Article

Effects of Higher Order Retarded Gravity on Galaxies

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Abstract: In a recent paper we have a shown that the flattening of galactic rotation curves can be explained by retardation. However, this will rely on a temporal change of galactic mass. In our previous work we have kept only second order terms of the retardation time in our analysis, while higher terms in the Taylor expansion where not considered. Here we consider analysis to all orders and show that indeed a second order analysis will suffice, and higher order terms can be neglected.

Keywords: Galaxies; rotation curves; gravitational retardation;

1. Introduction

Dynamics of large scale structures is inconsistent with Newtonian mechanics. This was notified in the 1930's by Fritz Zwicky [1], who pointed out that if more (unseen) mass would be present one would be able to solve the apparent contradiction. The phenomena was also observed in galaxies by Volders [2] who have shown that star trajectories near the rim of galaxies do not move according to Newtonian predictions, and later corroborated by Rubin and Ford [3–5] for spiral galaxies.

In as series of papers we have shown that those discrepancies can be shown to result from retarded gravity as dictated by the theory of general relativity [6–10]. Indeed in the absence of temporal density changes, retardation does not effect the gravitational force. However, density is not constant for galaxies, in fact there are many processes that change the mass density in galaxies over time. Mass accretion from the inter galactic medium and internal processes such as super novae leading to super winds [10] modify the density. In addition to those local processes there is a cosmological decrease of density due to the cosmic expansion. However, the later process is many orders of magnitude smaller than the former.

In previous analysis [6–10] the corrected gravitational force was evaluated assuming a second order approximation in the retardation time $\frac{R}{c}$, neglecting higher order terms without justification. Here we take into account all orders and show that a second order approximation is indeed sufficient.

2. Linear GR

Only in cases of extreme compact objects (black holes and neutron stars) and the very early universe we consider the solution of the full non-linear Einstein Equations [6]. In most cases of astronomical interest (including the galactic case) a linear approximation to those equations around the flat Lorentz metric $\eta_{\mu\nu}$ is used, such that:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} \equiv \text{ diag } (1, -1, -1, -1), \qquad |h_{\mu\nu}| \ll 1 \tag{1}$$

One **then** defines the quantity:

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}, \tag{2}$$

$\bar{h}_{\mu\nu} = h_{\mu\nu}$ for non diagonal terms. For diagonal terms:

$$\bar{h} = -h \Rightarrow h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}. \tag{3}$$

It was shown ([11] page 75, exercise 37, see also [12–14]) that for a proper gauge the Einstein equations are:

$$\Box\bar{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -\frac{16\pi G}{c^4}T_{\mu\nu}, \qquad \bar{h}_{\mu\alpha,}{}^{\alpha} = 0. \tag{4}$$

Equation (4) can be solved such that [15] :

$$\begin{aligned} \bar{h}_{\mu\nu}(\vec{x},t) &= -\frac{4G}{c^4}\int\frac{T_{\mu\nu}(\vec{x}',t-\frac{R}{c})}{R}d^3x', \\ t &\equiv \frac{x^0}{c}, \quad \vec{x} \equiv x^a \quad a,b \in [1,2,3], \\ \vec{R} &\equiv \vec{x}-\vec{x}', \quad R = |\vec{R}|. \end{aligned} \tag{5}$$

In [16–19] we explain why the symmetry between space and time is broken, which justifies the use of different notations for space and time. $\frac{4G}{c^4} \simeq 3.3 \times 10^{-44}$; is a tiny number, hence, in the above calculation one can take $T_{\mu\nu}$, to zeroth order in $h_{\alpha\beta}$. We now evaluate the affine connection in the linear approximation:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}\left(h_{\beta\mu,\nu} + h_{\beta\nu,\mu} - h_{\mu\nu,\beta}\right). \tag{6}$$

Notice that the affine connection has first order terms in $h_{\alpha\beta}$; hence, to the first order $\Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu}$ appearing in the geodesic equation, $u^{\mu}u^{\nu}$ must be taken to zeroth order. In which:

$$u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, u^a = \vec{u} = \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \vec{v} \equiv \frac{d\vec{x}}{dt}, \quad v = |\vec{v}|. \tag{7}$$

For velocities much smaller than the speed of light in vacuum:

$$u^0 \simeq 1, \qquad \vec{u} \simeq \frac{\vec{v}}{c}, \qquad u^a \ll u^0 \qquad \text{for} \quad v \ll c. \tag{8}$$

