# Quickest Transshipment in an Evacuation Network Topology

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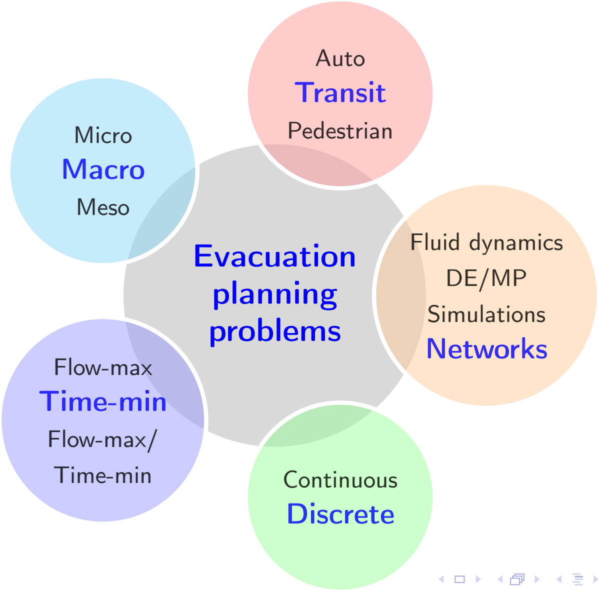
# Outline

# Evacuation planning problems



## Bus-based evacuation planning problem

$$
\begin{aligned}
&\text{Minimize } \mathcal{T}\text{max} && (1)\\
\text{such that } \mathcal{T}\text{max} &\geq \tau_{di} x_{ij}^{b1} + \sum_{r\in[R]} \tau_{to}^{br} + \sum_{r\in[R]} \tau_{back}^{br} \ \forall b \in [B] && (2)\\
\tau_{to}^{br} &= \sum_{i\in[Y]} \sum_{j\in[Z]} \tau_{ij} x_{ij}^{br} \ \forall b \in [B], r \in [R] && (3)\\
\tau_{back}^{br} &\geq \tau_{ij}[\sum_{k\in[Y]} x_{kj}^{br} + \sum_{l\in[Z]} x_{il}^{b,r+1} - 1], \ \forall b \in [B], r \in [R-1] && (4)\\
\sum_{i\in[Y]} \sum_{j\in[Z]} x_{ij}^{br} &\leq 1 \ \forall b \in [B], r \in [R] && (5)\\
\sum_{i\in[Y]} \sum_{j\in[Z]} x_{ij}^{br} &\geq \sum_{i\in[Y]} \sum_{j\in[Z]} x_{ij}^{b,r+1} \ \forall b \in [B], r \in [R-1] && (6)\\
\sum_{j\in[Z]} \sum_{b\in[B]} \sum_{r\in[R]} x_{ij}^{br} &\geq \alpha_i \ \forall i \in [Y] && (7)\\
\sum_{i\in[Y]} \sum_{b\in[B]} \sum_{r\in[R]} x_{ij}^{br} &\leq \beta_j \ \forall j \in [Z] && (8)\\
x_{ij}^{br} &\in \{0,1\} \ \forall \tau_{to}^{br}, \tau_{back}^{br}, \mathcal{T}\text{max} \in \mathbb{R} && (9)
\end{aligned}
$$

[4]

# An integrated network topology

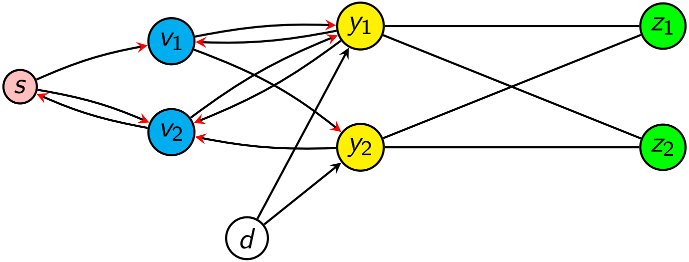



Figure 1: An integrated network topology.

