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“Null String” Gas Cosmology: 1st steps

Alexander Lelyakov [1,*]

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[1] Department of Theoretical Physics, Institute of Physics and Technology, V.I. Vernadsky Crimean Federal University, Simferopol 295007, Russia; lelyakov_a_p@cfuv.ru
* Correspondence: lelyakov_a_pl@cfuv.ru; Tel.: +7-978-757-0979

Abstract: This work is devoted to the study of the asymptotics of the gravitational field of primary particles with nonzero rest mass. These particles are structurally composed of two closed “null strings” (thin closed tubes of a massless scalar field) in the shape of a circle, and they are formed in a gas of null strings as a result of gravitational interaction. It is shown that on time scales much larger than the time of one complete cycle of oscillation of the null strings forming a particle, or at distances much larger than the dimensions of the region within which the oscillations of interacting null strings occur, the gravitational field of such a particle is described by the Minkowski metric. It is noted that with decreasing observation time or on distance scales that are commensurate with the size of primary particles, significant deviations of the gravitational field from the flat Minkowski space-time should appear in the gas of null strings.

Keywords: “Null String” Gas Cosmology (NSGC), primary particle, asymptotics of the gravitational field, structured massless scalar field.

1. Introduction

Observations carried out indicate that visible matter and detected radiation make only a small contribution to the total mass of the Universe. Evidence for the existence of dark matter comes for example from studies of gravitational lensing, cosmic microwave background radiation (CMB) or rotation curves of spiral galaxies [1-7]. According to the results of observations, dark matter plays a dominant role in the evolution and acceleration of the Universe. In this connection, the urgent task is to search for physical systems (models) for which the presence of dark matter is an intrinsic property.

One of such physical systems can be a “null string” gas, i.e., a multi-string system that structurally consists of gravitationally interacting closed thin tubes of a massless scalar field in the form of a circle. In works [8,9], a study of the influence of the shape of a closed null string on its gravitational properties was started. This study showed that closed null strings in the shape of a circle are stable when moving in an external gravitational field. That is, the shape of a closed null string in the form of a circle should be preserved when moving in an external gravitational field. It was noted that the observed manifestation of the action of an external gravitational field on such a null string can only be a change in its size (radius) or a change in the direction of its motion.

The possibility of taking into account the gravitational interaction in a gas consisting of closed null strings in the form of a circle was investigated in the works [10,11]. The result of these works was the ability to qualitatively take into account the mutual change in the trajectories of motion for two interacting null strings. The algorithm proposed to take into account the mutual influence is based on the study of the trajectories of the probe null strings in the gravitational field of the null string of the source. It should be noted that an interesting feature of the gravitational interaction in a gas of null strings is the possibility of the formation of coordinated motions of closed null strings, in which they perform oscillatory motions within a region limited in space. In this case, we can say that two gravitationally interacting null strings that oscillate inside a space-limited region form a primary particle with an effective nonzero rest mass.

Fig. 1 and Fig. 2 in a cylindrical coordinate system, qualitatively, show examples of trajectories of motion of two null strings, the gravitational interaction between which leads to oscillations of each null string inside a region limited in space (the repeating trajectories of motion of null strings forming the corresponding primary particles are bounded both in the variable z and in the variable ρ) [10,11]. The examples given are distinguished by the location in space of the regions within which the oscillations of the null strings that form the primary particles occur.

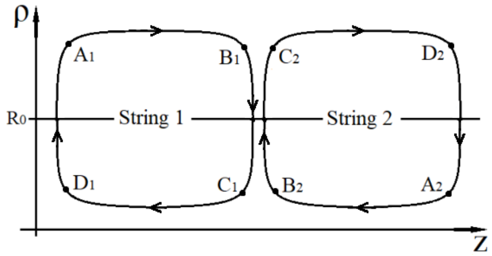

Figure 1. The figure shows an example of the trajectories of motion of two null strings forming a primary particle, the meeting surface for which is orthogonal to the ρ axis ($\rho = R_0$).

