

Proceedings

# Walking on polyhexes : surprising symmetry for certain Tori & Klein Bottles

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**Abstract:** Distance-based topological invariants, namely the topological roundness index and the Wiener number, show a new and somehow unexpected symmetry between toroidal and Klein bottle polyhexes. In this case the bottles are closed in the anti-parallel way along the zigzag edge of the hexagonal lattice. We report here that our computations point out that both cubic graphs are topologically indistinguishable for certain combination of  $x,y$  sizes. This means that an Escher's ant walking on the Klein bottle is no longer able to distinguish it from a same-size Torus by measuring the chemical distances of a node from all the others. Among other effects, this new topological similarity does transfer the translation invariance, that is a typical feature of the graphenic Tori, to the Klein bottle lattices. This size-induced phase transition connecting Klein bottles and toroidal cubic graphs represents a relevant topological behavior with uncharted mathematical and physical consequences. The non-trivial influence of the chirality of the bottle will be also numerically investigated showing a radically different behavior of the armchair Klein Bottles.

**Keywords:** Topological roundness; polyhexes; Klein Bottles; Tori

**Citation:** Ottorino O. Walking on polyhexes : surprising symmetry for certain Tori & Klein Bottles. *Symmetry* **2021**

Published: 7 August 2021

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