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Bound of the Non-Commutative Parameter Based on Gravitational Measurements†

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Abstract: In this paper, we investigate the four classical tests of general relativity in the non-commutative (NC) gauge theory of gravity. Using the Seiberg-Witten (SW) map and the star product, then we calculate the deformed components metric $\hat{g}_{\mu\nu}(r,\Theta)$ of Schwarzschild black hole (SBH). The use of this deformed metric enables us to calculate the gravitational periastron advance of mercury, red-shift, deflection of light and time delay in the NC spacetime. Our result for NC prediction of the gravitational deflection of light and time delay shows a new behavior than the classical one. As an application, we use a typical primordial black hole to give an estimation to the NC parameter Θ, where our result shows that$\Theta^{phy} \approx 10^{-34}\,m$ for the gravitational red-shift, deflection of light, and time delay at the final stage of inflation, and $\Theta^{phy} \approx 10^{-31}\,m$ for the gravitational periastron advance of some planets from our system solar.

Keywords: non-commutative gauge field theory; gauge field gravity; gravitational measurements.

1. Introduction

General relativity (GR) is considered one of the major scientific discoveries at the beginning of the 20th century, it describes an excellent relativistic description of gravity, which is one of the fundamental interactions that describe all phenomena in nature at the macroscopic scale. The success of this theory was due to the prediction of the experimental results of the first three tests, which are proposed by Albert Einstein in 1915, which were the periastron advance of mercury orbit, deflection of light, and the red-shift [1]. Later in 1964 I. Shapiro discovered and observed the time delay due to the presence of massive objects, which became another successful test of GR, which is so-called the fourth classical test of GR [2].

However, this theory remains unable to describe gravity at the quantum scale, this problem led to the emergence of many ideas. One of them adopts the same concept of quantum mechanics in quantization concerning the relations of commutation between the observables as $[\hat{x}_i, \hat{p}_j] = -i\hbar\delta_{ij}$, where in this theory the coordinates of spacetime $\hat{x}_\mu$are considering a non-commutative observable, and subject to the commutation relation between the coordinates themselves, namely:

where $\Theta_{\mu\nu}$ is an anti-symmetric real matrix of the NC parameter, which describes

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}, \tag{1}$$

the fundamental cell discretizing the spacetime, where the general idea of the NC geometry is that the quantization of the spacetime leads to the quantization of the gravity.
Moreover, in this theory the scalar product between two arbitrary functions $f(x)$ and $g(x)$ are changed to the star product

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$$(f*g)(x)=f(x)e^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}}g(x). \tag{2}$$

Recently, there has been a lot of research on determining a lower bound of the NC parameter and studies on quantum gravity effects, with several approaches [3-21]. Our aim is to estimate a lower bound on the NC parameter using the four classical experimental tests of GR inspired by the NC geometry based on the gauge theory of gravity and compared to the other results obtained by another approach of NC geometry [9,11]. In this study, we provide the NC corrections to the four classical predictions of the GR in the NC gauge theory of gravity. Firstly we obtain the NC periastron advance of orbit and we choose some planets of our solar system as examples for the numerical values of Θ, for the deflection of light, red-shift, and the time delay we use data of a typical primordial black hole at the early universe, where we use the scale factor to get a physical distance measured at any time [9,11], our result shows that the NC property of spacetime appears before Planck scale.

In this paper, we discussed the bound of the NC parameter in the NC gauge theory of gravity using the four classical tests of GR. A brief review of the NC gauge theory of gravity for the SBH metric is presented in Sec. 2. The estimation of the NC parameter for different experimental tests of GR in NC spacetime is obtained and discussed in Sec. 3. In Sec. 4 we present our conclusion.

