Can extra dimension pull space-time?

Vijay Singh

Department of Mathematical Sciences,
University of Zululand,
Kwa-Dlangezwa-3886, S. Africa

Plan of talk

1. Motivation
2. Model and field equations
3. Vacuum energy model
4. Summary

Motivation

- Observable Universe is $4D$, flat, isotropic and homogeneous.
- Unifying gravity with other fundamental forces is an active field of research (*Ishihara, Prog. Theor. Phys.* **72** *376 1984*).
- Among common origin of fundamental interactions higher dimensional Kaluza-Klein (K-K) theories is one of the possibility (*Appelquist and Chodos, PRD* **28** *272 1987*).
- Dimensions of space-time may be greater than observed $(3+1)$.
- Extra dimensions are not observed at present presumably because their typical dimensions are of Planck length.
- Higher dimensional theories are not restricted to particle physics, but extend to classical and modern cosmology (*Overduin and Wesson, Phys. Rep.* **283** *303 1997*).
- Early studies carried out in a variable-gravity theory in which there is only one extra dimension related to the rest mass (*Wesson, Astron. Astrophys.* **119** *145 1983, GRG* **16** *193 1984*).

- When an extra dimension is added to the $4D$ space-time, the theory becomes much richer in its physical consequences, **BUT** at the same time mathematical complications increase.
- It becomes difficult to obtain exact solutions of Einstein's equations, in general.
- Many attempted have been made to find solutions of homogenous and non-homogenous models in different context.
- $4D$ space-time of Einstein's theory can be regarded as embedded in $5D$ space-time (*Chodos and Detweiler, PRD* **21** *2167 1980*).
- Consider $5D$ analogs of the spatially-flat $4D$ FRW model of standard cosmology.
- Find some general and particular solutions of Einstein's equation in a very straight forward manner assuming the perfect fluid equation of state (EoS).

Model and the field equations

- $5D$ K-K line element

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2 + dz^2) + B^2 d\chi^2.$$

A & $B \rightarrow$ Functions of t.

- Energy-momentum tensor for the perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \qquad \mu, \nu = 0, 1, 2, 3, 4.$$

$\rho \rightarrow$ Energy density.
$p \rightarrow$ Pressure.

Field equations of $5D$ analogs of $4D$ spacetime

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu}, \qquad \mu, \nu = 0, 1, 2, 3, 4.$$

$8\pi G = 1 = c$.

- Manna and Bhui, (*Astrophys. Space Sci.*, **213** *299 1994*) obtained solutions of these equations for the metric we have considered assuming a time dependent EoS and **NO** excitation in the extra space, so that $p_4 = 0$.
- We consider that the pressure in the direction of extra space as usual $3D$ space, i.e., $p_4 = p$.

Evolution equations

$$
\begin{aligned}
3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} &= \rho, \\
2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} &= -p, \\
3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right)^2 &= -p.
\end{aligned}
$$

A 'dot' stands for $\frac{d}{dt}$.

- Three independent equations with four unknowns A, B, ρ and p.

Solutions

Assumption $\rightarrow$ universe is filled with vacuum energy

$$p = -\rho.$$

We find

$$B = \frac{\dot{A}}{\alpha}$$

$\alpha \Rightarrow$ Integration constant.

- It represents the most general solution of $5D$ K-K homogenous model filled with a perfect fluid.

