Multi-commodity Contraflow Problem on Lossy Network with Asymmetric Transit times

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IOCA-2021

Virtual, 27 Sept. to 10 Oct.

Outline

- Literature Review
- Flow Models
- Problem Status
- Solution Approaches
- Results

Literature Review

Lester Randolph Ford

Legends of dynamic network flow results

1958 and 1962

Delbert Ray Fulkerson

Single Commodity

- D.Gale (1959)
- K.Onaga (1966, 1967)
- W.L.Wilkinson (1971) E.Minieka (1974)
- B.Hoppe, E.Tardos (1994)
- L.Fleischer, E.Tardos (1998)
- S.Kim, S.Shekhar (2005)
- S.Rebennack, A.Arulselvan, L.Elefteriadou, P.M.Pardalos (2010)
- M.Gross, M.Skutella (2012)

Multi-commodity

- Jwell (1961)
- J.A.Tomlin (1966)
- L.Fleischer (2000)
- A.Hall, S.Hippler, M.Skutella (2007)
- L.Fleischer, M.Skutella (2007)
- M.Gross, M.Skutella (2012)
- T.N.Dhamala, S.P.Gupta, D.P.Khanal, U.Pyakurel (2020)
- U.Pyakurel, S.P.Gupta, D.P.Khanal, T.N.Dhamala (2020)

Flows

Static Flow

$f : A \to R_{\geq 0}$

Dynamic Flow

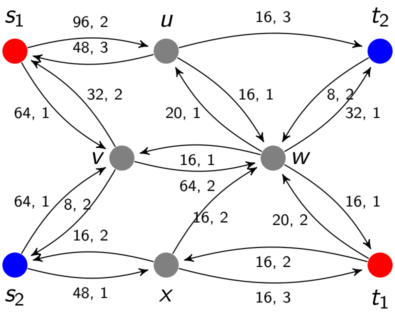
$\Phi : A \times \{0, 1, \ldots, T-1\} \to R_{\geq 0}$, flow enters e at time θ

Single-Commodity Flow

Multi-Commodity Flow

$\Phi^i : A \times \{0, 1, \ldots, T-1\} \to R_{\geq 0}, \ \forall \ i \in K$

Generalized Multi-commodity Network



capacity, transit time

$\mathcal{N} = (V, A, K, u, \tau, \rho, d_i, S_+, S_-, T)$

V = set of nodes

A = set of arcs

K = set of commodities

u = capacity of arc

τ = transit time of arc

ρ = gain or loss factor on arc

d_i = demand of commodity i

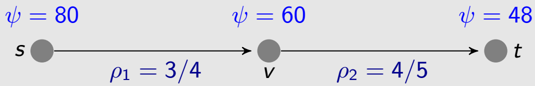
S_+ = set of source node s_i

S_- = set of sink node t_i

T = time horizon

Generalized Flow

Generalized flow problem is the generalization of classical flow problem by assigning flow multiplier on each arc.



$(\prod_{e \in p} \rho_e \psi = \rho_1 \rho_2 \psi)$

Classification

- $0 < \rho \leq 1, \forall e \in A$ (lossy network)
- ρ arbitrary (generalized network)
- $\rho = 1, \forall e \in A$ (classical network)

Transit time and loss factor along a path

- $\tau_p = \sum_{e \in p} \tau_e$
- $\rho_p = \prod_{e \in p} \rho_e$

Contraflow Configuration

Rebennack, Arulselvan, Elefteriadou, and Pardalos [2010], [6]

$$\text{Capacity } u_a = u_e + u_{e^r}$$

$$\text{Transit time } \tau_a = \begin{cases} \tau_e & \text{if } e \in A \\ \tau_{e^r} & \text{otherwise} \end{cases}$$

Assumption

Symmetric transit times on arcs ($\tau_e = \tau_{e^r}$)

Application

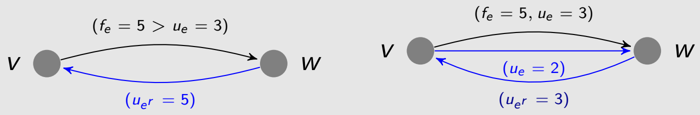
- It makes the traffic systematic and smooth
- It is a strategy used to evacuate the people in an emergency

