantum potential of this BEC gives rise to the effect of an observed DE in the form of a cosmological constant Λ. In fact, more intriguingly, we not only perceive a coveted unification of the CDM and the DE, as they stem out of the same source, but also that the visible baryonic matter content of the universe crucially backreacts on the metric to determine ultimately the evolutionary profile of the cosmic dark sector. Note that the positivity and the smallness of Λ here can be attributed to the fact that the quantum potential for the given wavefunction being treated as a classical potential (or part thereof) in a gravitational formulation is itself positive. On the other hand, although simple physical considerations, such as the positive-definiteness of the total energy density of the universe and the squared BEC mass (m^2), can assert the extent of the quantum backreaction (QB), the most stringent bound on the corresponding parameter ϵ comes from the legitimate demand that the BEC density should not exceed the total density of the universe at the present epoch. As a by-product, albeit of considerable importance, the upper bound on ϵ enables us to determine the lower bound on m. In particular, it is well-known that the preferred boson mass in the BEC, that can prevent the formation of small-scale structures in CDM via the uncertainty principle [57], is $m \sim 10^{-22}\,eV$. While the above stringent bound is found to be $\epsilon \lesssim \mathcal{O}(10^{-1})$, it is ensured thereby that the lower bound on m is well within the allowed mass range for the BEC (i.e. between 10^{-22} eV and 10^{-32} eV, the Hubble

value). Of course, on the whole, it must be pointed out here that it is the macroscopic wavefunction of the BEC, and the specific form of that wavefunction considered here, which have made it essential for all the above to work. Nevertheless, we have not contemplated on any deviation from the spatial flatness assumption, and have accepted the latter as an observational fact, while determining the bound on the QB parameter ϵ, or that on the BEC mass m. Moreover, we have not looked into the detailed aspects of the BEC, specifically, what its constituent bosons would be. Such an exploration is certainly worthy, from the point of view of deriving the density profiles of the CDM and comparing the same with the galaxy rotation data. Although there have already been some works in this direction [89], assuming the CDM to be solely due to the BEC, ample scope remains for garnering more evidence, once the effect of the quantum potential in emulating the DE, alongwith the backreaction of other constituents (such as baryons), are brought into consideration.

Acknowledgment

References

1. E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. **D 15** (2006) 1753.
2. L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations*, Cambridge University Press, United Kingdom (2010).
3. G. Wolschin, *Lectures on Cosmology: Accelerated expansion of the Universe*, Springer, Berlin, Heidelberg (2010).
4. S. Matarrese, M. Colpi, V. Gorini and U. Moschella, *Dark Matter and Dark Energy: A Challenge for Modern Cosmology*, Springer, The Netherlands (2011).
5. K. Bamba, S. Capozziello, S. Nojiri and S.D. Odintsov, Astrophys. Space Sci. **342** (2012) 155.
6. G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo and F. S. Queiroz, Eur. Phys. J. **C 78** (2018) no.3, 203.
7. L. F. Abbott and P. Sikivie, Phys. Lett. **B 120** (1983) 133.
8. J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. **B 120** (1983) 127.
9. M. Dine and W. Fischler, Phys. Lett. **B 120** (1983) 137.
10. M. S. Turner, Phys. Rev. **D 28** (1983) 1243.
11. R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80** (1998) 1582.
12. S. M. Carroll, Phys. Rev. Lett. **81** (1998) 3067.
13. E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. **D 57** (1998) 4686.
14. I. Zlatev, L.-M. Wang and P. J. Steinhardt, Phys. Rev. Lett. **82** (1999) 896.
15. S. Tsujikawa, Class. Quant. Grav. **30** (2013) 214003.
16. C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85** (2000) 4438.
17. C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. **D 63** (2001) 103510.
18. M. Malquarti, E. J. Copeland, A. R. Liddle and M. Trodden, Phys. Rev. **D 67** (2003) 123503.
19. R. J. Scherrer, Phys. Rev. Lett. **93** (2004) 011301.
20. S. Sur and S. Das, JCAP **0901** (2009) 007.
21. F. Piazza and S. Tsujikawa, JCAP **0407** (2004) 004.
22. J. S. Bagla, H. K. Jassal and T. Padmanabhan, Phys. Rev. **D 67** (2003) 063504.
23. G. Calcagni and A. R. Liddle, Phys. Rev. **D 74** (2006) 043528.
24. C. J. A. P. Martins and F. M. O. Moucherek, Phys. Rev. **D 93** (2016) 123524.
25. M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. **D 66** (2002) 043507.
26. L. Xu, J. Lu and Y. Wang, Eur. Phys. J. **C 72** (2012) 1883.
27. S. Nojiri and S. D. Odintsov, Phys. Lett. **B 631** (2005) 1.
28. S. Nojiri and S. D. Odintsov, Phys. Rev. **D 74** (2006) 086005.
29. T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82** (2010) 451.
30. T. Clifton et al., Phys. Rept. **513** (2012) 1.
31. L. Amendola, Phys. Rev. **D 62** (2000) 043511.
32. D. Comelli, M. Pietroni, and A. Riotto, Phys. Lett. **B 571** (2003) 115.
33. G. R. Farrar and P. J. E. Peebles, Astrophys. J. (2004) **604**.
34. R. G. Cai and A. Wang, JCAP **03** (2005) 002.
35. D. Bertacca, N. Bartolo and S. Mataresse, Adv. Astron. **2010** (2010) 904379.
36. D. Bertacca, M. Bruni, O. F. Piattella and D. Pietrobon, JCAP **1102** (2011) 018.

