Dark Matter: Reality or a Relativistic Illusion?

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In memory of Donald Lynden-Bell.

A great scholar, a wonderful collaborator and a true friend.

Contents:

Retardation

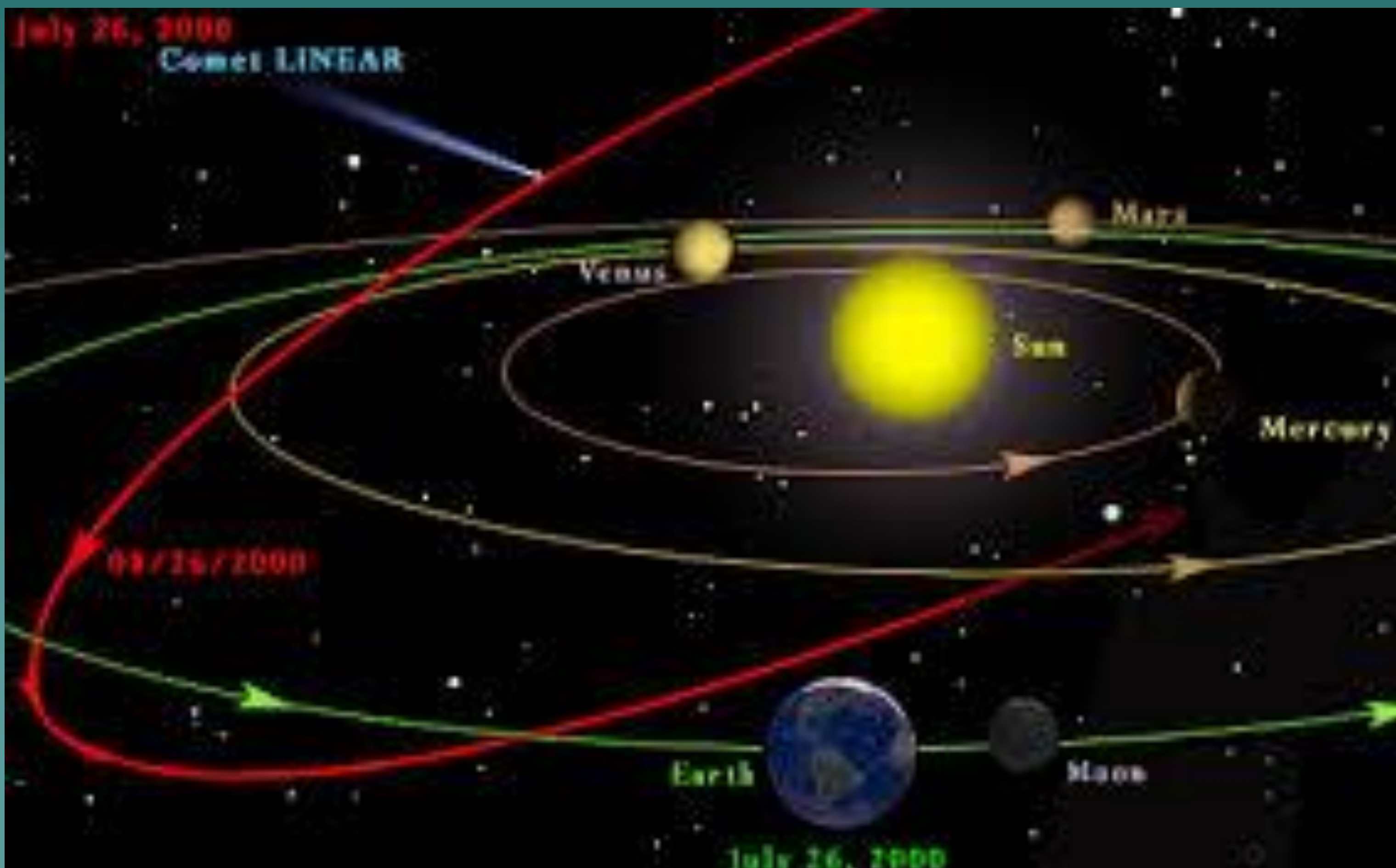
Relativity is a theory of the structure of space-time.

Special relativity was introduced in Einstein's famous 1905 paper: "On the Electrodynamics of Moving Bodies"

Maximum speed is finite: No physical object, message or field can travel faster than the speed of light in a vacuum.

Hence retardation, if someone at a distance R from me changes something I may not know about it for at least a retardation time of $\frac{R}{c}$.

Example


July 26, 2000
Comet LINEAR
Mars
Venus
Sun
Mercury
Earth
Moon
July 26, 2000

Question & Answer

Suppose somebody have eliminated the sun how much time will it take us to notice (gravitationally and electromagnetically)?

The distance from the earth to the sun is: 149.6 million km

The speed of light in vacuum is:

