engineering proceedings

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Proceedings

Quickest Transshipment in an Evacuation Network Topology

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Abstract: The quickest transshipment of the evacuees in an integrated evacuation network topology depends upon the evacuee arrival pattern in the collection network and their better assignment in the assignment network with appropriate traffic route guidance, destination optimization, and optimal route. In this work, the quickest transshipment aspect in an integrated evacuation network topology is revisited concerning a transit-based evacuation system. Appropriate collection approaches for the evacuees and their better assignment to transit vehicles for their quickest transshipment in such an embedded evacuation network are presented with their solution strategies.

Keywords: integrated network; evacuee arrival pattern; transit-vehicle assignment; quickest transshipment

Citation: Adhikari, I. M.; Dhamala, T. N.; Quickest Transshipment in an evacuation network topology . *Proceedings* **2021**, *68*, x. https://doi.org/10.3390/xxxxx

Published: date

1. Introduction

Evacuation planning problem deals with sending the maximum number of evacuees from sources to sinks in minimum time as efficiently as possible. The bus-based evacuation planning problem (BEPP) is an important tool for transit-based evacuation planning. The effectiveness of the solution of BEPP depends upon the evacuee arrival patterns at the pickup locations and their appropriate assignment to transit-vehicles in the available evacuation network [1–3].

The $\boldsymbol{NP}$-hard multi-depot, multi-trip BEPP was introduced and analyzed prominently in [4] which is closer to the split delivery multi-depot vehicle routine problem with inter-depot routes. However, if there is only one bus-depot, assuming that the bus pickups the same number of people that equals its capacity, the author in [5] has also proposed the BEPP for the evacuation of a region. Based on such BEPP, Pyakurel et al. [6] explored it to the transit-dependent. They all have considered that the evacuees have gathered themselves at different pickup locations and were silent about their arrival patterns.

In our work, we are focused on the new and better-suited form of arrival pattern of evacuees in the earliest arrival flow pattern. It will maximize the arrival of evacuees at every possible instance at the pickup locations with zero transit times from a source. We present a polynomial-time earliest arrival evacuee algorithm following the principle of temporally repeated flows to solve the earliest arrival evacuee problem with zero transit times and partial arc reversal capability. Such evacuees collected at different pickup locations of the primary sub-network are considered as the supplies during the subsequent vehicle assignment for the secondary sub-network. The partial arc reversal approach for the collection of evacuees also reduces the waiting instances at different pickup locations and helps to improve the solution. The assignment of transit-vehicles in such a general or a prioritized embedded network is also carried in a dominating solution approach for their quickest transshipment. The rest of the paper is organized as follows. In Section 2, we explain about the flow of evacuees. The network topology in Section 3. The integrated evacuation system related to the general and the prioritized network in Section 4 and 5, respectively. Section 6 concludes the paper.

2. Flow of evacuees

In an evacuation planning problem, the flow stands for either the evacuees or the evacuee carrying vehicles. An s-y flow of evacuees over time from source s to the sink y is a non-negative function f on $A \times R_+$ for given time $\boldsymbol{T} = \{0,1,\dots,T\}$ satisfying the flow conservation and capacity constraints (1-3). The inequality flow conservation constraints allow it to wait for flow at intermediate nodes, however, the equality flow conservation constraints force that flows entering an intermediate node must leave it.

$$\sum_{a\epsilon A_i^{in}} \sum_{\sigma=\tau_a}^{T} f\ (a,\ \sigma-\tau_a) - \sum_{a\epsilon A_i^{out}} \sum_{\sigma=0}^{T} f\ (a,\ \sigma)\ = 0\ ,\ \forall\ i \in V\backslash(S \cup Y), (1) \tag{1}$$

$$\sum_{a\epsilon A_i^{in}} \sum_{\sigma=\tau_a}^{\theta} f\ (a,\ \sigma-\tau_a) - \sum_{a\epsilon A_i^{out}} \sum_{\sigma=0}^{\theta} f\ (a,\ \sigma)\ \geq 0\ ,\ \forall\ i \in V\backslash(S \cup Y),\ \theta \in \boldsymbol{T}, (2) \tag{2}$$

$$0 \leq \mathrm{f}\,(\mathrm{a}, \theta) \leq u_a \quad \forall\ a \in A\ ,\ \ \theta \in \boldsymbol{T}. (3) \tag{3}$$