Hence, the current paper does not discuss the post-Newtonian approximation, in which matter travels at speeds close to the speed of light, but we do consider the retardation effects which are due to the finite propagation speed of the gravitational field. We emphasize that assuming $\frac{v}{c} << 1$ is not the same as stating $\frac{R}{c} << 1$ (with R being the typical size of a galaxy) since:

$$\frac{R}{c} = \frac{v}{c}\frac{R}{v} \tag{9}$$

now since in galaxies, $\frac{R}{v}$ is huge ($\frac{R}{v} \simeq 10^{15}$ seconds); it follows that, $\frac{v}{c}$ can be dismissed but not $\frac{R}{c}$, for which $\frac{R}{c} \simeq 10^{12}$ seconds. Inserting Equations (6) and (8) in the geodesic equation, we arrive at the approximate equation:

$$\frac{dv^a}{dt} \simeq -c^2\Gamma^a_{00} = -c^2\left(h^a_{0,0} - \frac{1}{2}h_{00,}{}^a\right) \tag{10}$$

Taking a standard matter $T_{\mu\nu}$, assuming $\rho c^2 \gg p$ and, taking into account Equation (8), we arrive at $T_{00} = \rho c^2$, while the remaining tensor components are much smaller. Therefore, $\bar{h}_{00}$ is larger than other components of $\bar{h}_{\mu\nu}$. Notice that it is not possible to deduce from the magnitudes of quantities that a similar difference exists between the derivatives of those quantities. Gauge conditions in Equation (4) lead to:

$$\bar{h}_{\alpha 0,}{}^{0} = -\bar{h}_{\alpha a,}{}^{a} \qquad \Rightarrow \bar{h}_{00,}{}^{0} = -\bar{h}_{0a,}{}^{a}, \quad \bar{h}_{b0,}{}^{0} = -\bar{h}_{ba,}{}^{a}. \tag{11}$$

Thus, the zeroth derivative of $\bar{h}_{00}$ (which contains a $\frac{1}{c}$) is of similar order as the spatial derivative of $\bar{h}_{0a}$. Also the zeroth derivative of $\bar{h}_{0a}$ (see equation (10)) is of similar order as the spatial derivative of $\bar{h}_{ab}$. However, we can compare spatial derivatives of $\bar{h}_{00}$ and $\bar{h}_{ab}$ and conclude that the former is larger. Taking into account equation (3) and the above consideration, we write equation (10) as:

$$\frac{dv^a}{dt} \simeq \frac{c^2}{4}\bar{h}_{00,}{}^{a} \Rightarrow \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}, \qquad \phi \equiv \frac{c^2}{4}\bar{h}_{00} \tag{12}$$

Thus, the gravitational potential ϕ can be estimated using Equation (5):

$$\begin{aligned} \phi &= \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' \\ &= -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x' \end{aligned} \tag{13}$$

and $\vec{F}$ is the force per unit mass. In the case that the mass density ρ does not depend on time, we may use the Newtonian instantaneous action at a distance. Please notice that it is improbable that ρ is static for a galaxy, as it accretes intergalactic medium gas.

3. Retardation Effects Beyond the Newtonian Approximation

The duration $\frac{R}{c}$ may be tens of thousands of years, but may be short with respect to the duration in which the galactic density changes considerably. Thus, we write a Taylor series for the density:

$$\rho(\vec{x}', t - \frac{R}{c}) = \sum_{n=0}^{\infty} \frac{1}{n!}\rho^{(n)}(\vec{x}', t)(-\frac{R}{c})^n, \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}. \tag{14}$$

By inserting Equations (14) into Equation (13), we will obtain:

$$\begin{aligned} \phi &= \phi_2 + \phi_{(n>2)} \\ \phi_2 &= -G\int \frac{\rho(\vec{x}', t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}', t) d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x' \\ \phi_{(n>2)} &= -G\sum_{n=3}^{\infty} \frac{(-1)^n}{n! c^n}\int R^{n-1}\rho^{(n)}(\vec{x}', t) d^3x' \end{aligned} \tag{15}$$

The Newtonian potential is the first term, the second term has null contribution, and the third term is the lower order correction to the Newtonian theory:

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x' \tag{16}$$

We will show later that $n > 2$ terms can be neglected thus the total force per unit mass can be approximated as:

$$\begin{aligned} \vec{F} &\simeq \vec{F}_N + \vec{F}_r \\ \vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R} d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\ \vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R} d^3x' \end{aligned} \tag{17}$$

$\vec{F}_N$ first introduced by Newton is attractive, however, the retardation force $\vec{F}_r$ can be either attractive or repulsive. Newtonian force decreases as $\frac{1}{R^2}$, however, the retardation force doe not depend on distance as long as the Taylor approximation given in Equation (14) holds. Below a certain distance, the Newtonian force dominates, but for larger distances the retardation force has the upper hand. Newtonian force can be neglected for distances significantly larger compared to the retardation distance:

$$R \gg R_r \equiv c\Delta t \tag{18}$$

Δt is a duration associated with the second order derivative of the density ρ. For $R \ll R_r$, retardation can be neglected and only Newtonian forces need to be considered; this is the situation in the solar system. As the galaxy accretes intergalactic gas, the galactic mass becomes larger thus $\dot{M} > 0$; however, the intergalactic gas is depleted, and thus the rate at which the mass is accreted decreases resulting in $\ddot{M} < 0$, hence we have an attractive retardation force.