2. Non-commutative corrections for Schwarzschild black hole

In our previous works [21], we used the tetrad formalism and both the star-product and the SW map [22] to construct the NC gauge theory for a static metric with spherical symmetric. One can use the perturbation form for the SW map to describe the deformed tetrad fields $\hat{e}_\mu^a$ as a development in the power of Θ up to the second-order, which can be obtained by following the same approach in Ref. [23]:

$$\begin{aligned}\hat{e}_\mu^a= {} & e_\mu^a-\frac{i}{4}\Theta^{\nu\rho}\left[\omega_\nu^{ab}\partial_\rho e_\mu^d+\left(\partial_\rho\omega_\mu^{ac}+R_{\rho\mu}^{ac}\right)e_\nu^d\right]\eta_{cd}+\frac{1}{32}\Theta^{\nu\rho}\Theta^{\lambda\tau}\Big[2\left\{R_{\tau\nu},R_{\mu\rho}\right\}^{ab}e_\lambda^c-\omega_\lambda^{ab}\left(D_\rho R_{\tau\nu}^{cd}+\right.\\ & \left.\partial_\rho R_{\tau\nu}^{cd}\right)e_\nu^m\eta_{dm}-\left\{\omega_{\nu,}\left(D_\rho R_{\tau\nu}+\partial_\rho R_{\tau\nu}\right)\right\}^{ab}e_\lambda^c-\partial_\tau\left\{\omega_{\nu,}\left(\partial_\rho\omega_\mu+R_{\rho\mu}\right)\right\}^{ab}e_\lambda^c-\omega_\lambda^{ab}(\omega_\nu^{cd}\partial_\rho e_\mu^m+\\ & \left(\partial_\rho\omega_\mu^{cd}+R_{\rho\mu}^{cd}\right)e_\nu^m)\eta_{dm}+2\partial_\nu\omega_\lambda^{ab}\partial_\rho\partial_\tau e_\mu^c-2\partial_\rho\left(\partial_\tau\omega_\mu^{ab}+R_{\tau\mu}^{ab}\right)\partial_\nu e_\lambda^c-\left\{\omega_{\nu,}\left(\partial_\rho\omega_\lambda+R_{\rho\lambda}\right)\right\}^{ab}\partial_\tau e_\mu^c-\\ & \left(\partial_\tau\omega_\mu^{ab}+R_{\tau\mu}^{ab}\right)\left(\omega_\nu^{cd}\partial_\rho e_\mu^m+\left(\partial_\rho\omega_\lambda^{cd}+R_{\rho\lambda}^{cd}\right)e_\nu^m\eta_{dm}\right)\Big]\eta_{bc}\ ,\end{aligned} \tag{3}$$

where $\hat{e}_\mu^a$ and ω_μ^{ab} are the tetrad field and the spin connection (gauge field), and:

$$\begin{gathered}\{\alpha,\beta\}^{ab}=\left(\alpha^{ac}\beta^{db}+\beta^{ac}\,\alpha^{db}\right)\eta_{cd},\ [\alpha,\beta]^{ab}=\left(\alpha^{ac}\beta^{db}-\beta^{ac}\,\alpha^{db}\right)\eta_{cd}\\ \mathrm{D}_\mu R_{\rho\sigma}^{ab}=\partial_\mu R_{\rho\sigma}^{ab}+\left(\omega_\mu^{ac}R_{\rho\sigma}^{db}+\omega_\mu^{bc}R_{\rho\sigma}^{da}\right).\end{gathered} \tag{4}$$

The deformed metric can be written as:

$$\hat{g}_{\mu\nu}=\frac{1}{2}\left(\hat{e}_\mu^a*\hat{e}_\nu^{b\dagger}+\hat{e}_\nu^a*\hat{e}_\mu^{b\dagger}\right)\eta_{ab}\ . \tag{5}$$

For the SBH solution we choose the following tetrad fields

$$\begin{gathered}e_\mu^0=\left(\sqrt{1-\frac{2m}{r}},0,0,0\right),\ e_\mu^1=\left(0,\frac{1}{\sqrt{1-\frac{2m}{r}}}sin\theta\ cos\phi,r\ cos\theta\ cos\phi,-r\ sin\theta\ sin\phi\right),\\ e_\mu^2=\left(0,\frac{1}{\sqrt{1-\frac{2m}{r}}}sin\theta\ sin\phi,r\ cos\theta\ sin\phi,r\ sin\theta\ cos\ \phi\right),\ e_\mu^3=\left(0,\frac{1}{\sqrt{1-\frac{2m}{r}}}cos\theta\ ,-r\ sin\theta\ ,0\right).\end{gathered} \tag{6}$$