$$A_1(t) = \frac{e^{-\sqrt{c_2}t}}{2}\left(2\gamma + \frac{e^{4\sqrt{c_2}t}}{c_2}\right)^{\frac{1}{2}},$$

$$A_2(t) = \frac{\left(2\gamma e^{2\sqrt{c_2}t} + e^{-2\sqrt{c_2}t}\right)^{\frac{1}{2}}}{2\sqrt{c_2}},$$

$$B_1(t) = \frac{e^{-\sqrt{c_2}t}\left(e^{4\sqrt{c_2}t} - 2c_2\gamma\right)}{2\alpha\left(2c_2\gamma + e^{4\sqrt{c_2}t}\right)^{\frac{1}{2}}},$$

$$B_2(t) = \frac{e^{-2\sqrt{c_2}t}\left(2c_2\gamma e^{4\sqrt{c_2}t} - 1\right)}{2\alpha\left(2c_2\gamma e^{2\sqrt{c_2}t} + e^{-2\sqrt{c_2}t}\right)^{\frac{1}{2}}},$$

where $\gamma = \alpha\beta$, $\alpha \neq 0$ and $c_2 > 0$ is constant of integration.

- $\rho = 6c_2 = -p \Rightarrow c_2$ acts as a cosmological constant.

Evolution of 3-space

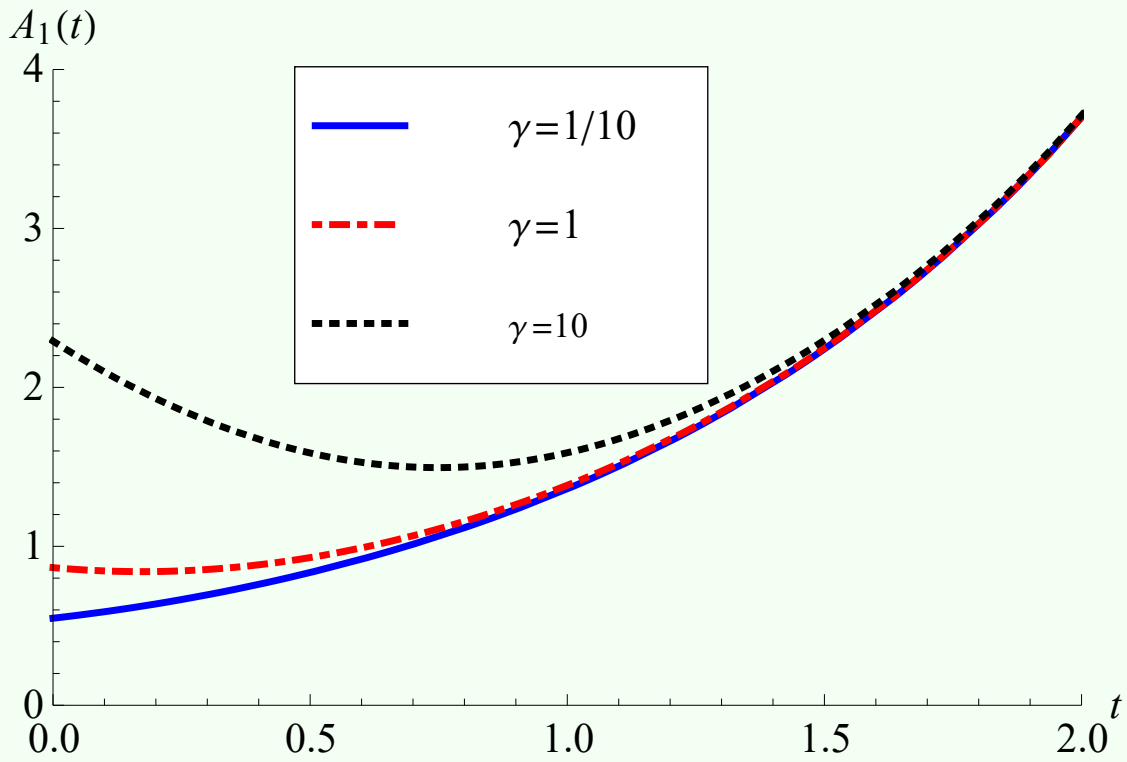

Figure: B *versus* t with $c_2 = 1$.

