entropy 2021

sciforum

Abstract

Basin Entropy: A New Method to Measure Unpredictability in Physical Systems †

Miguel A. F. Sanjuán

Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Madrid, Spain

† Presented at the Entropy 2021: The Scientific Tool of the 21st Century, 5–7 May 2021; Available online: https://sciforum.net/conference/Entropy2021/.

Published: 5 May 2021

In nonlinear dynamics, basins of attraction are defined as the set of points that, taken as initial conditions, lead the system to a specific attractor. This notion appears in a broad range of applications where multistability is present, which is a common situation in neuroscience, economy, astronomy, ecology, and other disciplines. Nonlinear systems often give rise to fractal boundaries in phase space, hindering predictability. When a single boundary separates three or more different basins of attraction, we call them Wada basins. Usually, Wada basins have been considered even more unpredictable than fractal basins. However, this particular unpredictability has not been fully unveiled until the introduction of the concept of basin entropy. The basin entropy provides a quantitative measure of how unpredictable a basin is. With the help of several paradigmatic dynamical systems, we illustrate how to identify the ingredients that hinder the prediction of the final state. The basin entropy together with two new tests of the Wada property have been applied to some physical systems such as experiments of chaotic scattering of cold atoms, models of shadows of binary black holes, and classical and relativistic chaotic scattering associated to the Hénon-Heiles Hamiltonian system in astrophysics.

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