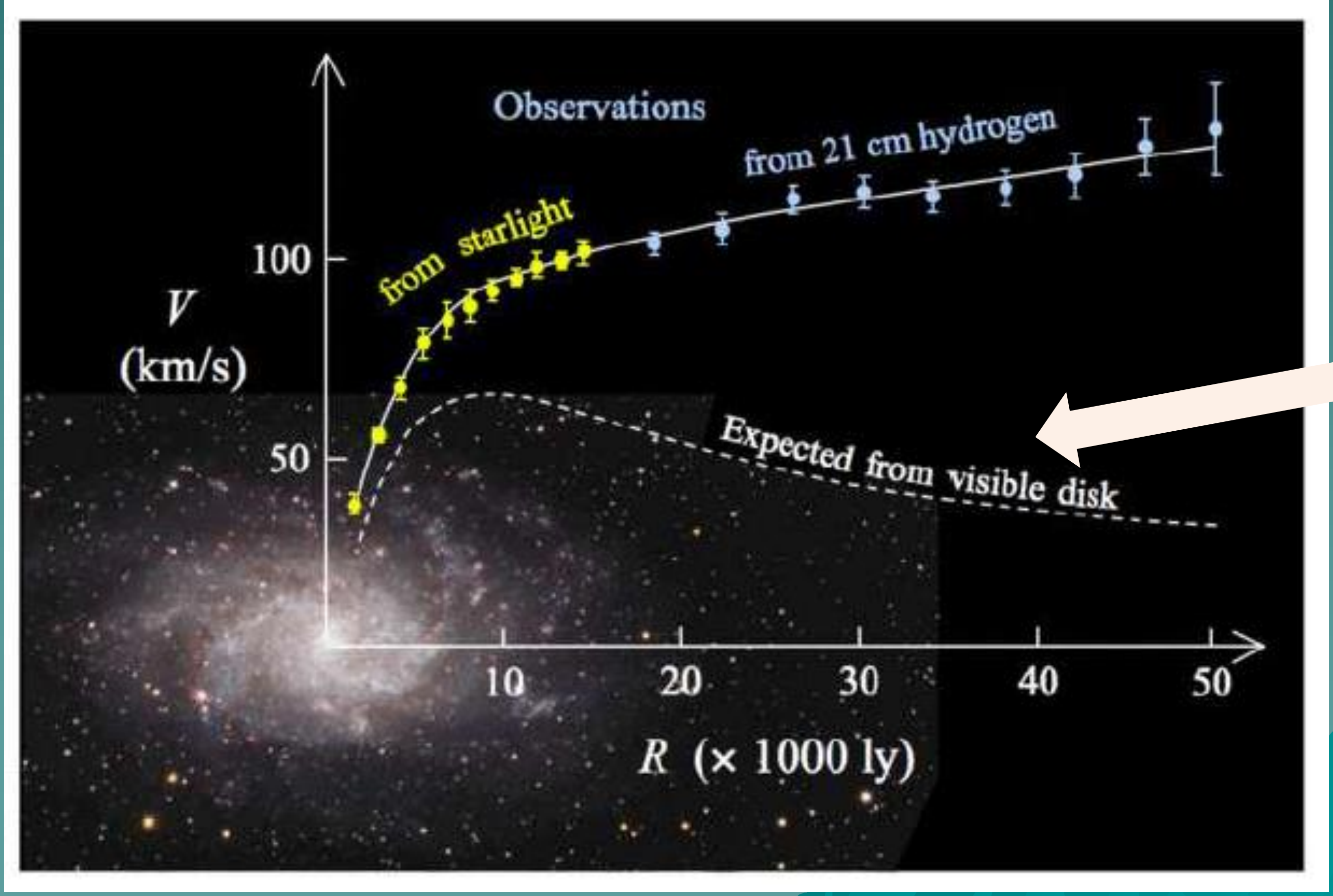
299 792 458 m / s

Hence retardation in this case is: 8.3 minutes.

Fact I - Retardation

Galaxies are huge physical systems having dimensions of many tens of thousands of light years. Thus any change at the galactic center will be noticed at the rim only tens of thousands of years later. Those retardation effects seems to be neglected in naïve galactic modelling used to calculate rotational velocities of matter in the rims of the galaxy and surrounding gas.

Fact II – Strange Rotation Curves



If you forget about retardation

Are those two facts connected?

General Relativity

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$T_{\mu\nu} = (p + \rho c^2) u_\mu u_\nu - p \; g_{\mu\nu}$$

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0$$

General Relativity

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R.$$

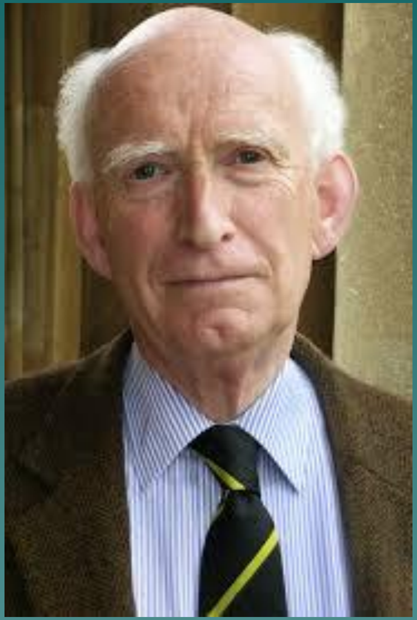
$$R^{\mu}_{\nu\alpha\beta} = \Gamma^{\mu}_{\nu\alpha,\beta} - \Gamma^{\mu}_{\nu\beta,\alpha} + \Gamma^{\sigma}_{\nu\alpha}\Gamma^{\mu}_{\sigma\beta} - \Gamma^{\sigma}_{\nu\beta}\Gamma^{\mu}_{\sigma\alpha}, \quad R_{\alpha\beta} = R^{\mu}_{\alpha\beta\mu}, \quad R = g^{\alpha\beta} R_{\alpha\beta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right), \qquad g_{\beta\mu,\nu} \equiv \frac{\partial g_{\beta\mu}}{\partial x^{\nu}}$$

How can one solve those complex, tensor, non-linear partial differential equations for the case of galaxies?

Answer:

- For most cases (galaxies included) it is not necessary to solve the full Einstein equation but only a linear approximation to them as only weak gravitational fields are involved.
- For some cases such as compact objects (black holes) and the very early universe (big bang) strong gravitational fields are involved and one needs to use the exact Einstein equations.



5 April 1935 – 6 February 2018

I would like to thank the late Professor Donald Lynden-Bell for this very important observation.

Linear Approximation (weak gravity)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} \equiv \text{ diag } (1, -1, -1, -1), \quad |h_{\mu\nu}| \ll 1$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}.$$

$$\Box \bar{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \qquad \bar{h}_{\mu\alpha,}{}^{\alpha} = 0.$$

Solution:

$$\bar{h}_{\mu\nu}(\vec{x},t) = -\frac{4G}{c^4}\int \frac{T_{\mu\nu}(\vec{x}',t-\frac{R}{c})}{R} d^3x',$$

$$t \equiv \frac{x^0}{c}, \quad \vec{x} \equiv x^a \quad a,b \in [1,2,3], \quad \vec{R} \equiv \vec{x} - \vec{x}', \quad R = |\vec{R}|.$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}.$$

$$\frac{4G}{c^4} \simeq 3.3 \ 10^{-44}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}\left(h_{\beta\mu,\nu} + h_{\beta\nu,\mu} - h_{\mu\nu,\beta}\right).$$

- If the size of the object is much smaller than the distance to the observer will result in a different approximation leading to the famous quadruple equation of gravitational radiation as predicted by Einstein and verified indirectly in 1993 by Russell A. Hulse and Joseph H. Taylor, Jr. for which they received the Nobel Prize in Physics. The discovery and observation of the Hulse-Taylor binary pulsar offered the first indirect evidence of the existence of gravitational waves.
- On 11 February 2016, the LIGO and Virgo Scientific Collaboration announced they had made the first direct observation of gravitational waves. The observation was made five months earlier, on 14 September 2015, using the Advanced LIGO detectors. The gravitational waves originated from the merging of a binary black hole system.
- The current work involves a near field application of gravitational radiation while previous art discusses far field results.