Behavior of extra dimension

When $\alpha > 0$

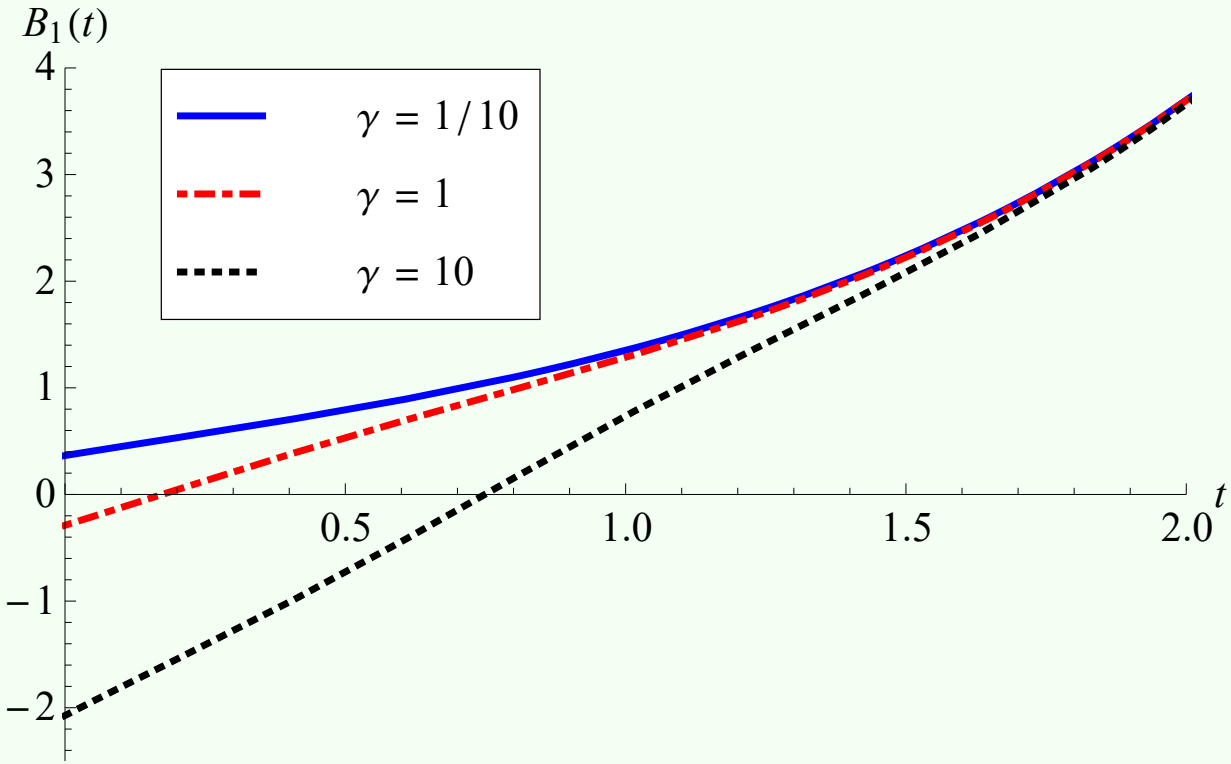

Figure: B *versus* t with $c_2 = 1 = \alpha$ and $\beta = -1$.