The sets of outgoing and incoming arcs corresponding to the node $i\ \epsilon\ V$ are denoted by, $A_i^{out} = \{a = (i,j)\ \epsilon\ A\}$ and $A_i^{in} = \{a = (j,i)\ \epsilon\ A\}$, respectively. Not stated otherwise, for all $y\ \epsilon\ Y$ and $s\ \epsilon\ S$, we assume that $A_i^{out} = A_i^{in} = \phi$ in the case without arc reversals. However, for s and y, the flow value be $\upsilon_\mathrm{f}\,(\mathrm{s}) > 0$ and $\upsilon_\mathrm{f}\,(\mathrm{y}) < 0$, respectively, where $\sum_{i\ \epsilon\ V} \upsilon_\mathrm{f}\,(\mathrm{i}) = 0$. If the supply and demand on sources and sinks $\upsilon_\mathrm{f}\,(\mathrm{i})$ is a fixed value for all $i\ \epsilon\ \{s,y\}$, then the earliest evacuee problem maximizes value $\upsilon_\mathrm{f}\,(\theta)$ for all $\theta\ \epsilon\ \boldsymbol{T}$, as in Equation (4) safisfying the constraints (1-3).

$$(\nu_f,\ \theta) = \sum_{a\epsilon A_s^{out}} \sum_{\sigma=0}^{\theta} f\ (a,\ \sigma) = \sum_{a\epsilon A_y^{in}} \sum_{\sigma=0}^{\theta} f\ (a,\ \ \sigma-\tau_a). (4) \tag{4}$$

The total amount out of the source s that reached to the pickup locations Y for all time up to $\theta' \in \boldsymbol{Z}_+$, with zero transit times $\tau_a = 0$, is given by,

$$|\nu_f|\theta' = \sum_{\sigma=1}^{\theta'} |value\ (Y,\theta)|.\ (5) \tag{5}$$

For the given time bound T, the value of Equation (5) becomes,

$$|\nu_f| = \sum_{\sigma=1}^{T} |value\ (Y,\theta)|. (6) \tag{6}$$

We consider a flow of evacuees over time problem with zero transit time function f: $A \times \boldsymbol{Z}_+ \rightarrow \boldsymbol{R}_+$.

3. Network topology

In an integrated evacuation scenario, we consider a network N, obtained by combining two of its components N_1 and N_2 representing a primary and a secondary sub-network, respectively. The first part N_1 contains directed two-way road segments and the partial arc reversals is applicable. The second part N_2 contains directed one-way road segments, connecting the bus depot to the pickup locations, and undirected edges connecting such pickup locations to the sinks for the bus routing. Evacuees collected at the pickup locations Y in $N_1 = (s, V, A, u_a, \tau_a, Y)$ are assigned to transit-buses in the appropriate route across N_2 and are finally sent to the sinks. Here, $V = \{v_1, v_2, v_3, \dots. v_n\}$ and $Y = \{y_1, y_2, y_3, \dots, y_n\}$ are the set of auxiliary nodes and the set of pickup locations, respectively. Set of arcs are denoted by
$A = \{a = (s, v) \cup (v, y) \colon v \in V, y \in Y\ \}$ where u_a and τ_a denotes the capacity and transit times for $a \in A$.

Also, in $N_2 = (d,\ Y,\ E, \tau_e, Z)$, d is the bus depot at which a set of transit-buses B having the homogeneous bus capacity are located initially and are assigned as required during the evacuation process. This node d does not play significant roles further on the solution procedure as the buses do not return to it even after the completion of the evacuation plan because of risks under threat. In an embedding, Y works as the supply nodes during the bus-assignment in N_2. The set of sinks is denoted by $Z = \{z_1, z_2, z_3, \ldots . z_n\}$. In this mixed sub-network, the set E consists of the one-way arcs e=(d, y) with $y \in Y$ and the undirected edges e=[y, z] with $z \in Z$. Here , τ_e is the transit times for $e \in E$ in N_2.

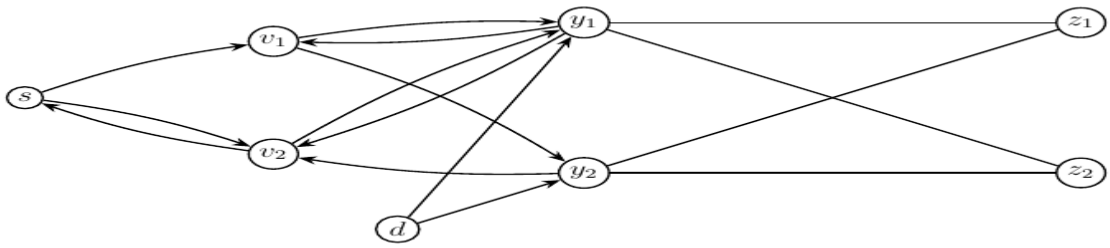

Figure 1. An integrated network topology consisting of primary and secondary sub-network in an embedding.

