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Evolution of Brans-Dicke Parameter with Deceleration Parameter within Generalized Brans-Dicke Theory †

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Abstract: In the present work, we have investigated the role of deceleration parameter in the evolution of the Brans-Dicke (BD) parameter. We have considered the generalized Brans-Dicke (GBD) theory where the BD parameter is assumed to be evolving with the BD scalar field. We have shown that, the GBD theory with a cosmological constant can suitably be expressed as an effective cosmic fluid within general relativity. The BD parameter contains two terms one of which may be non-evolving and the other part bears the time evolution. Also, we have shown that, for anisotropic cosmological models within the GBD theory, for the time-independent deceleration parameter, the cosmic anisotropy will not contribute to the evolution of the Brans-Dicke parameter. However, for models with a time-dependent deceleration parameter, the cosmic anisotropy affects the BD parameter as a whole.

Keywords: brans-dicke theory; brans-dicke parameter; deceleration parameter

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1. Introduction

In the context of modification of GR, scalar–tensor gravitation theories have played a major role. In order to investigate the cosmological aspects at different phases of cosmic evolution such as the cosmic coincidence, inflation, and the cosmic acceleration, the Brans–Dicke (BD) theory is considered as a successful scalar–tensor theory [1–7]. In the BD theory, a scalar field φ mediates the gravity and is coupled to the geometry through a coupling constant called Brans–Dicke parameter ω. The BD theory favours a time variation of the Newtonian gravitational constant coming through the evolution of the BD scalar field. Over a period of time, the BD theory has been tested through the experimental tests from solar system [8], the CMB data and large scale structure [9,10]. Also, the BD theory emerged as a low energy limit of many quantum gravity theories such as superstring theory or Kaluza–Klein theory [5].

In the usual BD theory proposed initially, the Brans-Dicke parameter is considered as a constant the value of which may be constrained from different observational tests. However there occurs a lot of controvery over its values. In some cases, the theories require a very high value such as $\omega > 40{,}000$, in some other cases it takes very small values such as $417 < \omega < 714$. Some recent studies also require negative values for the BD parameter. In some studies, it is found that, a small negative value of the BD parameter around $\omega < -1.5$ is required to explain the structure formation and the late time cosmic acceleration issue.

In the present work, we considered a generalized Brans-Dicke theory where the BD parameter is chosen to vary with the BD scalar field and there is a self interacting potential. In some previous studies we have shown the evolution of the BD parameter and obtained the range of values for the BD parameter to explain different scenario such as the Big Rip scenario, Bouncing scenario or a late time cosmic speed up phenomena [5–7]. Here, we are interested to investigate the time evolution of the BD parameter for different choices of the deceleration parameter. We consider an anisotropic Locally Rotationally Symmetric Bianchi I (LRSBI) metric to model the Universe and studied the effect of the anisotropic parameter on the evolutionary aspect of the BD parameter. The manuscript is organized as follows: In Section 2, a brief description of the formalism of generalized Brans-Dicke cosmology is presented. In Section 3, we obtain a general expression of the BD parameter in terms of the deceleration parameter to show their interdependence in the evolutionary behaviour. For different specific choices, we studied the evolution of the BD parameter. At the end, we summarized our results in Section 4.

2. Formalism of generalized Brans-Dicke theory

The action of the generalized Brans-Dicke theory within the Jordan frame is given by

$$S=\int d^4x\,\sqrt{-g}\,\left[\varphi R-\frac{\omega(\varphi)}{\varphi}\,\varphi^{,\mu}\varphi_{,\mu}-V(\varphi)+L_m\right], \tag{1}$$

where φ is the BD scalar field, $\omega(\varphi)$ is the Brans-Dicke parameter that evolves with the BD scalar field. $V(\varphi)$ is the self interacting potential and L_m is the matter Lagrangian.

