Virtual Model-Based Trajectory Optimization Algorithm for Aliquoting Robotic System

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Abstract: This work is devoted to the optimization of the trajectory of a robotic system for aliquoting biosamples, consisting of serial and parallel manipulators. Optimization consists of two stages. At the first stage, optimization constraints associated with the workspace, taking into account the ranges of permissible values of the angles of the drive rotational joints, the link interference and singularities. The workspace in the space of input and output coordinates is represented as a partially ordered set of integers. At the second stage, restrictions are formed related to the objects that are in the workspace during the aliquoting process, such as the body of the robotic system, test tubes and racks. The condition for excluding collisions of the manipulator with other objects is provided by geometric decomposition of objects and exclusion of areas corresponding to external objects from the set describing the workspace of the manipulator. Optimization is performed in the space of input coordinates. The objective function is proportional to the duration of movement along the trajectory. The possibility of evolutionary algorithms application for solving this problem is analyzed. An assessment of their performance is given. Optimization and export of the resulting trajectory are implemented in software, which allows you to verify the optimization results on a virtual model. The simulation results are presented.

Keywords: optimization; aliquoting system; workspace

1. Introduction

When automating technological processes in a wide range of industries, trajectory planning tasks are very important. The boundaries of the workspace, singularities, and intersections of robot links can be considered as obstacles. Currently, there are various methods for trajectory planning. Some methods are based on the spatial decomposition into geometric shapes of various shapes [1,2]. Discrete search methods allows using heuristic estimates of the perspective of trajectories, speeding up the solution of trajectory planning problems[3]. An important family of motion planning methods consists of potential field methods, originally developed for mobile robot navigation and real-time obstacle avoidance [4]. Sampling methods allow us to solve problems of planning the trajectory of movement, with a high level of complexity: for spaces with a large number of obstacles and mechanisms with a significant number of degrees of freedom [5]. The Probabilistic Roadmap Method [6] is widely applied for solving path finding problems in both local and global settings. One of the ways to reduce the number of vertices in a graph is the method of constructing randomized route networks based on scopes [7]. There are also currently a number of efficient stochastic methods that can be applied to trajectory planning of manipulators [8,9].

The application of evolutionary algorithms makes it possible to solve optimization problems, including such as planning a trajectory with high performance indicators. In this regard, within the framework of this paper, an original approach is proposed based on the application of evolutionary algorithms to find the optimal trajectory inside the workspace of robots, taking into account singularities, intersections of links and restrictions on drive coordinates. The optimization problem is solved using the example of a delta robot, which...
can be applied for a wide range of tasks, including biometric aliquoting [10]. During this task, it becomes necessary to avoid obstacles such as test tubes and racks.

2. Setting an Optimization Problem

Delta robot (Figure 1b) included in the robotic system for biomaterial aliquoting (Figure 1a) has 3 degrees of freedom and includes three kinematic chains RUU. In each of the chains, the drive rotary joint \( A_i \) is used to connect to the base. Universal joints \( C_i \) are used to connect to the moving platform. Universal joints \( B_i \) are used for connecting two links between each other. The end-effector is the center \( P \) of the mobile platform.

![Image](image1)

Figure 1. Robotic system: a) 3D model: 1—body, 2—DeLi manipulator, 3—dispensing head, 4—dispenser tip, 5—robot manipulator, 6—base of the robot manipulator, 7—workspace, 8—trolley, 9—tray for consumables, 10—rack with test tubes, b) delta-robot structure

The moving delta robot can move relatively freely along a certain arbitrary trajectory, taking into account the limitations determined by the workspace, taking into account singularities and intersections of robot links. The duration of robot movement should be minimized as much as possible. Since the duration of movement in time is determined by the duration of operation of robot drives required for the corresponding movement, it is advisable to optimize the trajectory in the space of input coordinates. So, for a delta robot, the input coordinates are the angles \( \theta_i \) of rotation of the drive rotary joints \( A_i \). An arbitrary trajectory can be represented as a set of movements (steps) in the space of \( m = 3 \) input coordinates. As a criterion function, it is proposed to use a function based on the Chebyshev metric, as well as the Euclidean metric, taken with some small weight coefficient \( \epsilon \) [11]:

\[
F = \sum_{i=1}^{n} \left( \max_{j \in \{1,2,\ldots,m\}} |\theta_{ij} - \theta_{i-1,j}| + \epsilon \sum_{j=1}^{m} (\theta_{ij} - \theta_{i-1,j})^2 \right) \rightarrow \min \tag{1}
\]

Optimization should be carried out with restrictions on the size of the workspace. In the framework of previous works, the authors proposed to use the representation of the workspace in the form of a partially ordered set of integers \( A \) [12]. Therefore, checking the opt-in restriction consists of two steps.