Based on the BEPP introduced by [4], authors in [5] has developed its simplified version for the evacuation of a region from a set of collection points to a set of capacitated shelters with the help of buses in minimum time assuming that the bus pick ups exactly the number of people that equals its capacity. During their solution on branch and bound framework, they have presented four different upper bounds and three lower bounds for time, three branching rules to minimize the number of branches, and two tree reduction strategies to avoid the equivalent branches. Among them, four upper bounds are constructed in polynomial-time complexity by four different heuristic algorithms, three are based on precomputed tour lists and the fourth one uses on iterative way without any precomputed tour lists and that dominates the rest concerning to evacuation duration and is considered as the dominating assignment approach [7].

Here, we introduce the earliest arrival evacuee problem respecting the partial arc reversal capability in N_1.

Problem 1. Given an evacuation sub-network $N_1 = (S, V, A, u_a, \tau_a, Y)$ with supplies at S, demands at Y, auxiliary nodes V, arc capacity u_a, and arc transit time τ_a for $a \epsilon A$. The quickest partial arc reversal transshipment problem is to find the quickest arrival of evacuees at Y with partial arc reversals capability.

Let the reversals of an arc $a = (i, j)$ be $a' = (j, i)$. Then the transformed network of N_1 consists of the modified arc capacities and constant transit times as,

$$u_{\bar{a}} = u_a + u_{a'} \quad \text{and} \quad \tau_{\bar{a}} = \tau_a \quad \text{if} \quad a \in A \quad \text{and is} \quad \tau_{a'} \text{ for otherwise.} \tag{7}$$

Here, an edge $\bar{a} \in \bar{A}$ in transformed network $\overline{N_1}$ if $a \vee a' \in N_1$. Concerning the auxiliary reconfiguration, it is allowed to redirect the arc in any direction with the modified increased capacity but with the same transit time in either direction. The remaining graph structure and data are unaltered.

Algorithm 1: Earliest arrival evacuee algorithm.

Input: A flow over time sub-network $N_1 = (s, V, A, u_a, \tau_a, Y)$ with $\tau_a = 0$ for each $a \in A$.

1. Construct a transformed network $\overline{N_1}$ to N_1 as in Equation (7).
2. Determine the maximum number of evacuees at every possible time instance at each Y from s as in [8].
3. For each $\theta \in T$, reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$, u_a replaced by 0 whenever $a \notin A$, in N_1, where c_a denotes the static flow value in each $a \in A$ for such network.

4. For each $\theta \in T$ and $a \in A$, if a is reversed, $\kappa_a = u_{\bar{a}} - c_{\bar{a}'}$ and $\kappa_{a'} = 0$. If neither a nor a' is reversed, $\kappa_a = u_a - c_a$, where κ_a is saved capacity of a, [9].

Output: Earliest arrival of evacuees at Y with $\tau_a = 0$ for each $a \in A$.

Theorem 1. The earliest arrival evacuee problem having zero transit times with partial arc reversal capability follows the principle of temporally repeated flows and can be solved in polynomial-time complexity.

Proof: Steps 1, 2, and 4 given by Algorithm 1 are solved in linear time. Its time complexity is dominated by the time complexity of computation of the earliest arrival evacuees at the pickup locations Y with zero transit times on each arc as in [8] in Step 2, which is solved in polynomial-time. Thus, it can be solved in polynomial-time complexity in N_1.

The flow over time problem having zero transit times that reached to each of the pickup locations determines the maximum number of evacuees at every possible time instance from the beginning in N_1. That means the earliest arrival of evacuees at Y from s with zero transit times in the transformed network follows the principle of temporally repeated flows which is equivalent to the solution with arc reversals capability in the original network [10].