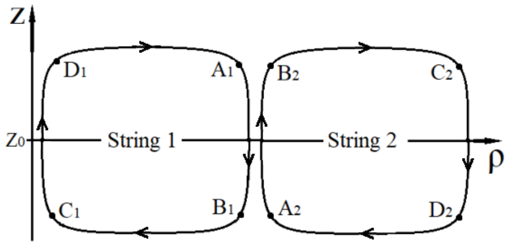

Figure 2. The figure shows an example of the trajectories of motion of two null strings forming a primary particle, the meeting surface for which is orthogonal to the z axis ($z = z_0$).

For the cases shown in Fig. 1 and Fig. 2, two gravitationally interacting closed null strings have the shape of a circle, they are located in parallel planes and move “towards” each other. For the situation shown in Fig. 1, the meeting surface (i.e., the surface on which two null strings “meet” when moving towards them) is $\rho = R_0$. For the situation shown in Fig. 2, the meeting surface is the surface $z = z_0$. In Fig. 1 and Fig. 2, the direction of motion for string 1 defines a sequence of points: A_1, B_1, C_1, D_1, ..., and for string 2, a sequence of points A_2, B_2, C_2, D_2, ... In this figures, the points on the trajectories of motion, which are designated by the same letter (but with different indices), correspond to the positions of null strings forming the primary particle for the same time value t .

If each primary particle shown in Fig. 1 and Fig. 2 is considered separately (i.e., outside the gas null strings), then the “lifetime” of such a particle will be unlimited. However, the “lifetime” of the primary particle located inside the gas of null strings, due to the gravitational interaction with external null strings, can be both long and very short. Long-term existence (“lifetime”) of primary particles is possible if they are combined into many-particle spatial structures.

Among such structures, the most interesting can be axially symmetric (“one-dimensional”) objects - “threads” and spherically symmetric domains - “macro” objects.

It should be noted that the possibility of combining primary particles (pairs of gravitationally interacting null strings) into complex many-particle structures depends on the direction of motion of the null strings involved in the formation of the primary particle [10,11]. To characterize the direction of motion of the null strings that form the primary particle, a vector orthogonal to the plane of the location of the null strings that form the primary particle can be constructed. To unite interacting null strings into complex multiparticle formations, for example, into a “one-dimensional” object (“thread”), the directions of the vectors that characterize the direction of movement of individual null strings (locally) must coincide.

For the case of combining primary particles into a spherically symmetric domain, the forming “threads” are located at the radii of such a domain, and null strings belonging to different “threads” form spherical surfaces (form layers of the domain). Fig. 3 shows two possibilities of such a combination (schematically). In Fig. 3, each vector corresponds to a null string (the vector determines the direction of motion of the null string, which is involved in the formation of the “thread”). Null strings that participate in the formation of a spherically symmetric domain are always located orthogonal to the direction of the corresponding vector [10,11].

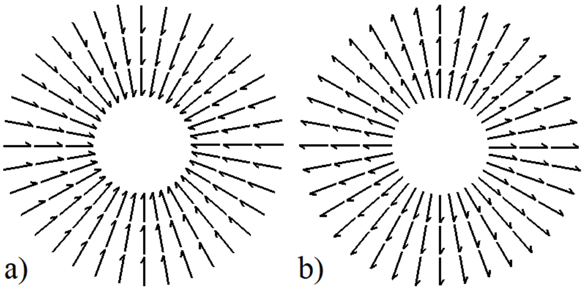

Figure 3. In the figure (a,b), two possibilities of combination of gravitationally interacting null strings into a spherically symmetric domain are schematically presented.

It can be noted that an interesting feature of such “macro” structures in a null string gas is the fundamental impossibility of having a fully formed (finished) structure. The following reasons can be named that lead to the impossibility of having a fully formed structure for “macro” objects (“macro” structures) in a gas of null strings:

- The presence of external (“free”) null strings and other “macro” objects in the gas;
- Possible movement of “macro” objects relative to each other;
- The existence for each null string of an “interaction zone”, outside of which the null strings do not experience the gravitational influence of each other [12-18].

The listed reasons should lead to a random (dynamic) change in the number of null strings that gravitationally “belong” to a specific (considered) “macro” object (specific “macro” objects). It is clear that due to the random change in the number of null strings that gravitationally “belong” to the considered “macro” formation, the spatial form of the “macro” object (domain) will also randomly (dynamically) change.