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}u^\mu u^\nu = 0$$

→ **Affine connection is first order.**

$u^\mu u^\nu$ **zeroth order**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

$$u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad u^a = \vec{u} = \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad \vec{v} \equiv \frac{d\vec{x}}{dt}, \quad v = |\vec{v}|.$$

$$u^0 \simeq 1, \qquad \vec{u} \simeq \frac{\vec{v}}{c}, \qquad u^a \ll u^0 \qquad \text{for} \quad v \ll c.$$

The resulting geodesic:

$$\frac{dv^a}{dt} \simeq -c^2\Gamma^a_{00} = -c^2\left(h^a_{0,0} - \frac{1}{2}h_{00,}{}^a\right)$$

$$\rho c^2 \gg p \quad \Rightarrow$$

$$\frac{dv^a}{dt} \simeq \frac{c^2}{4}\bar{h}_{00,}{}^a \Rightarrow \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}, \qquad \phi \equiv \frac{c^2}{4}\bar{h}_{00}$$

Back to Newton?

$$\phi = \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x'$$

If retardation is neglected or the density is static:

$$\phi = \phi_N = -G\int \frac{\rho(\vec{x}')}{R} d^3x'$$

However, retardation cannot be neglected on galactic scales and the density is not static as mass is accreted from the intergalactic medium:

Beyond the Newtonian Approximation

$$\rho(\vec{x}', t - \frac{R}{c}) = \sum_{n=0}^{\infty} \frac{1}{n!} \rho^{(n)}(\vec{x}', t)(-\frac{R}{c})^n, \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}.$$

$$\rho(\vec{x}', t - \frac{R}{c}) \simeq \rho(\vec{x}', t) - \rho^{(1)}(\vec{x}', t)\frac{R}{c} + \frac{1}{2}\rho^{(2)}(\vec{x}', t)(\frac{R}{c})^2.$$

$$[t - T_{max\ 2}, t + T_{max\ 2}].$$

$$R < c\, T_{max\ 2} \equiv R_{max}$$

Short Range!

$$\phi = -G\int \frac{\rho(\vec{x}',t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}',t)d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}',t)d^3x'$$

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}',t)d^3x'$$

$$\begin{aligned}
\vec{F} &= \vec{F}_N + \vec{F}_r \\
\vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}',t)}{R^2}\hat{R}d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\
\vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}',t)\hat{R}d^3x'
\end{aligned}$$

Obviously for small distances Newtonian forces dominate over Retardation forces but what happens for large distances were Newtonian forces decline and Retardation forces are not reduced?

Δt is the typical duration in which the density ρ changes.

$$R_r \equiv c\Delta t$$

Retardation distance

$$R \ll R_r$$

Newtonian regime

$$R \gg R_r$$

Retardation regime

For large distances:

$$\hat{R} \simeq \frac{\vec{x}}{|\vec{x}|} \equiv \hat{r}$$

$$\vec{F}_r = \frac{G}{2c^2}\hat{r}\int \rho^{(2)}(\vec{x}', t)d^3x' = \frac{G}{2c^2}\hat{r}\ddot{M}, \qquad \ddot{M} \equiv \frac{d^2M}{dt^2}.$$

Retardation forces can be repulsive or attractive.

$$\dot{M} > 0$$

Mass is accreted from the intergalactic gas.

$$\ddot{M} < 0.$$

The intergalactic gas is depleted.

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

The retardation force is attractive in the galactic scenario.

Alternative explanations for galactic rotation curves:

1. **Dark matter**
2. **MOND (Modified Newtonian Gravity)**

How can retardation effect be confused with a non existent "dark matter"?

$$-\frac{v_c^2}{r}\hat{r} = \vec{F}_d = -\frac{G M_d(r)}{r^2}\hat{r}$$

However, F_d is really F_r

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

→

$$M_d(r) = \frac{r^2|\ddot{M}|}{2c^2}$$

$$M_d(r) = 4\pi \int_0^r r'^2 \rho_d(r') dr', \qquad \frac{dM_d(r)}{dr} = 4\pi r^2 \rho_d(r)$$

$$\rho_d(r) = \frac{|\ddot{M}|}{4\pi c^2 r}$$

"dark matter" density decreases as $r^{-1.3}$ for the M33 galaxy.

E. Corbelli; P. Salucci (2000). "The extended rotation curve and the dark matter halo of M33". Monthly Notices of the Royal Astronomical Society. 311 (2): 441447. arXiv:astro-ph/9909252. doi:10.1046/j.1365-8711.2000.03075.x.

What can we learn from the incorrect MOND theory of gravity (but observationally fitting)?

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

$$\mu(\frac{a}{a_0}) = \frac{1}{1+(\frac{a_0}{a})^2}$$

Suggested by Milgrom and corrected for the relativistic case by Bekenstein

Assume small accelerations:

$$a = \frac{v^2}{r}$$

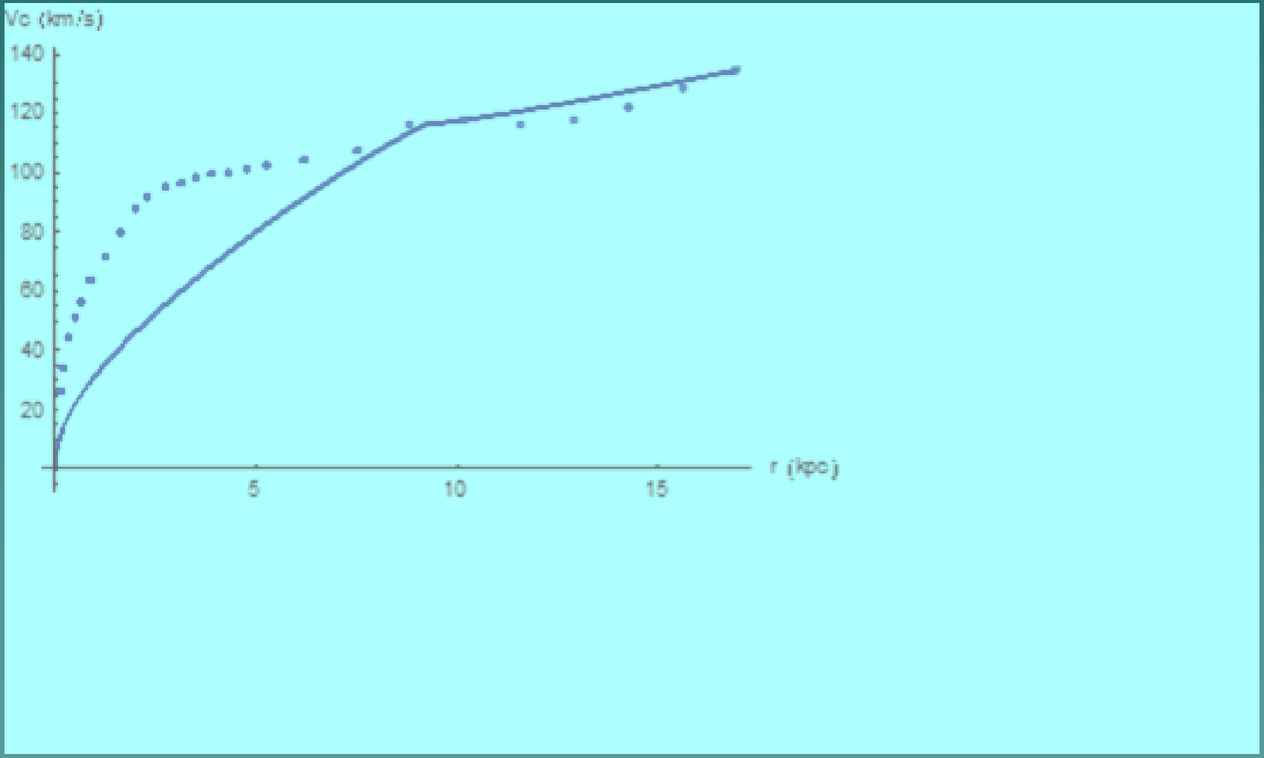
$$\vec{F}_M = -\frac{GMa_0^2}{v^4}\hat{r}$$

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

$$|\ddot{M}| = \frac{2Ma_0^2c^2}{v^4}.$$

Milgrom found $a_0 = 1.2\ 10^{-10} ms^{-2}$ to be most fitting to the data. The mass of the M33 galaxy is $9.95\ 10^{40} kg$ and the velocity far away from the galaxy is $179,000\ ms^{-1}$. We thus obtain $\ddot{M} \simeq 2.51\ 10^{17} kgs^{-2}$ and a ratio $\frac{|\ddot{M}|}{M} \simeq 2.52\ 10^{-24}\ s^{-2}$. This amounts to a typical accumulation acceleration time scale of $20,000$ years and retardation distance of $20,000$ light years