2.1. the First Stage. Definition of the Set \( B \) of Trajectory Coordinates in the Space of Integers.

For this purpose, an algorithm based on a modification of the algorithm is developed Bresenham’s algorithm [13], which assumes that the trajectory is represented as a polyline consisting of many segments. In this case, the coordinates must correspond to the covering set of the workspace, represented as a partially ordered set of integers, respectively,
they must be obtained taking into account the accuracy of the approximation $\Delta_j$ and the displacement $k_j$ along the $j$ coordinate axes by the formula

\[
x_i = \frac{\theta_{i,1} + k_1}{\Delta_1}, \quad y_i = \frac{\theta_{i,2} + k_2}{\Delta_2}, \quad z_i = \frac{\theta_{i,3} + k_2}{\Delta_2}
\]

(2)

2.2. the Second Stage. Checking Whether the Resulting Set $B$ a Belongs to the Workspace Set $A$.

Thus the optimization constraint condition has the form

\[
B_i \subset A, \quad i \in 1, \ldots, n
\]

(3)

where $n$ is the number of segments that make up the trajectory.

Thus, the optimization problem looks like this.

- parameters: coordinates of intermediate points of the trajectory $x_i, y_i, z_i, i \in 1, \ldots, (n - 1)$. For a delta robot, the coordinates are the rotation angles of the drive rotary joints, i.e. 

\[
[(x_i y_i z_i)^T = [(\theta_{i,1} \theta_{i,2} \theta_{i,3})]^T.
\]

- parameter change range: overall dimensions of the workspace in the space of input coordinates $\theta_{(i,j)} \in [\theta_{\text{min}}, \theta_{\text{max}}]$.

- criterion: the function $F$ calculated by formula (1).

- restriction: condition (3).

To increase the efficiency of optimization in the presence of obstacles, we transfer the optimization constraint to the criterion function

\[
F' = F + \sum_{i=1}^{n} \left( \theta_i \left( p_1 \sqrt{\sum_{j=1}^{m} (\theta_{i,j} - \theta_{i-1,j})^2} + p_2 \right) \right) \rightarrow \min
\]

(4)

where $p_1,p_2$ are the penalty coefficient, and $\theta_i$ is the Heaviside function:

\[
\theta_i = \begin{cases} 
0, & \text{if } B_i \subset A \\
1, & \text{otherwise}
\end{cases}
\]

(5)

We use the Genetic algorithm (GA), Particle Swarm Optimization (PSO), Grey Wolf Optimization (GWO) to solve the optimization problem.

3. Numerical Results

The workspace of the delta robot is limited by the range of permissible rotation angles in the hinges, the area of sign constancy The Jacobian and the condition that there are no intersections of links. The problem of determining the workspace $B_P$ for a delta robot in the space of output coordinates is considered by the authors in [10]. Figure 2 shows the results for the following delta robot parameters: $a = 600 \text{ mm}$, $c = 100 \text{ mm}$, $d = 350 \text{ mm}$, $e = 800 \text{ mm}$, $\Delta_j = 4^\circ$. 
Figure 2. Workspace virtual model of the delta robot: a) in \((x, y, z)\)coordinates, b) in coordinates \(\theta_i\) coordinates

An object was added as an obstacle with a parallelepiped shape (Figure 3a). The updated workspace virtual models are shown in Figure 3b, c.

Figure 3. Additional boundaries, related to the overall dimensions of the obstacle: a) obstacle C; b) The set \(Bp\) given; c) \(Bp\) taking into account; c) Set \(B\) given an obstacle

We perform trajectory optimization inside the workspace of a delta robot. A C++ software package has been developed for this purpose. Parallel computing is implemented using the OpenMP library. Visualization is performed by exporting an ordered set of integers describing the workspace in STL format and arrays of co-ordinates of trajectory points in JSON format, and then importing data in the Blender software package using developed Python scripts. Set the starting and ending points of the trajectory in the output coordinate space:

- \(x_{p1} = 250\) mm, \(y_{p1} = 250\) mm, \(z_{p1} = -500\) mm, \(x_{p2} = -270\) mm, \(y_{p2} = -270\) mm, \(z_{p2} = -450\) mm, and the number of vertices of the trajectory \(n = 3\). Accordingly, the number of optimization parameters \(p = 3n = 9\). The weight coefficient \(\epsilon = 0, 1\), the penalty coefficients \(p_1 = 5, p_2 = 500\).