Considering the spatial distributions of “macro” formations in a gas of null strings on different time scales, one can introduce the concepts of “substance” and “field”. Namely, the “macro” formations averaged over a long period of time form the “substance”, and the structures that are associated with the “macro” objects but whose “lifetime” less will form the “field”. Obviously, with a decrease in the observation time, various ordered structures (“particles”) will be observed in the “field”, which can be interpreted as interaction “particles”. Moreover, with decreasing observation time, the number of “particles” forming the “field” should increase, and the structure of the “particles” forming the “field” will become more diverse.

In this model, dark matter can be formed by extremely numerous and spatially diverse structures with nonzero rest mass, which are formed in a null string gas (gas of thin closed tubes of a massless scalar field) as a result of gravitational interaction. It is important to note that the “lifetime” of such structures in the gas can be extremely short, but their number and permanent formation of new ones can make a considerable contribution to the overall mass.

An interesting problem is the study of the gravitational field of such structures, in particular, the study of the gravitational field of two gravitationally interacting null strings in the form of a circle (primary particles with nonzero rest mass), the trajectories of which are shown in Fig. 1 and Fig. 2.

In the articles [17-21], the gravitational field of solitary closed null strings was investigated, the trajectories of which do not change over time. Namely, in the articles [20,21] a solution of the Einstein equations was found for a closed null string of constant radius, which moves along the z axis and at each moment of time t lies entirely in the plane orthogonal to this axis. In the articles [17-19], solutions are given that describe the gravitational field of a solitary closed null string, for the cases in which the null string radially increases or radially decreases its size (radius). One of the results of these papers is the statement that the flat Minkowski space cannot be considered as an asymptotic for such gravitational fields. On the other hand, in accordance with Petrov's classification [22], the gravitational field of particles with nonzero rest mass should be of the type D, respectively, the asymptotics of the gravitational field of particles with nonzero rest mass should be flat Minkowski space.

This article is devoted to the study of the asymptotics of the gravitational field of primary particles with nonzero rest mass. These particles are structurally composed of two closed null strings in the shape of a circle and they are formed in a gas of null strings as a result of gravitational interaction. In this work, a unit system is chosen in which the speed of light c = 1.

2. Construction of the quadratic form

We will assume that the source of the gravitational field are two gravitationally interacting null strings in the form of a circle (primary particles), the trajectories of which are shown in Fig. 1, and Fig. 2.

When constructing a quadratic form describing the field of two gravitationally interacting null strings, we can assume that the trajectory of motion of each of the interacting null strings consists of four segments (see Fig. 1, and Fig. 2). Moreover, on the segments $A_l B_l$ and $C_l D_l$ the null string moves with a constant radius. On the segments $B_l C_l$ and $D_l A_l$, the null string radially decreases or increases its size (radius).

It is important to note that on each of the given segments, the null strings that form the particle move in opposite directions ("towards" each other). So, for example, for the case shown in Fig. 1, on the segment $A_1 B_1$ string 1 moves in the positive direction of the z axis, and string 2, at the same time, on the segment $A_2 B_2$ moves in the negative direction of the z axis. On the segment $B_1 C_1$ string 1 radially decreases its size (radius), and string 2, at the same time, on the segment $B_2 C_2$ radially increases its size (radius).

If a closed null string, on a certain segment, moves along the z axis without changing the size (radius), then its "trajectory" of motion (world surface) is determined by the equalities

$$t = t_0 + \tau,\ \rho = R_0,\ \theta = \sigma,\ z = z_0 \mp \tau,\ \tau \in (\tau_0; \tau_1),\ \sigma \in [0; 2\pi], \tag{1}$$

where τ and σ are parameters on the world surface of the null string, τ_0 and τ_1 constants that determine the value of the τ parameter on the boundaries of the segment, R_0 null string radius, the constants t_0 and z_0, respectively, determine the initial time value and the initial position of the null string on the z axis. The motion of a closed null string in the negative direction of the z axis corresponds to the case $z = z_0 - \tau$. The motion of a closed null string in the positive direction of the z axis describes the case $z = z_0 + \tau$.