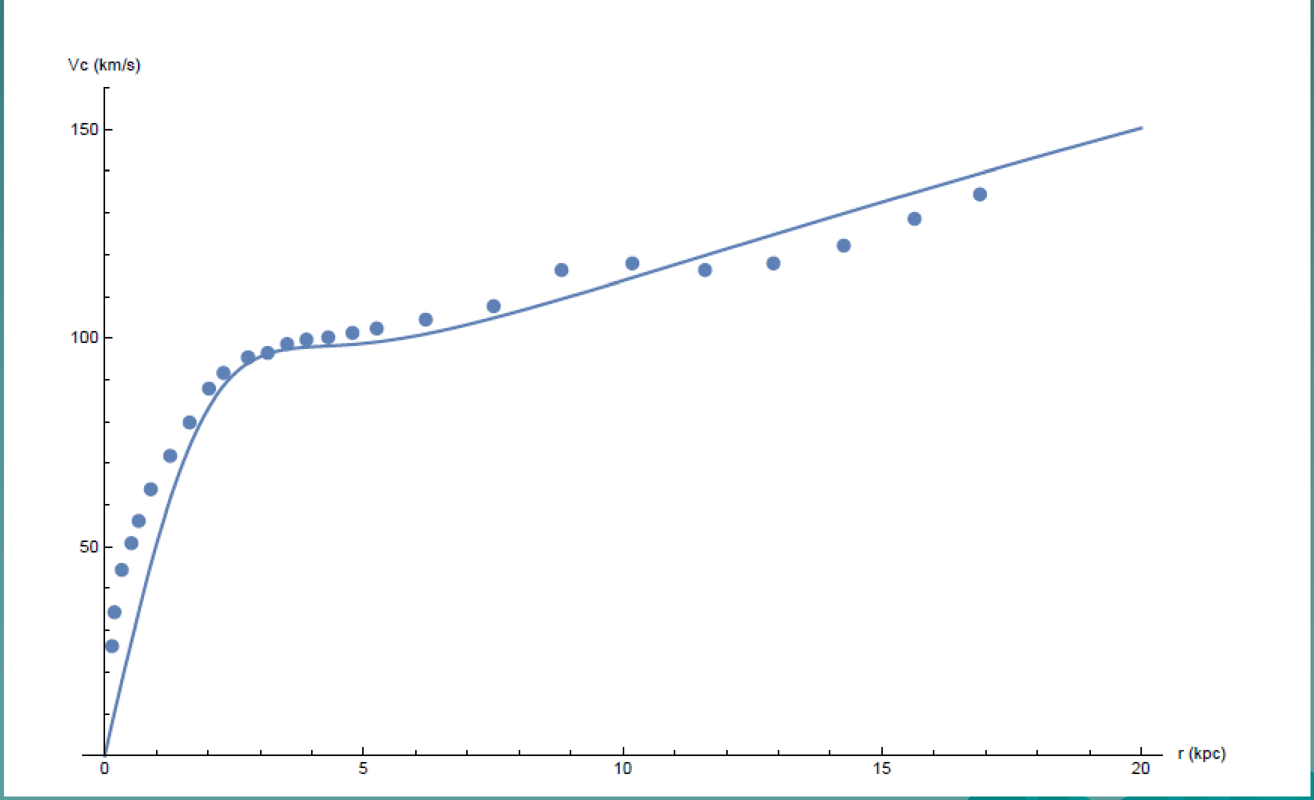
For Uniform Mass Distribution



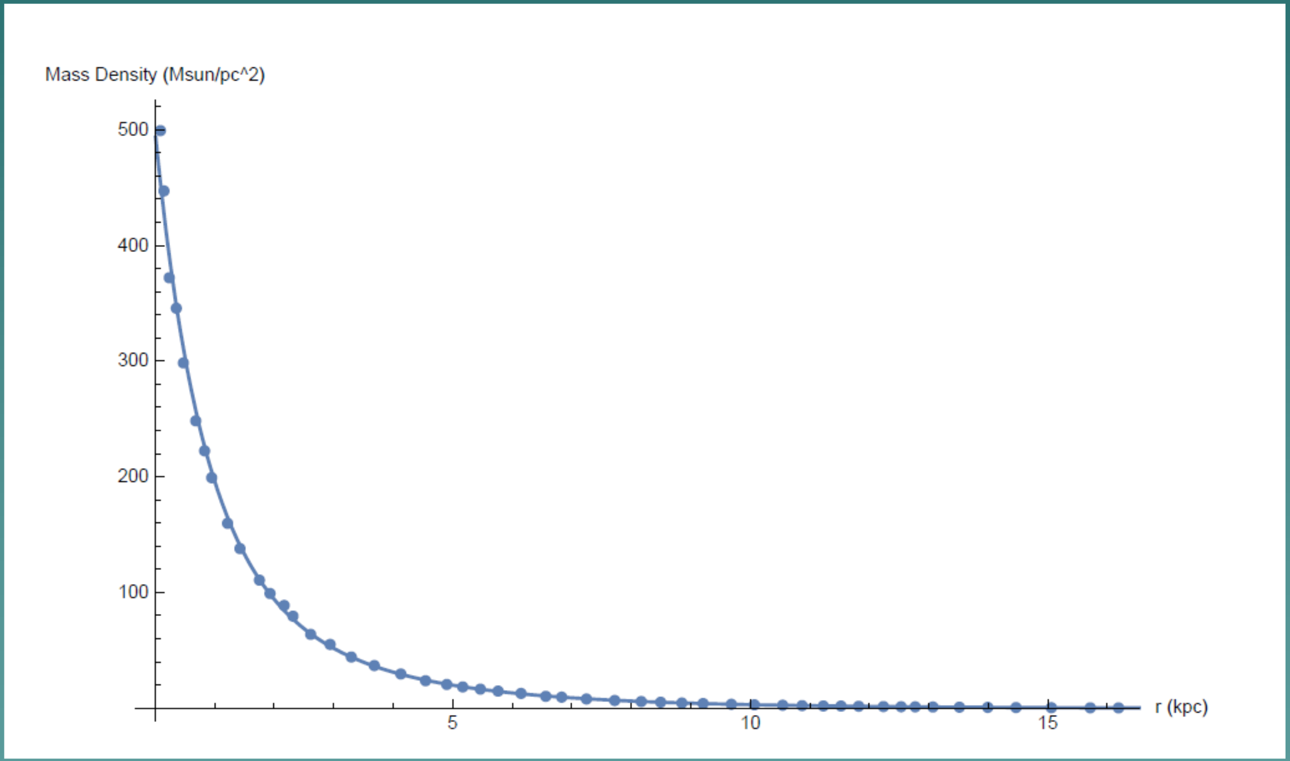
I thank Dr. Michal Wagman (PhD graduate in Ariel University) for extracting he point of M33 rotation curve.

A better profile and a better fit

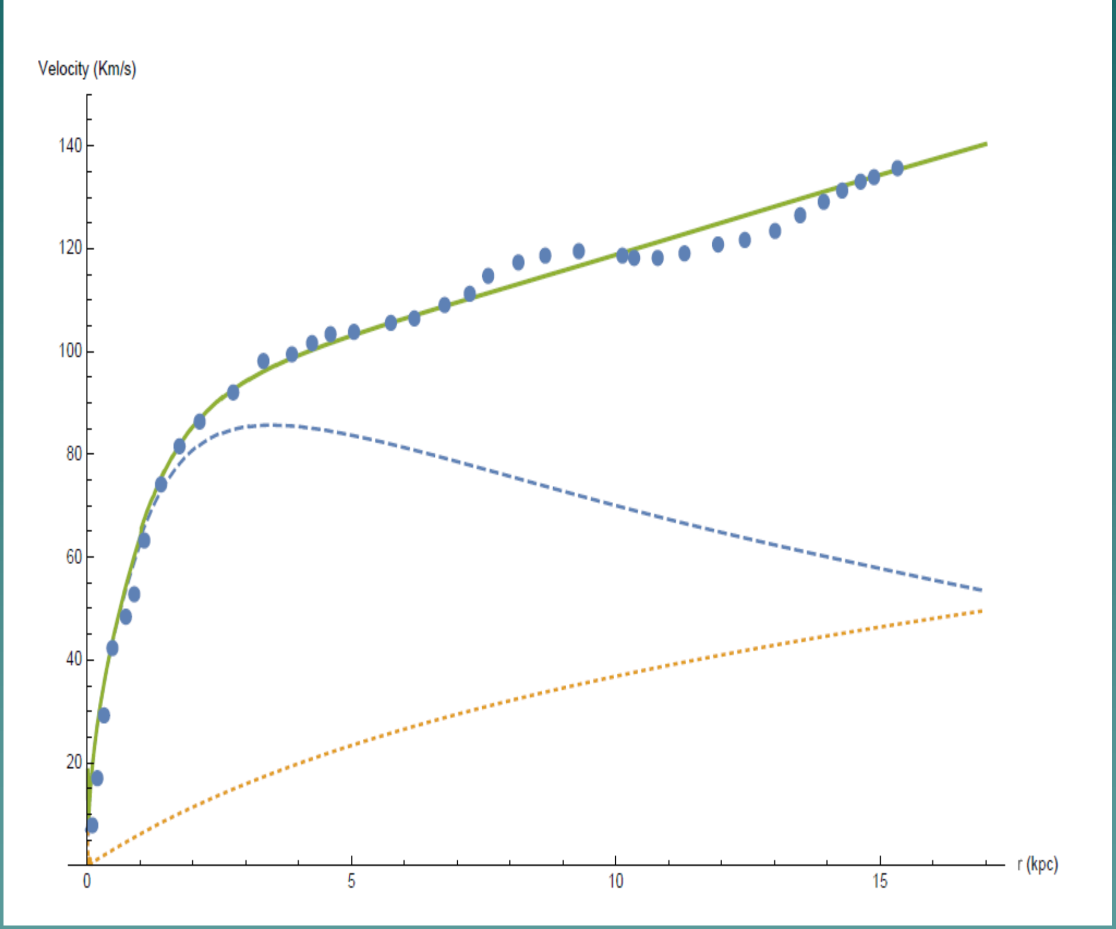
$$\rho(r) = \rho_c e^{-\frac{r^2}{R_G^2}}.$$