For an spatially homogeneous and anisotropic LRSBI Universe

$$ds^2=-dt^2+A^2dx^2+B^2(dy^2+dz^2), \tag{2}$$

and a perfect fluid distribution, $T_{\mu\nu}=(\rho+p)u_\mu u_\nu+pg_{\mu\nu}$, the field equations may be obtained as

$$\frac{\rho}{\varphi}=(2k+1)\xi^2H^2-\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2+3H\left(\frac{\dot\varphi}{\varphi}\right)\;-\frac{V(\varphi)}{2\varphi}, \tag{3}$$

$$-\frac{p}{\varphi}=2\xi\dot H+3\xi^2H^2+\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2+2\xi H\left(\frac{\dot\varphi}{\varphi}\right)+\frac{\ddot\varphi}{\varphi}-\frac{V(\varphi)}{2\varphi}, \tag{4}$$

$$-\frac{p}{\varphi}=(k+1)\xi\dot H+(k^2+k+1)\xi^2H^2+\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2+(k+1)\xi H\left(\frac{\dot\varphi}{\varphi}\right)+\frac{\ddot\varphi}{\varphi}-\frac{V(\varphi)}{2\varphi}. \tag{5}$$

The Brans-Dicke scalar field satisfies the Klein-Gordon equation

$$\frac{\ddot\varphi}{\varphi}+3H\frac{\dot\varphi}{\varphi}+\frac{\frac{\partial\omega}{\partial\varphi}\dot\varphi^2}{2\omega(\varphi)+3}+\frac{2V(\varphi)-\varphi\frac{\partial V}{\partial\varphi}}{2\omega(\varphi)+3}=\frac{\rho-3p}{2\omega(\varphi)+3}. \tag{6}$$

Here H is the Hubble parameter and is related to the directional Hubble parameters $\frac{\dot A}{A}$ and $\frac{\dot B}{B}$ as $H=\frac{1}{3}\left(\frac{\dot A}{A}+2\frac{\dot B}{B}\right)$. Assuming an anisotropic relationship among the directional Hubble parameter $\frac{\dot A}{A}=k\frac{\dot B}{B}$, the Hubble parameter may be expressed as $H=\frac{1}{\xi}\frac{\dot B}{B}$. The parameter ξ is related to the anisotropic parameter k as $\xi=\frac{3}{k+2}$. For an isotropic Universe, we have $k=\xi=1$.

The GBD theory may be recast as an effective cosmological fluid within general relativity that favours a phantom field dominated phase. In the absence of a self interacting potential, the field equations (3) –(5) may be expressed as

$$\frac{\rho+\rho_\varphi}{\varphi}=(2k+1)\xi^2H^2-\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2+3H\left(\frac{\dot\varphi}{\varphi}\right)\;-\frac{V(\varphi)}{2\varphi}, \tag{7}$$

$$-\frac{p+p_\varphi}{\varphi}=\left(\frac{k+3}{2}\right)\xi\dot H+\left(\frac{k^2+K+4}{2}\right)\xi^2H^2+\frac{\omega(\varphi)}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2+\left(\frac{k+3}{2}\right)\xi H\left(\frac{\dot\varphi}{\varphi}\right)+\frac{\ddot\varphi}{\varphi}-\frac{V(\varphi)}{2\varphi}, \tag{8}$$

where

$$\rho_\varphi = \frac{\omega(\varphi)\varphi}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2 - 3H\dot\varphi, \quad (9)$$

$$p_\varphi = -\frac{\omega(\varphi)\varphi}{2}\left(\frac{\dot\varphi}{\varphi}\right)^2 - \left(\frac{k+3}{2}\right)\xi H\,\dot\varphi - \ddot\varphi. \quad (10)$$

The effective equation of state parameter (EoS) for such an effective fluid becomes

$$\omega_{eff} = \frac{p_\varphi}{\rho_\varphi}. \quad (11)$$

3. Evolution of the Brans-Dicke parameter

The evolutionary aspect of the Brans-Dicke parameter may be obtained from the field Equations (3)–(5) as [11]

$$\omega(\varphi) = \left(\frac{\dot\varphi}{\varphi}\right)^{-2}\left[-\frac{\rho+p}{\varphi} - \frac{\ddot\varphi}{\varphi} + k\xi H\frac{\dot\varphi}{\varphi} - 2\xi\dot H + 2(k-1)\xi^2H^2\right]. \quad (12)$$