The deformed tetrad fields are calculated in Ref. [21], we follow the same steps to compute the deformed metric components of SBH in the equatorial plane $\theta=\frac{\pi}{2}$,

$$-\hat{g}_{00}=\left(1-\frac{2m}{r}\right)+\left\{\frac{m\left(88m^2+m\,r\left(-77+15\sqrt{1-\frac{2m}{r}}\right)-8r^2\left(-2+\sqrt{1-\frac{2m}{r}}\right)\right)}{16\,r^4(r-2m)}\right\}\Theta^2+O(\Theta^4), \quad \text{(a-7)}$$

$$\hat{g}_{11}=\frac{1}{\left(1-\frac{2m}{r}\right)}-\left\{\frac{m\left(12m^2+m\,r\left(-14+\sqrt{1-\frac{2m}{r}}\right)-r^2\left(5+\sqrt{1-\frac{2m}{r}}\right)\right)}{8\,r^2(r-2m)^3}\right\}\Theta^2+O(\Theta^4), \quad \text{(b-7)}$$

$$\hat{g}_{22}=r^2-\left\{\frac{m\left(m\left(10-6\sqrt{1-\frac{2m}{r}}\right)-\frac{8\,m^2}{r}+r\left(-3+5\sqrt{1-\frac{2m}{r}}\right)\right)}{16(r-2m)^2}\right\}\Theta^2+O(\Theta^4), \quad \text{(c-7)}$$

$$\hat{g}_{33}=r^2-\left\{\frac{5}{8}-\frac{3}{8}\sqrt{1-\frac{2m}{r}}+\frac{m\left(-17+\frac{5}{\sqrt{1-\frac{2m}{r}}}\right)}{16\,r}+\frac{m^2\sqrt{1-\frac{2m}{r}}}{(r-2m)^2}\right\}\Theta^2+O(\Theta^4), \quad \text{(d-7)}$$

where $m = GM$ denotes the mass of the SBH. It is clear that in the limit of $\Theta \to 0$, we obtain the commutative SBH solution.

3. Experemental test of GR in NC spacetime

In this section we present the NC corrections to the four classical tests of GR using the deformed SBH metric as background.

3.1. Gravitational periastron advance

In our previous work [21], we derive the expression of the angle deviation after one revolution in the NC SBH metric (7), using the perturbation form of the geodesic equation as in Ref. [24], then we found:

$$\Delta\phi=\frac{6\pi GM}{c^2\alpha(1-e^2)}+\pi\Theta^2\left\{\frac{\left({E_0}^2-m_0^2c^4\right)}{2GM\alpha(1-e^2)}+\frac{6\left(m_0^2c^2-(E_0/c)^2\right)}{\alpha^2(1-e^2)^2}+\frac{m_0^2c^2}{2\alpha^2(1-e^2)^2}\right\}, \quad (10)$$

where α, e denote the major semi-axis and the eccentricity of the movement. For numerical application we choose the problem of Mercury planet orbit, where the NC parameter is in the order:

$$\Theta^{phy}=\sqrt{\hbar\Theta}\approx 5{,}7876.10^{-31}\,m, \quad (11)$$

as we see the NC parameter Θ^{phy} is very small for the solar system, which means that our solar system is very sensitive to the NC parameter. For the other planets the lower bound on Θ^{phy} is shown in Table 1:

Table 1. Some observable values of orbital precession for different planets of our solar system, are show in columns 2. The prediction of the orbital precession in general relativity in column 3, in final column we give the lower bound for the non-commutative parameter Θ^{phy}.