My best fit



[3] Corbelli E., 2003, Monthly Notices of the Royal Astronomical Society 342(1): 199-207. https://arxiv.org/abs/astro-ph/0302318 DOI: 10.1046/j.1365-8711.2003.06531.x



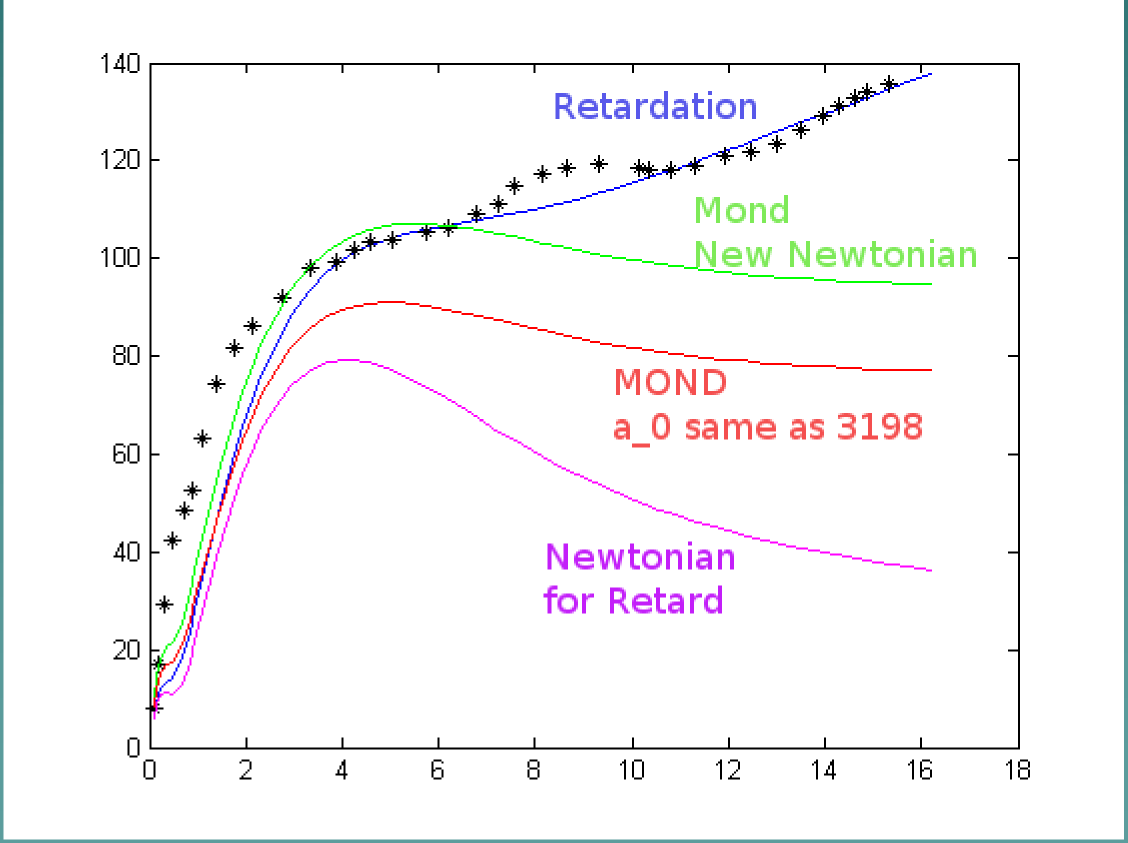
M/L =1

So M/L is not a fitting parameter as most authors assume.

the full line describe the complete rotation curve which is the sum of the dotted line describing the retardation contribution and the dashed line which is the Newtonian contribution.

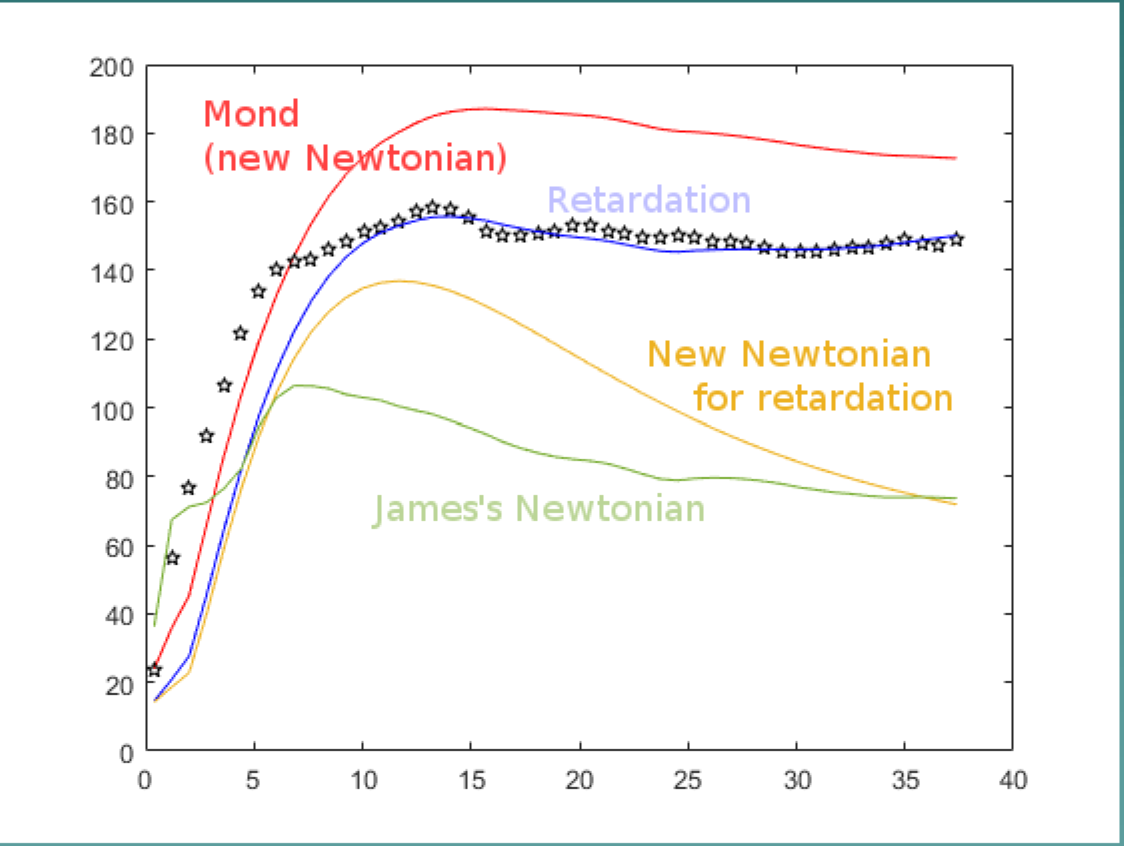
Work by Michal Wagman PhD student at Ariel University under my supervision.