4. Integrated evacuation network

For large scale disasters with a sufficiently large number of evacuees, all the evacuees may not arrive at Y at the same time, and it requests certain waiting time at Y before to start the bus assignment in N_2. Those who are delivered to Y earlier will have comparatively more waiting time. Meanwhile, for the evacuees, waiting at Y is comparatively better than to be at s. On the other hand, buses available at bus depot d request a certain time to be assigned to Y and are given by $\boldsymbol{\tau_{di}}$. Hence the effective waiting time in N can be denoted by $\boldsymbol{\Omega} = \mathbf{max}\{\boldsymbol{\omega_i}, \boldsymbol{\tau_{di}}\}$, for $\boldsymbol{\omega_i}$ be the waiting at $\boldsymbol{y_i} \in \boldsymbol{Y}$. To address this, the objective function given for the BEPP can be modified. So, for $\mathbf{T_{max}}$ the duration of evacuation overall vehicles under the constraint as in [5], the integrated evacuation planning problem can be reformulated as,

$$\text{Minimize } T_{max}, (8) \tag{8}$$

$$\text{Such that } T_{max} \geq \Omega + \textstyle\sum_{r \in R} \tau_{to}^{br} + \sum_{r \in R} \tau_{back}^{br} \quad \forall\ b \in B\ .\ (9) \tag{9}$$

Problem 2. Given $N = (s, d, V, A, E, u_a, \tau_a, \tau_e, Z)$, having supplies and demands at s and Z, respectively. The integrated evacuation planning problem in a prioritized embedding is to assign the vehicles for evacuees' transshipment with minimum clearance time.

Algorithm 2. Transit-vehicle assignment algorithm for minimum clearance time.

Input: An embedded evacuation network $N = (s, d, V, A, E, u_a, \tau_a, \tau_e, Z)$.

1. In $N_1 = (s, V, A, u_a, \tau_a, Y)$, consider Y as the sinks and determine the earliest arrival of evacuees for $\tau_a = 0$ at different Y from s, by using Algorithm 1.
2. Assign the transit-vehicles from d to $N_2 = (d, Y, E, \tau_e, Z)$ for the supplies provided by Step 1 at Y, as guided by the dominant vehicle assignment approach as in [7].
3. Stop, if all the supplies available at each of Y are fulfilled, respecting the capacity constraints of Z.
4. Otherwise, return to Step 2.

Output: Transit-vehicle assignment with the minimum clearance time from $s \rightarrow Z$.

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5. An integrated prioritized evacuation system

In a prioritized evacuation system as in [11,12], evacuees are collected from the disaster zone to the prioritized pickup locations of the primary sub-network in minimum time as the quickest transshipment by using the lex-max flow approach [13]. Considering such pickup locations as the sources, the available set of transit-buses are also assigned in the network to evacuate the evacuees safely to the sinks on a first-come-first-serve basis and is better-suited for the simultaneous flow of evacuees. Such an assignment is also carried in a dominating solution approach by adjusting the potential demands of the pickup locations to have minimum effecting waiting in the embedding. To have the quickest arrival of evacuees with partial arc reversals capability, we introduce a problem and design an algorithm as follows:

Problem 3. Given an evacuation sub-network $N_1 = (S,\ V, A,\ u_a, \tau_a, Y)$, with supplies at S, demands at Y, auxiliary nodes V, arc capacity u_a, and arc transit time τ_a for $a \in A$. The quickest partial arc reversal transshipment problem is to find the quickest arrival of evacuees at Y with partial arc reversals capability.

Algorithm 3. Quickest partial arc reversal transshipment algorithm.

Input: A dynamic sub-network $N_1 = (S,\ V,\ A,\ u_a,\ \tau_a,\ Y)$, with the supply and demand.

1. Construct a transformed dynamic sub-network $\overline{N_1}$ as in Equation (7).
2. Solve the quickest transshipment problem [13] in the transformed network of Step 1.
3. For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$, u_a replaced by 0 whenever $a \notin A$, in N_1, where c_a denotes the static $s - \text{y}$ flow value in each $a \in A$ for such sub-network.
4. For each $\theta \in \mathbf{T}$ and $a \in A$, if a is reversed, then $k_a = u_{\bar{a}} - c_{a'}$ and $k_{a'} = 0$. If neither a nor a' is reversed, then $k_a = u_a - c_a$ where k_a is saved capacity of a, [9].

Output: The quickest arrival of evacuees at Y in N_1 with partial arc reversal capability.

Theorem 2. For the quickest partial arc reversal transshipment in N_1, the quickest evacuee arrival problem can be computed in polynomial-time complexity via k minimum cost flow (MCF) computations in $O(\ k(MCF)(m,n))$ time, where $MCF(m,n) = O(m\ log\ n\ (m + n \log n))$ in a network having n nodes and m arcs.