For an anisotropic Universe with $k \neq 1$ and $\xi \neq 1$, we may obtain an associate evolution of the Brans-Dicke scalar field with the geometrical expansion of the Universe as

$$\frac{\dot\varphi}{\varphi} = (q-2)H\,, \quad (13)$$

where $q = -\frac{a\ddot a}{a^2}$ is the deceleration parameter, a being the mean radius parameter. It is obvious that, the evolution of the deceleration parameter decides the evolutionary aspect of the BD scalar field and consequently the evolution of the Brans-Dicke parameter. In the present work, in order to obtain the evolutionary aspect of the Brans-Dicke parameter, we consider some specific examples of the deceleration parameter and correlate the dynamics. Also, we will investigate the role of the anisotropic parameter on the evolution of the BD parameter. For brevity, we consider the barotropic equation of state $p = \omega_D\rho$, where ω_D is the Eos parameter and may be considered to be a constant for the present work.

From the continuity equation, we may get the energy density for a constant EoS parameter ω_D, as $\rho = \rho_0 a^{-3(1+\omega_D)}$. Consequently, we have $\rho + p = \rho_0(1+\omega_D)a^{-3(1+\omega_D)}$.

The Brans-Dicke parameter may be expressed in terms of the deceleration parameter as

$$\omega = \frac{1}{(q-2)^2}\left[\frac{\rho_0(1+\omega_D)a^{-3(1+\omega_D)}}{\varphi H^2} + \dot q H + \big(3+\xi(k+2)\big) + \{2\xi(k-1)(\xi-1)-6\}\right]. \quad (14)$$

For constant deceleration parameter, q_0, the above equation reduces to

$$\omega(H) = \frac{1}{(q_0-2)^2}\left[\frac{\rho_0(1+\omega_D)a^{-(q_0+3\omega_D+1)}}{\varphi_0 H^2} + \big(3+\xi(k+2)\big) + \{2\xi(k-1)(\xi-1)-6\}\right], \quad (15)$$

which has an evolving part and a non-evolving part. The behaviour of the evolving part comes from the matter field and the associated EoS parameter and the non-evolving part is controlled by the anisotropic parameter. One should note that, the evolving part of the Brans-Dicke parameter is not affected by the choice of the anisotropic parameters k and ξ. In obtaining the above equation (15), we have used the fact that, for a constant deceleration parameter the BD scalar field becomes $\varphi = \varphi_0 a^{(q_0-2)}$, where φ_0 is the value of the BD scalar field at the present epoch. Constant deceleration parameter can be obtained from power expansion and the exponential expansion of the Universe where the Hubble parameter may be written respectively as $H = \frac{1}{1+q_0}\frac{1}{t}$ and $H = H_0$. Substituting these expressions back into Euqation (15), we obtain the respective evolution of the BD parameter.

However, for time dependent evolution of the deceleration parameter, it is not possible to separate the BD parameter into its time evolving and non-evolving part. In fact, the evolution becomes an involved one. Consequently, the anisotropy dependence of the BD parameter is also involved. We may consider different time dependent deceleration parameter including some parametrized form to assess the time evolution of the Brans-Dicke parameter.

4. Conclusions

In the present work, we have studied the evolutionary aspect of the Brans-Dicke parameter with different choices of the deceleration parameter. We consider a generalized Brans-Dicke theory involving a BD parameter depending on the BD scalar field and a self interacting potential. Also, we consider an anisotropic Universe modelled through an LRSBI metric for the investigation. A general expression of the BD parameter is obtained in terms of the deceleration parameter which splits into two parts, one providing the evolutionary aspect and a non-evolving part when the deceleration parameter is chosen to be non-evolving. However, for an evolving deceleration parameter, the BD parameter becomes an involved one. The anisotropy parameter is found to affect the non-evolving part of the BD parameter for the choice of constant deceleration parameter. On the other hand, it is not possible to single out the evolving and non-evolving part of the BD parameter for time dependent deceleration parameter. Consequently, the anisotropy parameter affects the evolution of the BD parameter for the second choice.

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