$\mathscr{N} = \mathscr{N}_1 \cup \mathscr{N}_2$
$\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$
$\mathscr{N}_2 = (d, Y, E, \tau_e, Z)$

# Earliest arrivals of evacuees

$$\sum_{\sigma=\tau_a}^{T} \sum_{a \in A_i^{in}} f(a, \sigma - \tau_a) - \sum_{\sigma=0}^{T} \sum_{a \in A_i^{out}} f(a, \sigma) = 0, \ \forall \ i \notin \{s, y\} \quad (10)$$

$$\sum_{\sigma=\tau_a}^{\theta} \sum_{a \in A_i^{in}} f(a, \sigma - \tau_a) - \sum_{\sigma=0}^{\theta} \sum_{a \in A_i^{out}} f(a, \sigma) \geq 0, \forall i \notin \{s, y\}, \theta \in \mathbf{T} \quad (11)$$

$$0 \leq f(a, \theta) \leq u_a \qquad \forall \ a \in A, \ \theta \in \mathbf{T} \quad (12)$$

$$(\nu_f, \theta) = \sum_{\sigma=0}^{\theta} \sum_{a \in A_s^{out}} f(a, \sigma) = \sum_{\sigma=\tau_a}^{\theta} \sum_{a \in A_y^{in}} f(a, \sigma - \tau_a) \quad (13)$$

The total flow released from $s$ that reached to $Y$ up to $\theta' \in \mathbb{Z}_+$, with $\tau_a = 0$, is

$$|\nu_f|_{\theta'} = \sum_{\theta=1}^{\theta'} |\nu(Y, \theta)|. \quad (14)$$

For the given time bound $T$, the value in (14) is denoted by $|\nu_f| = \sum_{\theta=1}^{T} |\nu(Y, \theta)|$.
See in [3].

# Earliest arrival evacuee problem

Given $\mathcal{N}_1 = (s, V, A, u_a, \tau_a, Y)$. The earliest arrival evacuee problem is to find the earliest arrival of evacuees at the pickup locations $Y$ with partial arc reversal capability.

**Formulation of the transformed network:** Reversal arc for $a = (i, j)$ is $a' = (j, i)$. The transformed network of $\mathcal{N}_1$ consists of the modified arc capacities and constant transit times as,

$$u_{\tilde{a}} = u_a + u_{a'} \text{ and } \tau_{\tilde{a}} = \begin{cases} \tau_a & \text{if } a \in A \\ \tau_{a'} & \text{otherwise} \end{cases} \tag{15}$$

where an edge $\tilde{a} \in \tilde{A}$ in a transformed network, if $a \vee a' \in A$ in $\mathcal{N}_1$. The remaining graph structure and data are unaltered. In the transformed network $\mathcal{N}_1$, we have solved the earliest arrival evacuee problem with $\tau_a = 0$ on each arc as in [10] and saved all unused arc capacity as in [9].

**Algorithm 1:** Earliest arrival evacuee algorithm

**Input :** $\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$ with $\tau_a = 0$ for each $a \in A$.

1 Construct a transformed network $\tilde{\mathscr{N}_1}$ as in Equation (15).

2 Determine the maximum number of evacuees at every possible time instance at each $Y$ from $s$ as in [10].

3 For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$, $u_a$ replaced by 0 whenever $a \notin A$, in $\mathcal{N}_1$, where $c_a$ denotes the static $s - y$ flow value in each $a \in A$ for such sub-network.

4 For each $\theta \in \mathbf{T}$ and $a \in A$, if $a$ is reversed, $\kappa_a = u_{\tilde{a}} - c_{a'}$ and $\kappa_{a'} = 0$. If neither $a$ nor $a'$ is reversed, $\kappa_a = u_a - c_a$, where $\kappa_a$ is saved capacity of $a$, [8], [9].

**Result:** Earliest arrival of evacuees at $Y$ with $\tau_a = 0$ for each $a \in A$.

## Theorem 1

*The earliest arrival evacue problem with partial arc reversal capability follows the principle of temporally repeated flow at $\tau_a = 0$ and is polynomial-time solvable.*

**Sketch of proof:**

- Steps 1, 3, and 4 of Algorithm 1 are solved in linear time, its time complexity is dominated by the time complexity of Step 2.
- Earliest arrival transshipment computation to $Y$ with $\tau_a = 0$ on each arc is solved in polynomial-time in Step 2, [10].
- It follows the principle of temporally repeated flow at $\tau_a = 0$, [7].

# Transit-vehicle assignment problem

Let $\mathscr{N} = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$, provided with supplies and demands. The transit-vehicle assignment problem is to assign the vehicles for evacuees transshipment with minimum clearance time.