37. Y.-H. Li and X. Zhang, Phys. Rev. **D 89** (2014) 083009.
38. E. Guendelman, E. Nissimov and S. Pacheva, Eur. Phys. J. **C 76** (2016) 90.
39. Y. Fujii and K. Maeda, *The Scalar-Tensor Theory of Gravitation*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, United Kingdom (2003).
40. V. Faraoni, *Cosmology in Scalar-Tensor Gravity*, Kluwer Academic, Dordrecht (2004).
41. E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. **D 70** (2004) 043538.
42. T. Faulkner, M. Tegmark, E. F. Bunn and Y. Mao, Phys. Rev. **D 76** (2007) 063505.
43. B. Boisseau, H. Giacomini and D. Polarski, JCAP **1605** (2016) 048.
44. S. Sur and A. S. Bhatia, JCAP **1707** (2017) 039.
45. A. S. Bhatia and S. Sur, Int. J. Mod. Phys. **D 26** (2017) no.13, 1750149.
46. M. K. Sharma and S. Sur, *Growth of matter perturbations in an interacting dark energy scenario emerging from metric-scalar-torsion couplings*, e-Print arXiv: 2102.01525[gr-qc].
47. A. H. Chamseddine and V. Mukhanov, JHEP **1311** (2013) 13.
48. A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP **1406** (2014) 017.
49. L. Sebastiani, S. Vagnozzi and R. Myrzakulov, Adv. High Energy Phys. **2017** (2017) 3156915.
50. H. Ramo Chothe, A. Dutta and S. Sur, Int. J. Mod. Phys. **D 28** (2019) no.15, 1950174.
51. S. Sur, A. Dutta and H. Ramo Chothe, *Mimetic-Metric-Torsion with induced Axial mode and Phantom barrier crossing*, e-Print arXiv: 2007.04906[gr-qc].
52. S. Das, Phys. Rev. **D 89** (2014) 084068.
53. S. Das, R. K. Bhaduri, Class. Quant. Grav. **32** (2015) 105003.
54. S. Das, R. K. Bhaduri, Phys. News **40**, (2019) 81.
55. S. Das, R. K. Bhaduri, *On the quantum origin of a small positive cosmological constant*, e-Print arXiv: 1812.07647[gr-qc].
56. V. Sahni, L. Wang, Phys. Rev. **D 62**, (2000) 103517.
57. W. Hu, R. Barkana and A. Gruzinov, Phys. Rev. Lett. **85** (2000) 1158.
58. L. A. Urena-Lopez, JCAP **01** (2009) 14.
59. Bohua Li, *Cosmology with Bose-Einstein-Condensed Scalar Field Dark Matter*, M. A. Thesis, Univ. Texas (2013).
60. K. P. Sinha, C. Sivaram and E. C. G. Sudarshan, Found. Phys. **6** (1976) no.1, 65.
61. K. P. Sinha, C. Sivaram and E. C. G. Sudarshan, Found. Phys. **6** (1976) no.6, 717.
62. M. Yu. Khlopov, B. A. Malomed and Ya. B. Zeldovich, Mon. Not. Roy. Astron. Soc. **215** (1985) 575.