Planet	$\Delta\phi^{obs}\left(\frac{\text{arc-sec}}{\text{centry}}\right)$	$\Delta\phi^{GR}\left(\frac{\text{arc-sec}}{\text{centry}}\right)$	L. b of Θ^{phy} $(\times 10^{-31}m)$
Mercury	$42.9800 \pm 0{,}0020$	42.9805	≤ 05.7876
Venus	$8.6247 \pm 0{,}0005$	8.6283	≤ 04.5239
Earth	$3.8387 \pm 0{,}0004$	3.8399	≤ 04.0976

[1] The experemental data can be found in Ref [25,26].

As we see in Table 1, the lower bound of Θ^{phy} is in the same order for the planet's orbit of our solar system $\Theta^{phy}\sim 10^{-31}m$.

3.2. Deflection of light

Another successful experimental test of GR is the gravitational deflection of light which is predicted by Albert Einstein in his theory of gravity. In which the light is deflected from its original path when it passes near a strong gravitational field, the formula that describes this phenomenon is given by [1], and the NC expression can be read as:

$$\Delta\hat{\phi} = 2\int_b^\infty \frac{1}{r\sqrt{\hat{g}_{00}(r)}}\left(\frac{r^2}{b^2}\left|\frac{\hat{g}_{00}(b)}{\hat{g}_{00}(r)}\right| - 1\right) dr - \pi, \tag{12}$$

this integral can be computed after expanding our expression on the first order in m/r and we stop in the second order of Θ, with some calculations we get:

$$\Delta\hat{\phi} = \frac{4GM}{c^2 b} - \frac{5GM}{6c^2 b^3}\Theta^2. \tag{13}$$

As we see the first term represents the GR prediction and the second term represents the NC corrections to the gravitational deflection of light, where this correction should be smaller than the accuracy of the measurements [27], to estimate Θ for this phenomenon we use the radius $(r \sim b \approx 1.5 \times 10^{-3}\ \mathrm{m})$ and the mass $(GM \sim 5 \times 10^{-4}\ \mathrm{m})$ of a typical micro black hole, so we get

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leq 5{,}7.\,10^{-34}\, m, \tag{14}$$

where α is the scale factor at the end of inflation, we multiplying our results by α^2, because we use the space-space (Θ^{ij}) NC parameter [12] to obtain a physical results. It is worth to note that, the NC geometry change the behavior of the angle deviation, as we see when $b \to 0$, the NC term dominates and the behavior of $\Delta\hat{\phi} \propto -\frac{1}{b^3}$ changes to the negative one.

3.3. Gravitational red-shift

The third success experimental test of GR, which is the gravitational red-shift, where the shift in the spectral of light due to gravity is given in [1], and its NC form can be computed using the NC deformed metric (a-7),

$$\hat{z} = \sqrt{\left|\frac{\hat{g}_{00}(r_2)}{\hat{g}_{00}(r_1)}\right|} - 1. \tag{15}$$

For an asymptotic observer $r_2 \to \infty$, measured the red-shift for the NC SBH is given by $\hat{z}$:

$$\hat{z} = z\left(1 - \left(\frac{z+1}{z}\right)\left[\frac{\left(88GM^2 + GM\, r_1\left(-77+15\sqrt{1-\frac{2GM}{r_1}}\right) - 8r_1^2\left(-2+\sqrt{1-\frac{2GM}{r_1}}\right)\right)}{32r^3(r_1-2GM)^2}\right]\Theta^2\right), \tag{16}$$

where $z = \left(\left(1 - \frac{2GM}{r_1}\right)^{-1/2} - 1\right)$ is the red-shift that is predicted by the GR. We use the same data of micro black hole with accuracy of the measurements [28], then we get the bound on the Θ^{phy} parameter:

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leq 2{,}09.\,10^{-34}\, m, \tag{17}$$