Integrated model for the embedded network: Let $\omega_i$ be the waiting at $y_i \in Y$ with the effective waiting, $\Omega = \max\{\omega_i, \tau_{di}\}$ for $\mathscr{N}$, then it can be reformulated as,

$$\text{minimize } \mathcal{T}_{\max} \tag{16}$$

$$\text{such that } \mathcal{T}_{\max} \geq \Omega + \sum_{r \in [R]} \tau_{to}^{br} + \sum_{r \in [R]} \tau_{back}^{br} \; \forall b \in [B] \tag{17}$$

$$\text{with the constraints } (3 - 9) \tag{18}$$

**Algorithm 2:** Transit-vehicle assignment algorithm for minimum clearance time.

**Input** : An embedded network $\mathscr{N} = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$.

1 In $\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$, determine the earliest arrival of evacuees for $\tau_a = 0$ at different $Y$ from $s$, by using Algorithm 1.

2 Assign the transit-vehicles from $d$ as guided by the dominant assignment approach as in [1] to $\mathscr{N}_2 = (d, Y, E, \tau_e, Z)$ for the supplies provided by Step 1, [4].

3 Stop, if all the supplies at each of $Y$ are fulfilled, respecting the capacity constraints of $Z$.

4 Otherwise, return to Step 2.

**Result:** Transit-vehicle assignment with minimum clearance time: $s \rightarrow Z$.

## Example 2

### Integrated approach in $\mathscr{N}$

- $|s| = +12$, $|y_1| = \mp 8$, $|y_2| = \mp 4$, $|z_1| = -6$, $|z_2| = -10$.
- $s \to v_1 \to y_1$, $s \to v_2 \to y_1$, $s \to v_1 \to y_2$.
- $|v(f)| = 1$, initially, temporally repeated.
- $|v_f|_{(y_1, y_2)} = (8, 4)$, T=4.
- If arc reversal, the same is at T=2.
- For $|B| = 1$, $\mathcal{T}_{\max}$= 41 and 39, respectively.
- For $|B| \geq 12$, $\mathcal{T}_{\max}$= 7 and 5, respectively.

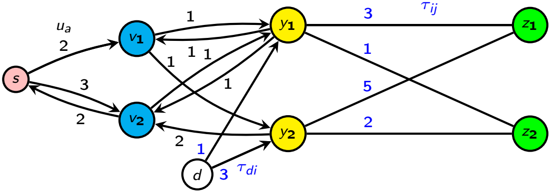



Figure 2: An instance of an integrated network $\mathscr{N}$

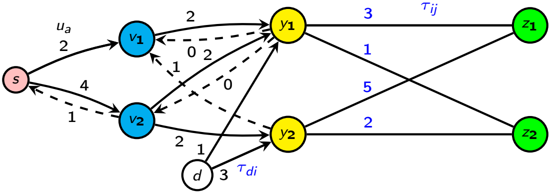



Figure 3: $\mathscr{N}$ with partial arc reversal

# An integrated prioritized evacuation network

- A flow value is said to be lexicographic if it is compared according to the rank of the terminals for a prioritized network.
- Let $\mathscr{N} = (S, V, A, u_a, \tau_a, Z)$ be a prioritized network with priority $t_1, t_2, \cdots, t_n;\ t_i \in S \cup Z$. Let

$$|f|_t := \begin{cases} \sum_{a \in A_t^+} f_a, & t \in S \text{ is a source} \\ \sum_{a \in A_t^-} f_a, & t \in Z \text{ is a sink} \end{cases}$$

  be the out/in flow value from/in the source/sink, respectively.
- For $f^1$, $f^2$ be the terminal respecting flows, then $f^1 \geq_L f^2$ if $\exists\, l \in \{0, 1, \cdots, k-1\} : \forall\, i \in \{1, 2, \cdots, l\} : |f^1|_{t_i} = |f^2|_{t_i} \wedge |f^1|_{t_{l+1}} > |f^2|_{t_{l+1}}$ or $\forall i \in \{1, 2, \cdots, k\} : |f^1|_{t_i} = |f^2|_{t_i}$.
- The maximum flow respecting the rank of the terminals is the lex-max.

# Quickest partial arc reversal transshipment problem

Given $\mathcal{N}_1 = (S, V, A, u_a, \tau_a, Y)$ with supplies at $S$, demands at $Y$. The quickest partial arc reversal transshipment problem is to find the quickest arrival of evacuees at $Y$ with partial arc reversals capability.

**Algorithm 3:** Quickest partial arc reversal transshipment algorithm.

**Input** : Let $\mathcal{N}_1 = (S, V, A, u_a, \tau_a, Y)$, with the supply and demand.

1 Construct a transformed dynamic sub-network $\overline{\mathcal{N}}_1$ as in Equation (15).

2 Solve the quickest transshipment problem in $\overline{\mathcal{N}}_1$ of Step 1, [5].

3 For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ iff $c_a > u_a$, $u_a$ replaced by 0 whenever $a \notin A$, in $\mathcal{N}_1$, where $c_a$ denotes the static $s - y$ flow value in each $a \in A$ for such sub-network.