If, on some segment, a closed null string radially increases or decreases its size (radius) while being on the surface $z = z_0$, then the "trajectory" of motion is determined by the equalities

$$t = t'_0 + \tau,\ \rho = R_0 \mp \tau,\ \theta = \sigma,\ z = z_0,\ \tau \in (\tau'_0; \tau'_1),\ \sigma \in [0; 2\pi], \tag{2}$$

where the constants t_0' and R_0, respectively, determine the initial time value and the initial radius of the null string. The case $\rho = R_0 - \tau$ describes radial compression, and the case $\rho = R_0 + \tau$ describes the radial expansion of a closed null string. It is assumed that

$$R_0 - \tau_1' = r_0 \geq 0,\ R_0 + \tau_0' = r_0' \geq 0. \qquad (3)$$

In the equalities (3), the constant r_0' is the radius of the null string at the initial moment of time ($t = \tau_0'$) for the case of radial expansion, and r_0 is the radius of the null string at the final moment of time ($t = \tau_1'$) for the case of radial compression.

Since for trajectories of a null string (1), (2), the shape of a null string is preserved during motion (remains a circle), the metric functions $g_{mn} = g_{mn}(t, \rho, z)$. Then, using the invariance of the quadratic form with respect to the inversion of θ on $-\theta$, we obtain $g_{02} = g_{12} = g_{32} = 0$.

It can also be noted that for the cases shown in Fig. 1, and Fig. 2, on time scales much larger than the time of one full cycle of oscillation of the null string (or at distances for which the geometric dimensions of the particles shown in Fig. 1 and Fig. 2 can be neglected), the functions of the required quadratic form must be invariant under the simultaneous inversion $t \to -t$, $z \to -z$, i.e.,

$$g_{mn}(t, \rho, z) = g_{mn}(-t, \rho, -z). \qquad (4)$$

The consequence of (4) is $g_{01} = g_{31} = 0$. Also, using the freedom to choose coordinate systems in GR, we partially fix it by the requirement $g_{03} = 0$. Thus, the quadratic form for the problem being solved can be represented as

$$dS^2 = e^{2\nu}(dt)^2 - A(d\rho)^2 - B(d\theta)^2 - e^{2\mu}(dz)^2, \qquad (5)$$

where $\nu,\ \mu,\ A,\ B$ are functions of the variables $t,\ \rho,\ z$.

3. Null string motion

The motion of a null string in a pseudo-Riemannian space-time is determined by the system of equations [23]

$$x^{\alpha}{}_{,\tau\tau} + \Gamma^{\alpha}_{pq} x^{p}{}_{,\tau} x^{q}{}_{,\tau} = 0, \qquad (6)$$

$$g_{\alpha\beta} x^{\alpha}{}_{,\tau} x^{\beta}{}_{,\tau} = 0, \quad g_{\alpha\beta} x^{\alpha}{}_{,\tau} x^{\beta}{}_{,\sigma} = 0, \qquad (7)$$

where $g_{\alpha\beta}$ is the metric tensor, and Γ^{α}_{pq} are Christoffel symbols for the external space-time, $x^{p}{}_{,\tau} = \partial x^p / \partial \tau$, $x^{\beta}{}_{,\sigma} = \partial x^{\beta} / \partial \sigma$, the indices $\alpha,\ \beta,\ p,\ q$ take values 0, 1, 2, 3, and the functions $x^{\alpha}(\tau, \sigma)$ determine the motion trajectory (the world surface) of the null string.

Since the trajectories (1) and (2) simulate the motion of gravitationally interacting null strings, on the corresponding segments of the trajectories shown in Fig. 1, and Fig. 2, then the functions $x^{\alpha}(\tau, \sigma)$, $\alpha = 0, \ldots, 3$, must be particular solutions of the equations of motion of a null string. In this case, the analysis of the equations of motion can give additional restrictions on the functions of the quadratic form (5).

For trajectories (1), equations (7) lead to the equality

$$e^{2\nu} \equiv e^{2\mu}, \qquad (8)$$

and for trajectories (2), to the equality

$$e^{2\nu} \equiv A. \qquad (9)$$

For (8), (9), the quadratic form (5) takes the form

$$dS^2 = e^{2\nu}\left((dt)^2 - (d\rho)^2 - (dz)^2\right) - B(d\theta)^2, \qquad (10)$$

where ν, B are functions of the variables $t,\ \rho,\ z$.