Partial Contraflow

Pyakurel, Gupta, Khanal, and Dhamala [2020], [5]

Reversal Technique

- Arc $e^r = (w, v)$ is reversed iff either $f_e > u_e$, where $e = (v, w)$ or there is $f_e \geq 0$ along the arc $e = (v, w) \notin A$. If $u_a > f_e$ then the arc e^r is reversed partially and capacity of remaining arcs e^r are saved
- If $f_e > u_e$ and $u_a = f_e$, then the arc e^r is reversed completely
- If $f_e < u_e$ neither e nor e^r is reversed



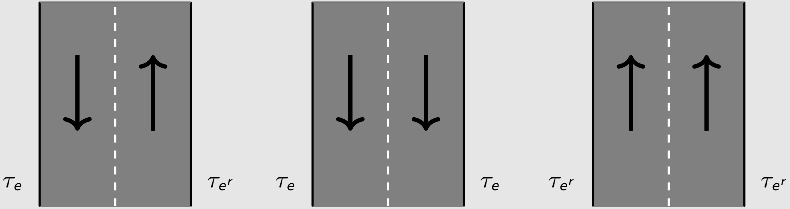
Contraflow Configuration

Assumption

Asymmetric transit times on arcs

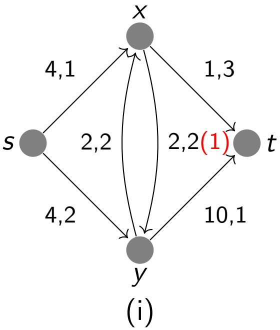
Gupta, Pyakurel, and Dhamala [2021], [2]

- $u_a = u_e + u_{e^r}$
- The transit time of auxiliary arc τ_a is taken as transit time of non-reversed arcs as shown in Figures (b) and (c)
- In the case of a single direction $\tau_a = \tau_e = \tau_{e^r}$



(a) (b) (c)

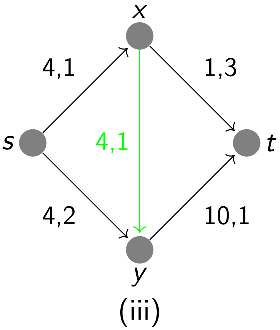
Contraflow with symmetric Transit Times



Paths	Time	F. before LR	T. F.	F. after LR	T. F.
$s-y-t$	3	4	12	4	12
$s-x-y-t$	4	2(3)	4(6)	4	8
$s-x-t$	4	1	2	-	-
Total			18		20

T = 6, T.F.=Total Flow, LR = Lane Reversals

Contraflow with Asymmetric Transit Times



Paths	Time	Flow (x, y)	T. F.	Flow (y, x)	T. F.
$s - y - t$	3	4	12	4	12
$s - x - y - t$	3	4	12	-	-
$s - x - t$	4	-	-	1	2
Total			24		14

T = 6, T.F.=Total Flow

Flow Model with Lane Reversals

$$\max \sum_{e \in B_d} \sum_{\delta=0}^{T-\tau_e-1} \rho_e \Phi_e^i(\delta) \tag{1}$$

$$\sum_{e \in B_v} \sum_{\delta=0}^{T-\tau_e-1} \rho_e \Phi_e^i(\delta) - \sum_{e \in A_v} \sum_{\delta=0}^{T-1} \Phi_e^i(\delta) \quad = \quad 0, \quad v \notin \{s, t\} \tag{2}$$

$$\sum_{e \in B_v} \sum_{\delta=0}^{\theta-\tau_e} \rho_e \Phi_e^i(\delta) - \sum_{e \in A_v} \sum_{\delta=0}^{\theta} \Phi_e^i(\delta) \quad \geq \quad 0, \ \forall \ \theta \in \mathbb{T}, \ v \neq s \tag{3}$$

$$0 \leq \Phi_e^i(\theta) \quad \leq \quad u_e \ + \ u_{\overleftarrow{e}}, \ \forall \ e \in A, \ \theta \in \mathbb{T} \tag{4}$$