63. M. Y. Khlopov, A. S. Sakharov and D. D. Sokoloff, Nucl. Phys. **B** (Proc. Suppl.) **72** (1999) 105.
64. T. Fukuyama and M. Morikawa, Prog. Theo. Phys. **115** no.6 (2006) 1047.
65. J. W. Moffat, *Spectrum of cosmic microwave fluctuations and the formation of galaxies in a modified gravity theory*, e-Print arXiv: astro-ph/0602607.
66. X. Z. Wang, Phys. Rev **D64** (2001) 124009.
67. C. G. Boehmer and T. Harko, JCAP **06** (2007) 025.
68. T. Harko and G. Mocanu, Phys. Rev. **D 85** (2012) 084012.
69. P. Sikivie, Int. J. Mod. Phys. **A 25** (2010) 554.
70. P-H. Chavanis, Astron. Astrophys. **537** (2012) A127.
71. G. Dvali and C. Gomez, Fortsch. Phys. **61** (2013) 742.
72. H. Ziaeeapour, Springer Proc. Phys. **145** (2014) 297, e-Print arXiv: 1112.3934[hep-ph].
73. B. Kain and H. Y. Ling, Phys. Rev. **D 85** (2012) 023527.
74. A. Suárez, V. Robles and T. Matos, Astrophys. Space Sci. Proc. **38** (2014) 107, e-Print arXiv: 1302.0903[astro-ph.CO].
75. Z. Ebadi, B. Mirza and H. Mohammadzadeh, JCAP **11** (2013) 057.
76. M. Dwornik, Z. Keresztes and L. A. Gergely, *Recent Development in Dark Matter Research*, Eds. N. Kinjo, A. Nakajima, Nova Science Publishers (2014) 195, e-Print arXiv: 1312.3715[gr-qc].
77. M. Dwornik, Z. Keresztes, E. Kun and L. A. Gergely, Adv. High Energy Phys. **2017** (2017) 4025386.
78. D. Bettoni, M. Colombo and S. Liberati, JCAP **02** (2014) 004.
79. S. Gielen, Class. Quant. Grav. **31** (2014) 155009.
80. H-Y. Schive, T. Chiueh, T. Broadhurst, Nature Phys. **10** (2014) 496.
81. S. Davidson, Astrop. Phys. **65** (2015) 101.
82. M. Cadoni, R. Casadio, A. Giusti, W. Mück and M. Tuveri, Phys. Lett. **B 776** (2018) 242.
83. M. Cadoni, R. Casadio, A. Giusti and M. Tuveri, Phys. Rev. **D 97** (2018) 044047.
84. M. Brack and R. K. Bhaduri, *Semiclassical Physics*, Westview Press (2003).
85. M. Grether, M. de Llano and G. A. Baker Jr., Phys. Rev. Lett. **99** (2007) 200406.
86. S. Fujita, T. Kimura and Y. Zheng, Found. Phys. **21** (1991) no.9, 1117.
87. H. E. Haber and H. A. Weldon, Phys. Rev. Lett. **46** (1981) 1497.
88. S. Das, M. K. Sharma and S. Sur, in preparation.
89. E. Kun, Z. Keresztes, S. Das and L. A. Gergely, Symmetry **10** (2018) 520.