Note that on each of the segments: $A_l B_l$, $B_l C_l$, $C_l D_l$, $D_l A_l$, $l = 1, 2$ (see Fig. 1, Fig. 2), the null strings forming the primary particle move towards each other (in opposite directions). Then, the functions defining the trajectories of the interacting null strings, for each of the segments, must simultaneously satisfy the equations of motion (6).

For the case of motion of a closed null string of constant radius in the negative direction of the z axis, equations of motion (6), for (1), (10), lead to the only equation

$$\nu_{,t} - \nu_{,z} = 0 . \quad (11)$$

For the case of motion of a closed null string of constant radius in the positive direction of the z axis, equations of motion (6), for (1), (10), lead to the equation

$$\nu_{,t} + \nu_{,z} = 0 . \quad (12)$$

The joint solution of equations (11), (12) is

$$\nu = \nu(\rho) . \quad (13)$$

Equations of motion (6), taking into account (2), for the case of radial compression and radial expansion of a closed null string located in the plane lead to the equations

$$\nu_{,t} - \nu_{,\rho} = 0 , \quad (14)$$

$$\nu_{,t} + \nu_{,\rho} = 0 . \quad (15)$$

The joint solution of equations (14), (15) is

$$\nu = \nu(z) . \quad (16)$$

The consequence of the equalities (13), (16) is $\nu = \nu_0 = const$. Without loss of generality, we can fix the value of the constant $\nu_0 = 0$, then we finally get

$$e^{2\nu} = 1 . \quad (17)$$

4. Einstein's equations solution

If functions $x^l(\tau,\sigma)$ define the trajectory of motion (world surface) of a null string, which moves in a pseudo-Riemannian space-time with the metric tensor g_{mn} . Then the components of the energy-momentum tensor of such a null string are determined by the equalities [23]

$$T^{mn}\sqrt{-g} = \gamma \int d\tau d\sigma x^m{}_{,\tau} x^n{}_{,\tau} \delta^4\left(x^l - x^l(\tau,\sigma)\right), \quad (18)$$

where indexes m, n, l take values 0,1,2,3, $g = |g_{mn}|$, $\gamma = const$.

According to (18) outside the null string, i.e. in the region where $x^l \neq x^l(\tau,\sigma)$, $l = 0,...,3$, all components of the null string energy-momentum tensor are identically zero. In this article, we investigate the asymptotics of the gravitational field of primary particles (i.e., we are looking for a solution in empty space surrounding the primary particle). Then, without loss of generality, we can assume that all components of the energy-momentum tensor of gravitationally-interacting null strings in the investigated region are equal to zero.

In the investigated region of space-time, the Einstein system of equations, for the quadratic form (10), (17), can be represented as

$$\left(\frac{B_{,t}}{B}\right)_{,t} + \frac{1}{2}\left(\frac{B_{,t}}{B}\right)^2 = 0 , \quad (19)$$

$$\left(\frac{B_{,z}}{B}\right)_{,z} + \frac{1}{2}\left(\frac{B_{,z}}{B}\right)^2 = 0 , \quad (20)$$

$$\left(\frac{B_{,\rho}}{B}\right)_{,\rho} + \frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2 = 0 , \quad (21)$$

$$\left(\frac{B_{,z}}{B}\right)_{,t} + \frac{1}{2}\frac{B_{,t}}{B}\frac{B_{,z}}{B} = 0 , \quad (22)$$

$$\left(\frac{B_{,\rho}}{B}\right)_{,t}+\frac{1}{2}\frac{B_{,t}}{B}\frac{B_{,\rho}}{B}=0, \tag{23}$$

$$\left(\frac{B_{,\rho}}{B}\right)_{,z}+\frac{1}{2}\frac{B_{,\rho}}{B}\frac{B_{,z}}{B}=0. \tag{24}$$

The solution to equations (19) - (24) can be represented as

$$B=\left(\beta_1 t+\beta_2 z+\beta_3 \rho\right)^2, \tag{25}$$

where β_1, β_2, and β_3 are constants.