Problem Status

Generalized flow over time problem is $\mathcal{NP}$-hard.
[Gross and Skutella (2012)]
*PARTITION

Problems

Problem

Consider a lossy network $\mathcal{N} = (V, A, u, \tau, \rho, K, S_+, S_-, T)$ *with orientation-dependent transit times on arcs having proportional loss factor* $\rho_a = 2^{\lambda \tau_a}, \lambda < 0$ *on each arc. The maximum GDMCCF problem on a lossy network is to obtain the maximum amount of flow that can be sent from* $s_i - t_i$, *paths* $\forall i \in K$ *with a minimum loss, if the direction of the arc can be reversed at time zero without reversal costs.*

The GDMCCF with Non-Symmetric Arc Parameters

Input: A dynamic multi-commodity lossy network $\mathcal{N} = (V, A, u, \tau, \rho, K, S_+, S_-, T)$ with constant and non-symmetric transit times, i.e., $\tau_e \neq \tau_{e^r}$, for some $e \in A$
Output: The maximum GDMCCF

1. Construct the corresponding auxiliary network by adding two-way capacities on $\mathcal{N}_a = (V, A_a, u_a, \tau_a, \rho_a, K, S_+, S_-, T)$ as
$$u_a = u_e + u_{e^r},$$
$$\tau_a := \begin{cases} \tau_e & \text{if arc } e^r \text{ is reversed in direction of } e \\ \tau_{e^r} & \text{if arc } e \text{ is reversed in direction of } e^r. \end{cases}$$
2. Compute commodity dependent paths $s_i - t_i, \forall i \in K$.
3. Compute generalized multi-commodity flow with capacity u_a, transit time and symmetric proportional loss factor $\rho_a = 2^{\lambda \tau_a}, \lambda < 0$, on auxiliary network using algorithm of Gross and Skutella [4] from $s_i - t_i, \forall i \in K$.
4. For each time $\theta \in \mathbb{T}$, reverse $e^r \in A$ up to the capacity $\Psi_e - u_e$ iff $\Psi_e > u_e$, u_e replaced by 0 whenever $e \notin A$, $\forall i$, where $\Psi_e = \sum_{i=1}^{k} \Psi_e^i$.
5. For each $e \in A$, if e^r is reversed, $s_c(e^r) = u_a - \Psi_e$ and $s_c(e) = 0$. If neither e nor e^r is reversed, $s_c(e) = u_e - \Psi_e > 0$, where $s_c(e)$ is the saved capacity of e.
6. Flow $\Psi^i = \sum_{p \in P} \Psi_p^i$. Therefore total flow $\Psi = \sum_{i=1}^{k} \Psi^i$.

Theorems

Lemma

The solution of the GDMCCF problem obtained by algorithm is feasible.

Theorem

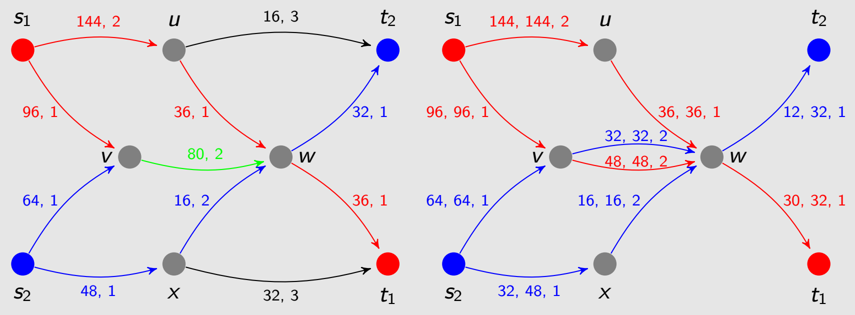
Algorithm computes the solution of GDMCCF problem optimally.

Corollary

A GDMCCF problem can be computed in a pseudo polynomial-time complexity.