4. Higher order terms

Comparing equation (31) to equation (82) of [9] it follows that:

$$g^{(2)}(t) = g^{(2)}(0)e^{\frac{t}{\tau}} \tag{19}$$

Hence:

$$g^{(n)}(t) = g^{(2)}(0)\tau^{2-n}e^{\frac{t}{\tau}} = g^{(2)}(t)\tau^{2-n}, \qquad n > 2 \tag{20}$$

And also:

$$\rho^{(n)}(\vec{x},t) = \rho^{(2)}(\vec{x},t)\tau^{2-n}, \qquad M^{(n)}(t) = M^{(2)}(t)\tau^{2-n}, \qquad n > 2 \tag{21}$$

Thus according to equation (15) we have the following correction to the retardation potential:

$$\phi_{(n>2)} = -G\sum_{n=3}^{\infty}\frac{(-1)^n}{n!c^n\tau^{n-2}}\int R^{n-1}\rho^{(2)}(\vec{x}',t)d^3x'. \tag{22}$$

The deviation from the second order approximation is more pronounced for large r for which $R \simeq r$ which is the case we consider here, thus:

$$\phi_{(n>2)} \simeq -G\sum_{n=3}^{\infty}\frac{(-1)^n}{n!c^n\tau^{n-2}}r^{n-1}\int \rho^{(2)}(\vec{x}',t)d^3x' = -\frac{G\ddot{M}(t)\tau^2}{r}\sum_{n=3}^{\infty}\frac{1}{n!}\left(\frac{-r}{c\tau}\right)^n. \tag{23}$$

Now using the well known identity:

$$\sum_{n=3}^{\infty}\frac{\alpha^n}{n!} = e^{\alpha} - (1 + \alpha + \frac{1}{2}\alpha^2) \tag{24}$$

We may write equation (23) as a closed expression instead of an infinite sum:

$$\phi_{(n>2)} \simeq -\frac{G\ddot{M}(t)\tau^2}{r}\left(e^{-\frac{r}{c\tau}} - 1 + \frac{r}{c\tau} - \frac{1}{2}\left(\frac{r}{c\tau}\right)^2\right). \tag{25}$$

For $r \ll c\tau$ it is quite clear that the term in the parenthesis of equation (25) vanishes, since:

$$\lim_{\alpha\to 0}\sum_{n=3}^{\infty}\frac{\alpha^n}{n!} = \lim_{\alpha\to 0}\left(e^{\alpha} - (1 + \alpha + \frac{1}{2}\alpha^2)\right) = 0. \tag{26}$$

Hence $\phi_{(n>2)}$ can be neglected if indeed $r \ll c\tau$ for the relevant measurements of the M33 rotation curve, that is up to about $r < 20$ kpc. Now τ is dependent according to equation (81) of [9] on the density gradient of the inter galactic medium (IGM) and the typical velocity in this medium. Although those values are not known precisely we may assume that $v_z \sim 100$ km /s and the typical gradient is the same as the gradient of the optical disk luminosity that is $\frac{1}{k} \sim 0.1$ kpc. Thus $\tau \sim 10^6$ years, and $\tau c \sim 300$ kpc, making the second order approximation used so far reasonable.

5. Conclusions

The phenomena of retardation is ubiquitous in physics, and follows directly from the Lorentz symmetry group. Hence, any system that is invariant under the Lorentz transformation will exhibit retardation phenomena. Those include physical systems related to classical electromagnetism [20–23] General Relativity [6–9], but also to other Lorentz invariant theories such as conformal gravity [24–26].

Dark matter being a major candidate to explain galactic rotation curves has only a slim chance to being found, given that accelerator experiments, as using the Large Hadron Collider was unable to find any super symmetric particles, not only of the community's favorite form of dark matter, but also the form of it that is mandated in string theory, a theory that also suggests a quantized version of Einstein gravity.

We have shown that at least on the galactic scale dark matter is not needed [6–10], as the needed dynamics can be explained by a retarded gravitational potential when a near field approximation is used. We remark that the analysis of far field leading to gravitational waves [27] was corroborated in recent years by observations [28,29].

A justification for the second order Taylor series approximation which we used in previous works is given here for the first time, showing that indeed higher order terms can be safely neglected.

Finally we mention other approaches to the galactic rotation curves problem, which suggest to overcome the problem by changing the laws of gravity. Such approaches are Milgrom's MOND [30] and Mannheim's conformal gravity [24–26]. Unfortunately those approaches seem to contradict both general relativity (which is supported by a large body of observational evidence) and recent observations. Indeed van Dokkum et al. [31] have shown that there are galaxies with Newtonian rotation curves, excluding the possibility of a universal modification to the laws of gravity which prevail in every galaxy. Thus either van Dokkum's galaxy is devoid of "dark matter" or it has a small retardation depletion effect as the gas around it has not yet depleted (or fully depleted).

Given the negative results from accelerator experiments regarding dark matter, retardation theory seem to be the only valid option.

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