3.3. Time delay (Shapiro effect)

The fourth successful classical test of the GR is discovered by I.Shapiro [1,2], which is so-called gravitational time delay and also the Shapiro effect, where this phenomenon studies the necessary time for a radar signal emitting from one point to another one traveling near a massive object and returning to the emitting point. Supposing now radar traveling from point

$$\Delta\hat{t} = 2\left[\hat{t}(r_1, b) + \hat{t}(r_2, b) - \sqrt{b - r_1} - \sqrt{b - r_2}\right], \tag{12}$$

where

$$\hat{t}(r, b) = \int_b^r \frac{1}{\hat{g}_{00}(r')}\left(1 - \frac{b^2\hat{g}_{00}(r')}{r'^2\hat{g}_{00}(b)}\right) dr', \tag{12}$$

we expand our expression on the first order in m and we stop at the second order in Θ, after some calculations we obtain:

$$\hat{t}(r, b) = \sqrt{r^2 - b^2} + 2GMln\left(\frac{r + \sqrt{r^2 - b^2}}{b}\right) + GM\left(\frac{r - b}{r + b}\right)^{1/2} - \frac{GM(3b - 4r)\sqrt{r^2 - b^2}}{4b^2 r(r + b)}\Theta^2, \tag{16}$$

we take in consideration that $r_1 \ll b$ and $r_2 \ll b$, we obtain the full expression of the time delay in the NC spacetime:

$$\Delta\hat{t} \approx 4GM\left[ln\left(\frac{4\, r_1\, r_2}{b^2}\right) + 1\right] - \frac{4GM}{b^2}\Theta^2, \tag{12}$$

The same note in the behavior of time delay in the NC spacetime, when $b \to 0$, the NC correction term dominates and the behavior of $\Delta\hat{\phi} \propto -\frac{1}{b^2}$ becomes negative, and that means the NC geometry removes the divergent behaviors.

For a numerical application, we take the same micro black hole data and the ratio $\frac{4\, r_1\, r_2}{b^2}$ is considered in the same order as in our solar system scale, with accuracy of the measurements [29], so we get:

$$\Theta^{phy} = \sqrt{\alpha^2\Theta^2} \leq 2{,}57.10^{-34}\, m, \tag{17}$$

4. Conclusion

In this paper, we investigate the four classical tests of GR in the NC spacetime. As a background for our calculation, we use a deformed SBH metric via the NC geometry using the SW maps and the star product.

As a first step, we obtain a correction to the periastron advance of mercury up to the second-order in Θ [21], where our results show that the NC parameter is close to the Planck scale $\Theta^{phy}{\sim}10^{-31}m$ and the extend of this application to other planets of our solar system shows that $\Theta^{phy}$is of the same order acts as a fundamental constant of the solar system. Then we compute the correction to the light deflection, red-shift and time delay in the NC spacetime, for application we choose a data of a microscopic black hole at the early universe.

Our results show that, the experimental test which uses a radio wave or a light give us a bound on the NC parameter in the order of $\Theta^{phy}{\sim}10^{-34}m$, where our results are smaller than the one obtained in Ref. [12,16] because we use a different approach. But for the orbital motion of a massive particle (planet), the bound on Θ is in the order of $\Theta^{phy}{\sim}10^{-31}m$, and it's remarkable that our result is close to the one obtained by using the classical mechanics in NC flat spacetime as Ref. [4,5]. This result indicates that the macroscopic system is very sensitive to the NC parameter. It is essential to mention that through the study of black holes thermodynamics in NC spacetime the bound on the NC parameter $\sqrt{\theta}$ has been obtained in some papers as in Ref. [6-9] which is expected to be $\sqrt{\theta}{\sim}10^{-1}.\, l_p$, using the point-like structure for the matter with the NC Gaussian distribution and gauge theory. But in this work and our privious works [21,30], we show that the lower bound of $\Theta^{phy}$is limited before the Planck scale between $(10^{-31}m - 10^{-35}m)$, where we use black hole thermodynamics [21] and the four classical tests of GR, which confirmes that the NC property of spacetime appears before Planck scale.

The use of the NC gauge theory of gravity enables us to obtain good results on the bound of the NC parameter. This theory needs more work, it may have a bright future to describe quantum gravity.

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