Behavior of extra dimension

When $\alpha < 0$

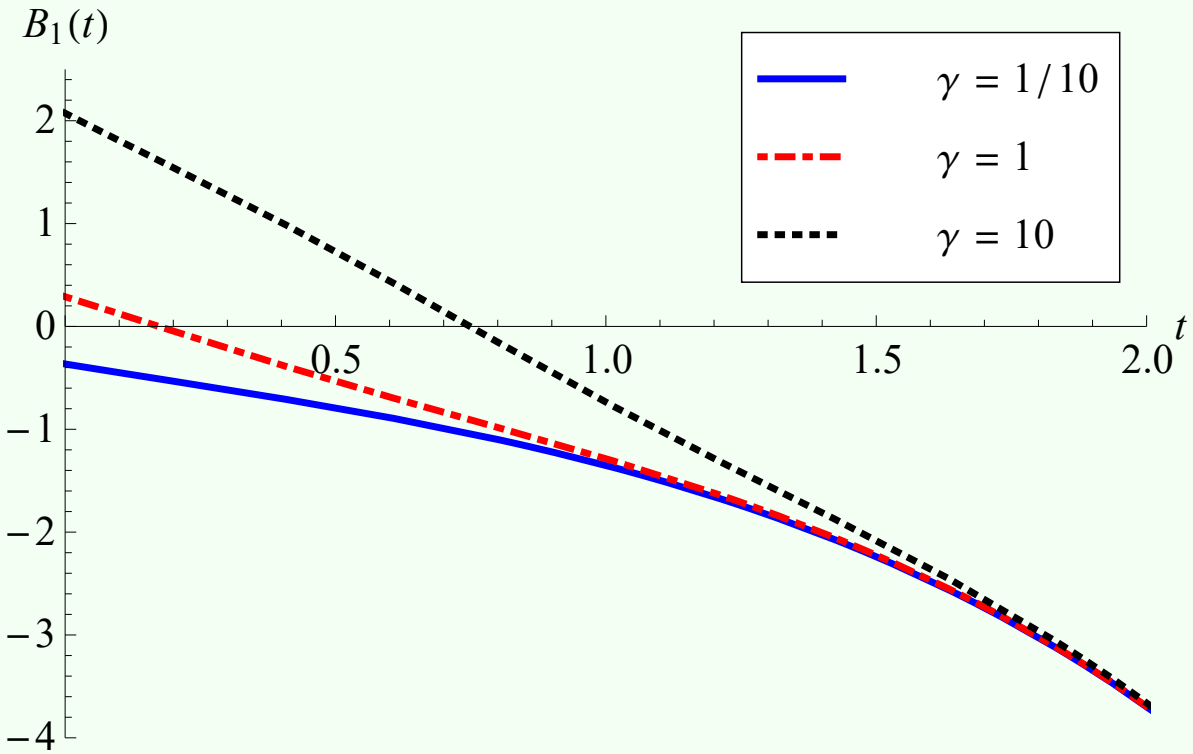

Figure: B *versus* t with $c_2 = 1 = \beta$ and $\alpha = -1$.

Deceleration parameter

$$q_1 = -\frac{16c_2^4\gamma^4 - 56c_2^2\gamma^2 e^{8\sqrt{c_2}t} + e^{16\sqrt{c_2}t}}{\left(4c_2^2\gamma^2 + e^{8\sqrt{c_2}t}\right)^2}$$