Proof. Steps 1, 3, and 4 related to the arc reversal capability as in Algorithm 3 are solved in linear time. So their time complexity is dominated by the time complexity of the computation of the quickest evacuee arrival in N_1 and is solved in polynomial-time in $\text{O}(\ \text{k}(\text{MCF})(\text{m}, \text{n})$ where $MCF(m,n) = O\big(m \log n\,(m + n \log n)\big)$ in a network having n nodes and m arcs as in [14]. (10)

□

Transit-buses having uniform capacity Q are assigned from d which are sufficiently nearer to Y in N_2 on the first-come-first-serve basis. Such assignment begins only after $\alpha_1 \geq \text{Q}$ for α_1 be the number of evacuees arrived at the highest pickup demand. For the subsequent assignments, the effective waiting instance ψ is almost negligible.

Buses are assumed to pick up their full capacities. For this, the potential demands of the pickup locations are adjusted to be the integral multiple of busloads. Let the potential demand of the pickup location $y_k \in Y$ be $\alpha(y_k)$. For $\lfloor\,.\,\rfloor$ be the floor function, the demands can be adjusted to be $\alpha'(y_k)$ by using the following demand adjustment.

$$\alpha'(y_k) = \left\lfloor \frac{\alpha(y_k) + \sum_{q=1}^{k-1}[\alpha(y_k) - \alpha'(y_k)]}{Q} \right\rfloor . Q \ \ (10) \tag{11}$$

But if the $\boldsymbol{k^{th}}$ pickup location is the last one with the least priority, then it is taken as,

$$\alpha'(y_k) = \alpha(y_k) + \sum_{q=1}^{k-1} \{\alpha(y_q) - \alpha'(y_q)\} \text{ (11)} \quad (12)$$

Then the integrated evacuation planning problem, under the similar constraints as above, can be reformulated as ;

$$\text{Minimize} \quad \mathrm{T}_{max} \text{ (12)} \quad (13)$$

$$\text{such that} \tau_{max} \geq \psi + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br} \quad \forall\ b \in B, \text{(13)} \quad (14)$$

Constraint (13) needs T_{max} to be greater than or equal to the maximum travel cost incurred by all buses and is to be maximized in (12).

In an integrated approach, the quickest transshipment of the evacuees at Y in N_1 in the form of lex-max dynamic flows with respect to the adjusted demands are assigned to the transit-buses in N_2. For this we introduce,

Problem 4. Given an evacuation network $N = (S,\ V,\ A,\ u_a,\ \tau_a,\ Y,\ d,\ u_e,\ \tau_e,\ Z)$ having supplies and demands at s and Z respectively. The integrated evacuation planning problem in a prioritized embedding is to assign the vehicles for evacuees' transshipment with minimum clearance time.

Algorithm 4. An integrated evacuation planning algorithm in a prioritized embedding.

Input: An embedding $N = (S,\ V,\ A,\ u_a, \tau_a,\ Y,\ d,\ u_e,\ \tau_e,\ Z)$, with given supply and demand.

1. Consider $N_1 = (S,\ V,\ A,\ u_a, \tau_a,\ Y)$ having their pickup locations be Y.
2. Construct a priority ordering of Y assigning the highest priority to the nearest from .
3. Determine the arrival of evacuees at Y of N_1 from S using Algorithm 3.
4. Assign the transit-buses from d to Y in $N_2 = (d,\ Y,\ u_e, \tau_e,\ Z)$ for the supplies obtained in Step 3, to the nearest sink Z, on the first-come-first-serve basis.
5. Begin the assignment with $\alpha_1 \geq Q$ for α_1 be the collection of evacuees at Y provided by Equation (10).
6. Stop, if all the supplies at each Y be fulfilled, respecting the capacity constraints of .
7. Otherwise, return to Step 4.

Output: Transshipment of evacuees finally to Z in minimum clearance time.

6. Conclusions

Different network structures, models, algorithms, and their solution strategies are integrated and extended to achieve the quickest transshipment of the evacuees in an integrated network. Assignment of transit vehicles in such embeddings are carried out in domination solution approach for the minimum evacuation time.

Corresponding to an integrated network topology, specific arrival patterns are considered in the collection network. In such network, we use the concept of partial arc reversals which is beneficial to increase the flow values of evacuees by decreasing their collection time and is also favorable to have the minimum clearance time of the evacuees. The unused and saved arcs can be used for logistics and emergency facility. A prioritized primary network is considered to collect the evacuees in lex-max flow approach as the quickest transshipment and are assigned in the secondary sub-network in such prioritized embedding. It is better-suited and a novel approach for the simultaneous assignment with minimum delay in the embedding.

Author Contributions: Conceptualization, I.M.A. and T.N.D.; writing—original draft preparation, I.M.A.; writing—review and editing, T.N.D.; supervision, T.N.D. Both authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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