Parameters of the GA algorithm: the number of individuals in the initial population \(H = 250\), the number of generations \(W = 200\), the number of crosses at each iteration \(S_{GA} = 125\), the number of possible values of each of the parameters \(g = 2^{25}\), the probability of mutation \(p_m = 70\%\).

Parameters of the GWO algorithm: \(H = 250, W = 200\), number of new individuals at each iteration \(S_{GWO} = 1000\).

Parameters of the PSO algorithm: \(H = 250, W = 200\), number of groups \(G=2\), values of free parameters \(\alpha=0.7, \beta=1.4, \gamma=1.4\).

Optimization for each of the tests was performed in four stages. At the first stage, the range of parameters was changed to the ranges corresponding to the overall dimensions of
the workspace for each of the coordinates. The parameter ranges at each subsequent stage were reduced by a factor of $10^2$. At the same time, the center of the ranges corresponded to the best result obtained at the previous stage. The optimization results are shown in Table 1. The GA algorithm showed the best average value of the criterion function.

Table 1. Results table, mm.

<table>
<thead>
<tr>
<th>Trials</th>
<th>GA</th>
<th>GWO</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>152,499</td>
<td>149,852</td>
<td>149,737</td>
</tr>
<tr>
<td>2</td>
<td>151,070</td>
<td>150,136</td>
<td>130,477</td>
</tr>
<tr>
<td>3</td>
<td>131,368</td>
<td>130,477</td>
<td>149,778</td>
</tr>
<tr>
<td>4</td>
<td>137,201</td>
<td>150,394</td>
<td>130,385</td>
</tr>
<tr>
<td>5</td>
<td>149,876</td>
<td>150,427</td>
<td>149,674</td>
</tr>
<tr>
<td></td>
<td>Average. value</td>
<td>143,679</td>
<td>144,272</td>
</tr>
</tbody>
</table>

To justify the feasibility of using the criterial modified Chebyshev metric as a criterion function, we analyze the values of the trajectory length for various metrics and the criterion function for one of the tests. Table 3 shows the results of trajectory optimization for Test 3. The table shows that despite the fact that the trajectory obtained as a result of PSO optimization by the algorithm is 0.2% shorter in length than the trajectory obtained by the GA algorithm, however, the length in accordance with the Chebyshev metric and, accordingly, the positioning duration for the PSO trajectory is 14% longer. We can conclude that it is advisable to apply the Chebyshev metric when planning a trajectory.

Table 2. Trajectory optimization results for test 3, mm.

<table>
<thead>
<tr>
<th>Trajectory according to test 3</th>
<th>GA</th>
<th>GWO</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path length</td>
<td>168,101</td>
<td>163,734</td>
<td>167,705</td>
</tr>
<tr>
<td>Chebyshev length (estimation of positioning duration)</td>
<td>114,558</td>
<td>114,104</td>
<td>133,008</td>
</tr>
<tr>
<td>Criterion function ($\alpha = 0.1$)</td>
<td>131,368</td>
<td>130,477</td>
<td>149,778</td>
</tr>
</tbody>
</table>

Figure 9 shows the trajectories for Test 3 inside the workspace virtual model. As can be seen from the figure, the PSO algorithm found only a local minimum of the criterion function for avoiding the obstacle.

Figure 4. Trajectories inside the virtual workspace model: GA - red, GWO - blue, PSO - green.
4. Conclusion

The application of heuristic algorithms made it possible to solve the problem of planning a trajectory for a 3D workspace, represented as a partially ordered set of integers. It is shown that the planning of the trajectory in the space of input coordinates and the use of the Chebyshev metric as part of the criterion function makes it possible to reduce the duration of positioning from the initial to the final point of the trajectory. As part of future work, the choice of parameters of optimization algorithms will be performed to achieve the best convergence rates. Various modifications of heuristic algorithms, including hybrid ones, will be applied. A comparative analysis of the effectiveness of the developed methodology with existing methods of trajectory planning will be carried out.

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Abbreviations
The following abbreviations are used in this manuscript:

- GA: Genetic algorithm
- PSO: Particle Swarm Optimization
- GWO: Grey Wolf Optimization

References