M33

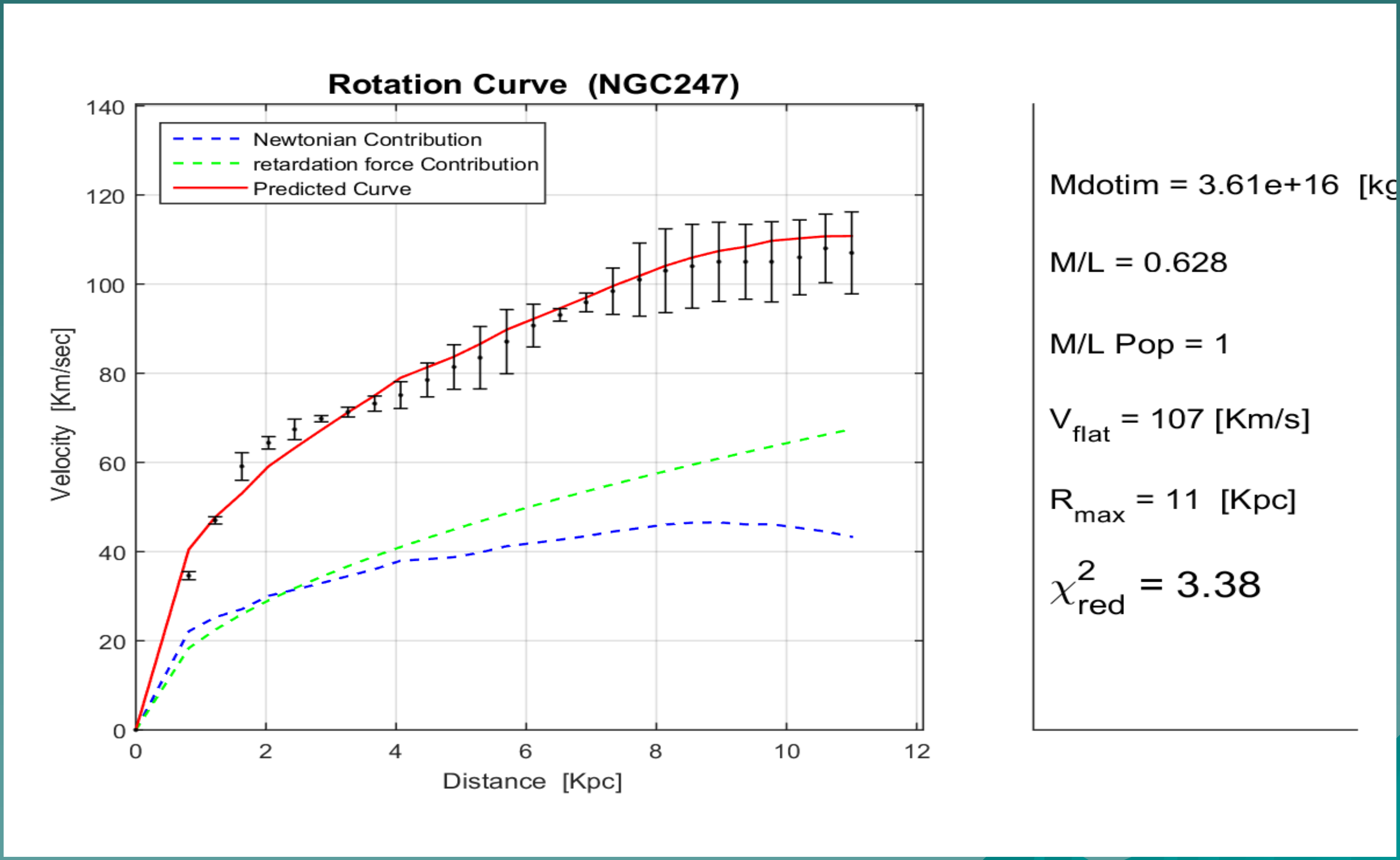


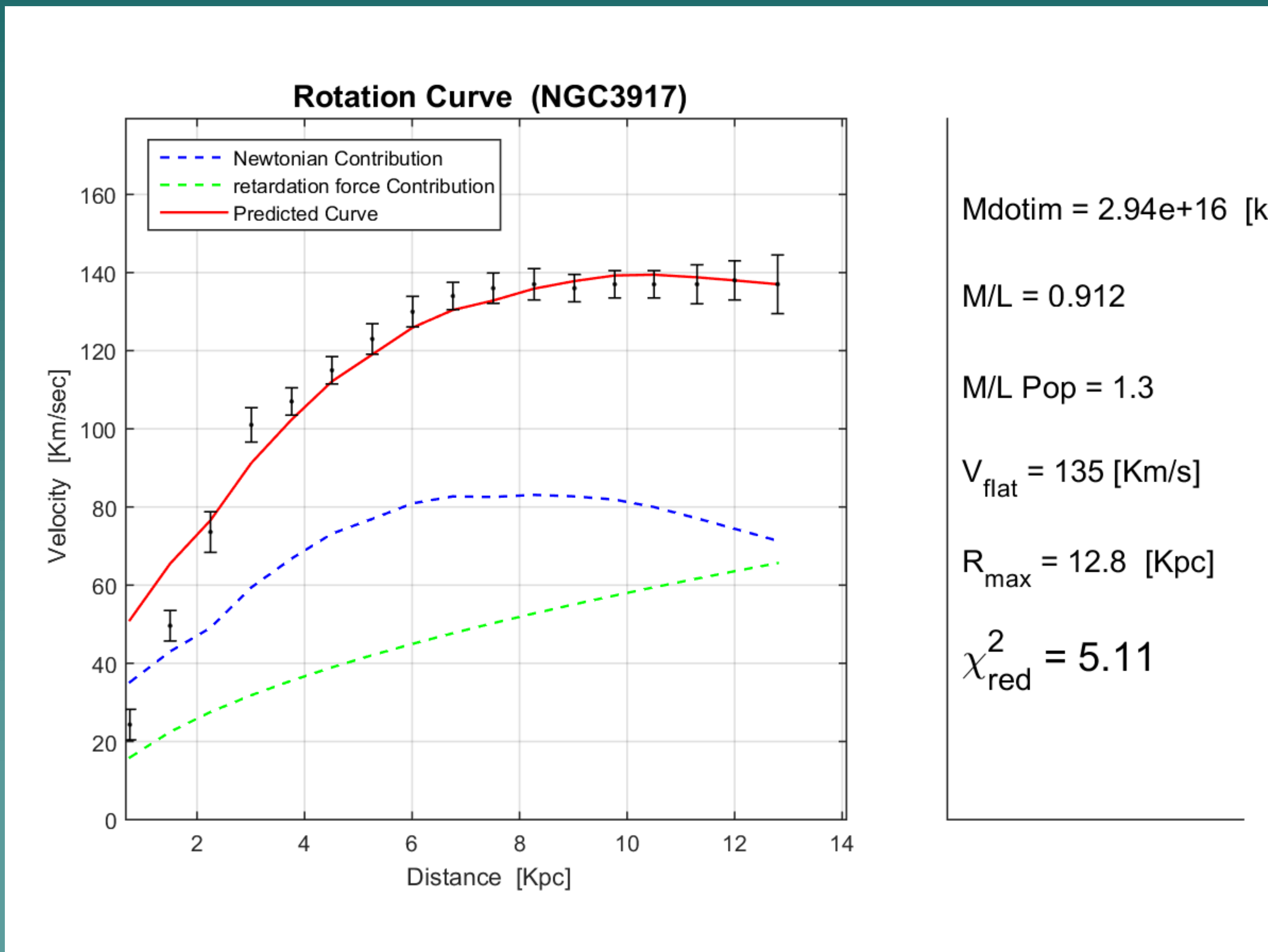
Work by Michal Wagman PhD student at Ariel University under my supervision.

NGC 3198



Work by Tomer Zimmerman & Roy Gomel
PhD students at Tel Aviv University




Rotation Curve (NGC3917)
Newtonian Contribution
retardation force Contribution
Predicted Curve
Velocity [Km/sec]
Distance [Kpc]
0
20
40
60
80
100
120
140
160
2
4
6
8
10
12
14
Mdotim = 2.94e+16 [kg
M/L = 0.912
M/L Pop = 1.3
V_{flat} = 135 [Km/s]
R_{max} = 12.8 [Kpc]
χ^2_{red} = 5.11

Michal's Results

(1) Galaxy Name	(2) Galaxy Type	(3) Mass Model	(4) Scale Lenght(pc)	(5) Σ_0 $(M_\odot pc^- 2)$	(6) $\frac{\ddot{M}}{M}\left(\frac{1}{t^2}\right)$	(7) b	(8) M/L ratio $(M_\odot/L_\odot)$	(9) MOND a_0
NGC 3198	Sbc	Thick	3848	110	$-6.88 \cdot 10^{-25}$	0.27*	4.6	$0.5 \cdot 10^{-10}$
238 G-24	Sb	Double	4100	602	$-1.12 \cdot 10^{-24}$	0.35	0.9	$1.2 \cdot 10^{-10}$
308-G23	Sb	Double	2198	812	$-1.29 \cdot 10^{-24}$	0.35	1.3	$1 \cdot 10^{-10}$
286-G16	Sb	Double	3821	665	$-1.46 \cdot 10^{-24}$	0.35	0.75	$1.2 \cdot 10^{-10}$
N3917	Scd	Double	2600	60	$-1.5 \cdot 10^{-24}$	0.69	9	$0.8 \cdot 10^{-10}$
123-G23	Sc	Double	3675	493	$-2 \cdot 10^{-24}$	0.9	0.7	$1.2 \cdot 10^{-10}$
362-G11	Sc	Double	2108	572	$-3.2 \cdot 10^{-24}$	0.9	0.9	$1.2 \cdot 10^{-10}$
N3949	Sbc	Double	995	916	$-4.8 \cdot 10^{-24}$	0.9	2	$1.2 \cdot 10^{-10}$
M33	Sc	Double	1315	944	$-6.23 \cdot 10^{-24}$	0.9	0.6	$0.8 \cdot 10^{-10}$
NGC 300	Sc	Double	1700	354	$-8.17 \cdot 10^{-24}$	0.9	2.5	$0.7 \cdot 10^{-10}$