Since the function B must be invariant under the simultaneous inversion of $t \to -t$, $z \to -z$, the consequence of (4) is

$$\beta_1=\beta_2=0. \tag{26}$$

Also, without loss of generality, you can fix the value of the constant

$$\beta_3=1. \tag{27}$$

For constant values (26), (27), we find

$$B=\rho^2. \tag{28}$$

Applying (17), (28), for (10), we obtain the expression for the required quadratic form

$$dS^2=dt^2-d\rho^2-\rho^2 d\theta^2-dz^2. \tag{29}$$

4. Discussion

It should be noted once again that the found solution (29) should be considered only as the asymptotics of the gravitational field of particles structurally consisting of gravitationally interacting null strings. It is incorrect to say that null strings forming particles, the trajectories of which are shown in Fig. 1, and Fig. 2, move in the Minkowski space-time, because when constructing the solution (29) we applied a number of approximations:

- The trajectory of motion of each of the interacting null strings (see Fig. 1, and Fig. 2) consists of four segments. On two of these segments, the null string moves with a constant radius, and on two more segments, the null string radially decreases or radially increases its size (radius). This approximation allowed us to obtain the equalities (8), (9), and (17). However, this approximation can be considered correct only at distances much larger than the size of the region inside which oscillations of interacting null strings occur;
- Symmetry of metric functions with respect to the simultaneous inversion $t \to -t$, $z \to -z$ (equality (4)), the application of which made it possible to reduce the quadratic form to the form (5), can be considered correct only on time scales much larger than the time of one full cycle of oscillation of null strings forming a particle, or at distances much larger than the dimensions of the region inside which oscillations of interacting null strings occur.

It is important to note that in the case when the observation time scale is comparable to the time of one complete cycle of oscillation of the null strings that form the primary particle. Or in the case when the size of the study region is comparable to the size of the region inside which the oscillations of the null strings that form the primary particle occur. Then, in these cases, in a gas of null strings, significant deviations of the gravitational field from the flat Minkowski space-time should be observed.

5. Conclusions

It is interesting that since by the term "null string'" we mean a thin tube of a massless scalar field, then, in fact, physical processes in a gas of null strings are processes in a structured massless scalar field. The gravitational (curvature) interaction between the structural elements of this field (thin tubes) leads to the formation of primary particles that have a nonzero rest mass. By interacting, such particles can combine into various multiparticle structures (“macro” objects). The “lifetime” of both primary particles and “macro” objects formed in a structured scalar field can be different, but the result of their existence is the appearance of mass in the scalar field. Since the interaction between primary particles depends on the “direction” of motion of the null strings forming the particle (it depends on the direction of the vector that characterizes the “direction” of motion of the null strings forming the primary particle). The consequence of gravitational (curvature) interaction in a structured scalar field is the appearance of a vector field, the formation of which is associated with the peculiarity of interaction between various global structures of the scalar field. In this case, time-stable “macro” objects are sources of structures that form a “field”, and also are sources of an external vector field.

It seems interesting to consider a structured massless scalar field (a gas of thin closed tubes) as the initial matter of the Universe. With such a choice, it may become possible to reduce all known types of interaction to various realizations of curvature (gravitational) interaction between elements (formed structures) of such a gas. In fact, it becomes possible to combine the concepts of “substance” and “field”. The advantages of such a model include its four-dimensionality, as well as the obvious quantum properties of “macro” structures and “macro” processes that can be observed in such a gas. In essence, a future theory describing processes in a gas of thin tubes of a massless scalar field can be considered as a possible example of a hybrid theory. Since this theory should be partly quantum (for “macro” structures and “macro” processes) and partly classical (motion of the “null strings” forming a gas).

To my friend and colleague Marina Glumova. She fought bravely, but lost her personal battle with COVID-19.