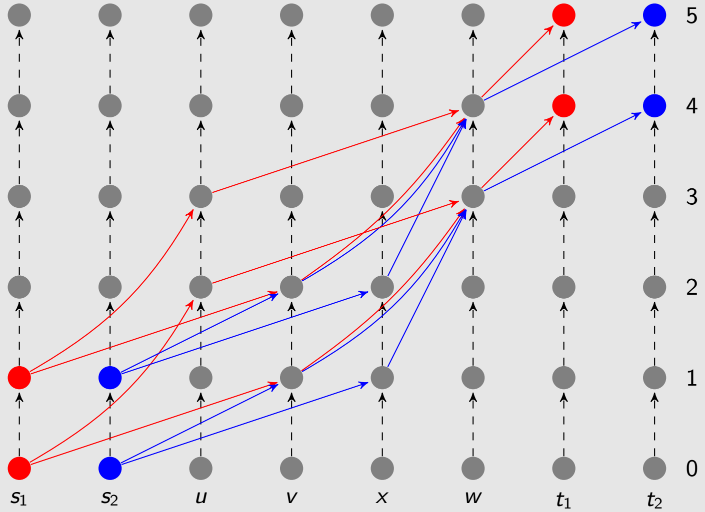
Example

Consider a multi-commodity flow problem in a lossy network as shown in Figure (a). The arcs, for example, (v, w) and (w, v), represent the two-way road segments between nodes v and w. Each arc contains asymmetric capacity, orientation-dependent transit time, and proportional loss factor. The capacity of each edge of Figure (b) is obtained by adding two-way capacities, and the transit time is taken as non-reversed arc. The objective is to maximize the flow of sum of commodities from s_i to t_i for all $i \in K$.



(a) capacity, transit time (b) flow, capacity, transit time

Example Contd.



(c) time-expanded network

Flow before Lane Reversals

Table: Maximum GDMCF before partial contraflow for T = 6

Path	Flow from s_i	ρ along path	Transit time	Flow at t_i
Commodity-1				
$s_1 - u - w - t_1$	64	$1/4\times1/2\times1/2=1/16$	4	$2\times4=8$
$s_1 - v - w - t_1$	64	$1/2\times1/4\times1/2=1/16$	4	$2\times4=8$
Commodity-2				
$s_2 - v - w - t_2$	64	$1/2\times1/2\times1/4=1/16$	4	$2\times 4=8$
$s_2 - x - w - t_2$	32	$1/2\times1/2\times1/4=1/16$	4	$2\times 2=4$
Total				28

Flow after Lane Reversals

Table: Maximum GDMCF after partial contraflow for T = 6

Path	Flow from s_i	ρ along path	Transit time	Flow at t_i
Commodity-1				
$s_1 - u - w - t_1$	144	1/4×1/2×1/2=1/16	4	2×9=18
$s_1 - v - w - t_1$	96	1/2×1/4×1/2=1/16	4	2×6=12
Commodity-2				
$s_2 - v - w - t_2$	64	1/2×1/2×1/4=1/16	4	2× 4=8
$s_2 - x - w - t_2$	32	1/2×1/2×1/4=1/16	4	2× 2=4
Total				42

Flow Increment

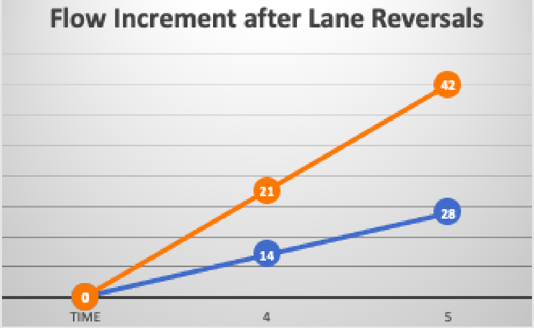


Figure: Comparison of GDMCF on lossy network before and after LR.

Percentage of flow increased with lane reversals$= 14/28 \times 100 = 50$.

Conclusion

- By flipping the orientation of lanes and taking the transit time of the non-flipped arc, the capacity of the lanes will be increased that amplifies the flow value and reduces the time horizon that reflects the situation of contraflow of uneven road topology in the real sense
- We only developed analytical solution
- We want to implement the algorithm in the real life scenario

Acknowledgments

University Grants Commission, Nepal

DAAD "Graph Theory and Optimization with Applications in Industry and Society (GraThO) "

Nepal Mathematical Society

Central Department of Mathematics, Tribhuvan University, Kirtipur

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Thank You !!!

धन्यवाद

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