4 For each $\theta \in \mathbf{T}$ and $a \in A$ , if $a$ is reversed, $\kappa_a = u_{\overline{a}} - c_{a'}$ and $\kappa_{a'} = 0$. If neither $a$ nor $a'$ is reversed, $\kappa_a = u_a - c_a$, where $\kappa_a$ is the saved capacity of $a$, [8].

**Output:** The quickest arrival of evacuees at Y in $\mathcal{N}_1$ with partial arc reversal capability.

### Theorem 3

*The quickest partial arc reversal transshipment problem can be computed in polynomial-time complexity via $\delta$ minimum cost flow (MCF) computations in $O(\delta m \log n(m + n \log n))$ on the network with n nodes and m arcs.*

**Sketch of proof:**

- Steps 1, 3, and 4 related to the arc reversal capability as in Algorithm 3 are solved in linear time.
- The time complexity is dominated by the time complexity of the computation of the quickest evacuee arrival at $Y$ in $\mathcal{N}_1$ and is solved in polynomial-time with $\delta$ MCF computations in $O(\delta \mathrm{MCF}(m, n))$, where $\mathrm{MCF}(m, n) = O(m \log n(m + n \log n))$ on a network having $n$ nodes and $m$ arcs as in [6].

# Assignment of vehicles in an embedding

- Let $\alpha_1$ be the No. of evacuees arrived at the highest pickup demand.
- Assignment begins only after $\alpha_1 \geq Q$, for uniform bus capacity Q.
- Adjusted demands $\alpha'(y_k)$ to be the integral multiple of busloads by using the following demand adjustment principle, [2],

$$\alpha'(y_k) = \left\lfloor \frac{\alpha(y_k) + \sum_{q=1}^{k-1}[\alpha(y_q) - \alpha'(y_q)]}{Q} \right\rfloor .Q \; \forall k \in 1, 2, \dots, n-1 \quad (19)$$

$$\alpha'(y_k) = \alpha(y_k) + \sum_{q=1}^{k-1}[\alpha(y_q) - \alpha'(y_q)], \text{ for the last pickup location} \quad (20)$$

# Assignment of transit-vehicles in a prioritized embedding

## Integrated evacuation planning problem in a prioritized embedding

Given a prioritized network $\mathcal{N} = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$, having supplies and demands at $s$ and $Z$, respectively. The integrated evacuation planning problem in a prioritized embedding is to assign the vehicles for evacuees transshipment with minimum clearance time.

**Integrated model for the prioritized embedding:** For $\Psi$ be the effective waiting in such embedding, it can be reformulated as:

$$\text{minimize } \mathcal{T}_{\max} \tag{21}$$

$$\text{such that } \mathcal{T}_{\max} \geq \Psi + \sum_{r \in [R]} \tau_{to}^{br} + \sum_{r \in [R]} \tau_{back}^{br} \; \forall b \in [B] \tag{22}$$

$$\text{with the constraints } \quad (3-9) \tag{23}$$

Constraint (22) needs $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all buses and is to be minimized in (21). Other constraints are the same as in Equations (3-9).

**Algorithm 4:** Integrated evacuation planning algorithm in a prioritized embedding.

**Input** : $\mathcal{N} = (S, V, A, u_a, \tau_a, Y, d, u_e, \tau_e, Z)$, with given supply, demand.

1 Consider $\mathcal{N}_1 = (G, u_a, S, \tau_a, Y)$, having their pickup locations be $Y$.

2 Construct a priority ordering of $Y$, the highest priority to the nearest.

3 Determine the arrival of evacuees at $Y$ of $\mathcal{N}_1$ from $S$ using Algorithm 3.

4 Assign transit-buses from $d$ to $Y$ in $\mathcal{N}_2 = (d, Y, u_e, \tau_e, Z)$ for the supplies of Step 3, as guided by the dominant assignment approach, on the first-come-first-serve basis, provided by [4].

5 Begin the assignment with $\alpha_1 \geq Q$, $\forall\ \alpha'(y_k)$ provided by Equation (19).

6 Stop, if all $\alpha'(y_k)$ be fulfilled, respecting Equation (20) the $\nu(Z)$.

7 Otherwise, return to Step 4.

**Output:** Transshipment of evacuees to $Z$ in minimum clearance time.

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# Suggestions are welcome

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