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References

1. Gariazzo, S.; Escudero, M.; Diamanti, R.; Mena, O. Cosmological searches for a noncold dark matter component. Phys. Rev. D 2017, 96, 043501.
2. Hsueh, J.W.; Enzi, W.; Vegetti, S.; Auger, M.W.; Fassnacht, C.D.; Despali, G.; Koopmans, L.V.; McKean, J.P. SHARP-VII. New constraints on the dark matter free-streaming properties and substructure abundance from gravitationally lensed quasars. Mon. Not. R. Astron. Soc. 2020, 492, 3047-3059.
3. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland; R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. 1998, 116, 1009-1038.
4. Martinelli, M.; Tutusaus, I. CMB tensions with low-redshift H0 and S8 measurements: Impact of a redshift-dependent type-Ia supernovae intrinsic luminosity. Symmetry 2019, 11, 986.
5. Collett, T.E.; Oldham, L.J.; Smith, R.J.; Auger, M.W.; Westfall, K.B.; Bacon, D.; Nichol, R.C.; Masters, K.L.; Koyama, K.; van den Bosch, R. A. Precise Extragalactic Test of General Relativity. Science 2018, 360, 1342-1346.
6. Di Valentino, E.; Melchiorri, A.; Mena, O.; Vagnozzi, S. Nonminimal dark sector physics and cosmological tensions. Phys. Rev. D 2020, 101, 063502.
7. Ren, X.; Zhao, L.; Abdukerim, A.; Chen, X.; Chen, Y.; Cui, X.; Fang, D.; Fu, C.; Giboni, K.; Giuliani, F.; et al. Constraining Dark Matter Models with a Light Mediator at the PandaX-II Experiment. Phys. Rev. Lett. 2018, 121, 021304.

8. Lelyakov, A.P.; Osokin, C.S. Solution of the Einstein Equations for Axially Symmetric Closed Null String, Conserving Its Shape. Acta Phys. Pol. B 2017, 48, 2045-2069.
9. Lelyakov, A.P.; Haneychuk, O.V. Solution of Einstein's Equations for a Closed Null String with Axial Symmetry that Is Radially Increasing Its Size. Mosc. Univ. Phys. Bull. 2020, 75, 18-25.
10. Lelyakov, A. Gravitational Interaction in a Null String Gas and Its Possible Consequences. Universe 2020, 6, 142.
11. Lelyakov, A.P. Mass Generation by a Massless Scalar Field. Gravit. Cosmol. 2020}, 26, 358-372.
12. Lelyakov, A.P. Dynamics of a null string in the gravitational field of a closed null string of constant radius. Gravit. Cosmol. 2015, 21, 309-318.
13. Lelyakov, A.P. Peculiarities of null string motion in the gravitational field of a radially expanding or radially collapsing multistring system. Gravit. Cosmol. 2017, 23, 50-62.
14. Lelyakov, A.P. Dynamics of a test null string in the gravitational field of a null string domain that radially changes its size. Int. J. Mod. Phys. A 2020, 35, 2040011.
15. Lelyakov, A.P.; Osokin, C.S. On the Possibility of the Formation of Particles with a Nonzero Rest Mass in the "Gas" of Null-strings. Acta Phys. Pol. B 2019, 50, 767-793.
16. Lelyakov, A.P. Influence of the Gravitational Field of a Null String Domain on the Dynamics of a Test Null String. Gravit. Cosmol. 2019, 25, 319-330.
17. Lelyakov, A.P.; Karpenko, A.S. Dynamics of a probe null string in the gravitational field of a closed null string radially collapsing in the plane. Theoret. Math. Phys. 2017, 190, 140-153.
18. Lelyakov, O.P.; Karpenko, A.S.; Babadzhan, R.D.O. Dynamics of a Non-Rotating Test Null String in the Gravitational Field of a Closed "Thick" Null String Radially Expanding or Collapsing in the Plane z = 0. Ukr. J. Phys. 2014, 59, 1114-1125.
19. Lelyakov, O.P.; Karpenko, A.S.; Babadzhan, R.D.O. Scalar-Field Potential Distribution for a Closed "Thick" Null String Moving in the Plane z=0. Ukr. J. Phys. 2014, 59, 547-554.
20. Lelyakov, A.P. Solution to the Einstein equations for a "spread" closed null string of constant radius. Gravit. Cosmol. 2015, 21, 200-207.
21. Lelyakov, O.P. Scalar Field Potential Distribution for a "Thick" Null String of Constant Radius. Ukr. J. Phys. 2011, 56, 296-302.
22. Petrov, A.Z. New Methods in General Relativity Theory; Nauka: Moscow, Russia, 1966.
23. Roshchupkin, S.N.; Zheltukhin, A.A. Friedmann universes and exact solutions in string cosmology. Class. Quantum. Grav. 1995, 12, 2519-2524.