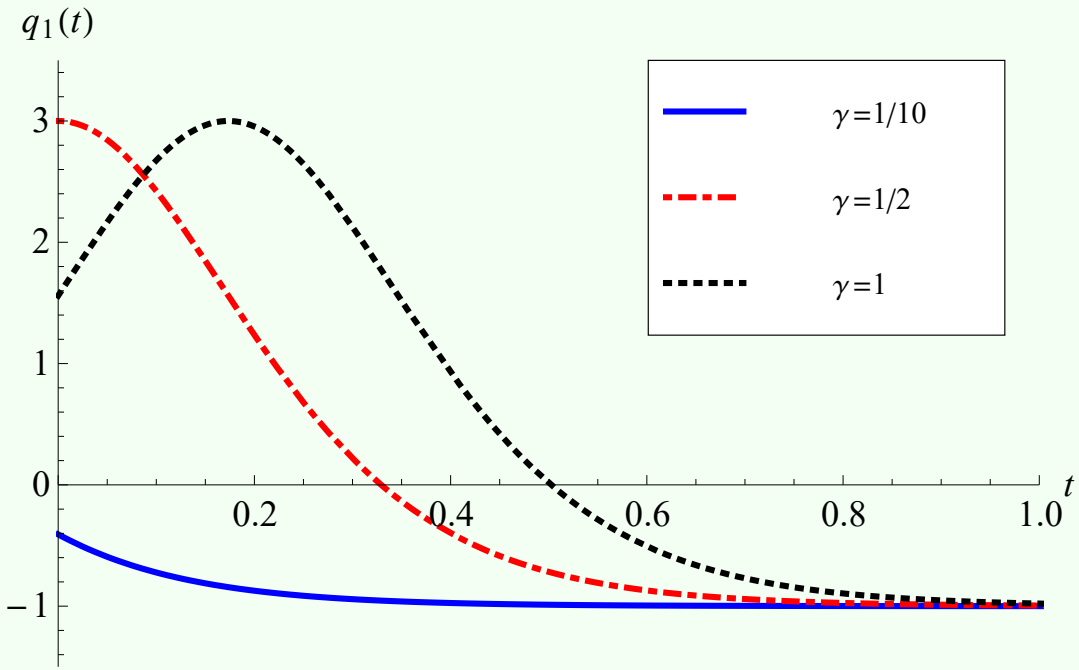

Figure: q_1 *versus* t for different values of γ with $c_2 = 1$.

Cause of transition from dec. to acc.

- Possibility I: Matter manifestation due to extra space?

$$
\begin{aligned}
3\left(\frac{\dot{A}}{A}\right)^2 &= \rho+\rho_1, \\
2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2 &= \rho-p_1, \\
\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2 &= \rho,
\end{aligned}
$$

where $\rho_1 = -3\frac{\dot{A}\dot{B}}{AB}$ and $p_1 = \frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}$, can be regard as the energy density and pressure of the matter manifested from the geometry of extra space.

$$\rho_1 = -\frac{3c_2\left(4c_2^2\gamma^2+12c_2\gamma e^{4\sqrt{c_2}t}+e^{8\sqrt{c_2}t}\right)}{\left(2c_2\gamma+e^{4\sqrt{c_2}t}\right)^2},$$

$$p_1 = \frac{c_2\left(12c_2^2\gamma^2+4c_2\gamma e^{4\sqrt{c_2}t}+3e^{8\sqrt{c_2}t}\right)}{\left(2c_2\gamma+e^{4\sqrt{c_2}t}\right)^2}.$$

- c_2 must be negative for $\rho_1 > 0$, which contradicts as $c_2 > 0$.
- The hypothesis of manifestation of matter from the geometry of the extra dimension is **REJECTED**.
- However, $p_1 > 0$, which shows that the extra dimension generates some attraction force similar to gravity.
- This attraction force causes past deceleration.

Extra dimension contracts or expands?

- If $\beta = 0$, then $A_1 \propto e^{\sqrt{c}_2 t}$, $B_1 \propto \frac{e^{\sqrt{c}_2 t}}{\alpha}$ and $q_1 = -1$.
- Ever accelerating model that evolves isotopically.
- Early decelerated phase is possible only for non zero values of β.
- A non-vanishing β causes anisotropy.
- The dynamics of the deceleration parameter remains unaffected for positive or negative values of α.
- Therefore, it is difficult to say that the early deceleration is caused whether by the contraction or expansion of extra space.
- Shrinking fifth dimension is physically more relevant.
- Whatever the possibility could be but **extra dimension causes the transition from decelerating to accelerating universe**.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Summary

- General solutions of $5D$ K-K model filled with vacuum energy.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Thank You!

- General solutions of $5D$ K-K model with perfect fluid EoS.
- The model describes transition from decelerated to accelerated phase.
- Extra dimension causes early decelerated phase.
- An alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Further interaction: gtrcosmo@gmail.com

Thank You!

- eneral solutions of $5D$ K-K model with perfect fluid Eo
- he model describes transition from decelerated to accel
  hase.
- xtra dimension causes early decelerated phase.
- n alternative resolution of a sudden change from deceleration to acceleration.
- Anisotropic at early times but becomes isotropic at late times.
- Straight forward method to solve the field equations can make it useful in future similar studies.

Further interaction: gtrcosmo@gmail.com