UGCA 442	Sm	Double	1400	92	$-1.46 \cdot 10^{-23}$	1.67*	1	$0.25 \cdot 10^{-10}$

Rotation Curve fitting data for 11 galaxies. Columns are: (1) Name of the galaxy, (2) Galaxy type, (3) Mass model used for calculation where Double is short for double infinitesimally thin exponential disk model, Thick is short for exponential disk with a distribution in the z-axis. (4) Exponential disk model scale-length given in parsecs, (5) Central mass density, (6) mass flux change-to-mass ratio, (7) star birthrate of the galaxy type from Portinari et al and Kennicutt et al ; data retrieved from Kennicutt et al is denoted with an asterisk*. (8) mass-to-light ratio (9) MOND acceleration fitting parameter

Conclusion

We show that ”dark matter” and ”MOND” effects are explained in the framework of standard GR as effects due to retardation without assuming any exotic matter or modifications of the theory of gravity.

What will happen if the mass outside the galaxy is totally depleted or not yet depleted?
In this case retardation force should vanish. This was indeed reported recently for the galaxy NGC1052-DF2.

Pieter van Dokkum, Shany Danieli, Yotam Cohen, Allison Merritt, Aaron J. Romanowsky, Roberto Abraham, Jean Brodie, Charlie Conroy, Deborah Lokhorst, Lamiya Mowla, Ewan OSullivan & Jielai Zhang "A galaxy lacking dark matter" Nature volume 555, pages 629632 (29 March 2018) doi:10.1038/nature25767.

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