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Abstract: Conventional digital holography uses the technique of combining two coherent light fields 10 and the numerical reconstruction of the recorded hologram leads to the object amplitude and phase 11 information. In spite of significant developments on the DH with coherent light, complex field im-12 aging with arbitrary coherent source is also desired for various reasons. Here, we present a possible 13 experimental approach for holography with the incoherent light. In case of incoherent light, the 14 complex spatial coherence function is a measurable quantity and the incoherent object holograms 15 are recorded as the coherence function. Thus, to record complex spatial coherence a square Sagnac 16 radial shearing interferometer is designed with the phase-shifting approach. The five-step phase-17 shifting method helps to measure the fringe visibility and the corresponding phase, which jointly 18 represents the complex coherence function. The inverse Fourier transform of complex coherence 19 function helps to retrieve the object information. 20

Keywords: Digital holography; Interference; Coherence; Phase-shifting

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1. Introduction

Digital holography (DH) is a technique of interference of two waves both spatially 25 and temporally coherent [1]. The recorded interference pattern contains both amplitude 26 and phase information which can be further reconstructed on digitally processing of the 27 recorded hologram [2, 3]. The coherent source is utilized to achieve the interference in the 28 DH, but the coherent imaging system suffers from speckle noise and edge effects [4, 5]. 29 Whereas, the incoherent imaging systems both spatially and temporally incoherent, such 30 as broad band or a light emitting diode (LED) do not suffer from speckle noise and also 31 cost effective. Therefore, over the coherent imaging systems the incoherent imaging sys-32 tems are preferred for certain applications. 33

The interference of incoherent light reflected or emitted from an object result in inco-34 herent digital holograms. Many known methods of recording incoherent holograms are 35 based on the self-interference principle which uses the property that each incoherent 36 source point is acting as an independent scatterer and is self-spatially coherent and hence, 37 it can generate an interference pattern with light coming from its mirror imaged point [6-38 8]. These techniques record the incoherent hologram in the form of intensity patterns. The 39 other way to record an incoherent hologram is in the form of two-point spatial complex 40 coherence function [9, 10]. These methods are based on the van Cittert-Zernike (VCZ) the-41 orem which connects the far-field complex coherence function with the incoherent source 42 intensity distribution. 43 In this paper, we present an experimental approach to record an incoherent object 1 hologram based on the VCZ theorem and present some initial results. In the first part of 2 the set up the object information is recorded as a two-point complex spatial coherence 3 function and in the later part a square Sagnac radial shearing interferometer with fivestep phase-shifting technique is designed to obtain the complex coherence function. The digital inverse Fourier transform of complex coherence function helps to retrieve the object information. 7

2. Principle

The principle of coherence holography is based on the existing similarity between the 9 VCZ theorem and the diffraction integral. The VCZ theorem connects the coherence func-10 tion of the incoherent source with its intensity distribution through the Fourier transform 11 relation. Consider a resolution target (number '5') incoherently illuminated by a yellow 12 LED having spectral scattering density $\eta(\tilde{r}; \lambda)$ where λ is the mean wavelength of the 13 LED and \tilde{r} is the transverse coordinate, respectively. As the source is incoherent each 14 point of LED is acting as random scatterer having instantaneous phase $\phi(\tilde{r}, t)$. The spec-15 tral component of the field is given by $A(\tilde{r}; \lambda) = \sqrt{\eta(\tilde{r}; \lambda)} \exp[i\phi(\tilde{r}, t)]$ corresponding to 16 λ . The field is Fourier transformed using a lens L having a focal length f and at Fourier 17 plane, a single realization of the field is represented as 18

$$E(\mathbf{r}) = \iint A(\tilde{\mathbf{r}}; \lambda) \exp\left(-\frac{i2\pi}{\lambda f}(\mathbf{r}, \tilde{\mathbf{r}})\right) d\tilde{\mathbf{r}}$$
(1) 19

where r is the coordinate of Fourier plane.



Figure 1. Geometry for recording information of spatial coherence function

The coherence function at the Fourier plane is given by 24

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2) = \langle E^*(\boldsymbol{r}_1) E(\boldsymbol{r}_2) \rangle \tag{2}$$

where the asterisk represents complex conjugate and angle bracket denotes the ensemble average.

From Eq. (1), we can write

$$\Gamma(\mathbf{r}_{1},\mathbf{r}_{2}) = \frac{\kappa}{\lambda^{2}f^{2}} \iint \sqrt{\eta(\tilde{\mathbf{r}}_{1};\lambda)\eta(\tilde{\mathbf{r}}_{2};\lambda)} \langle exp[i(\phi(\tilde{\mathbf{r}}_{2},t) - \phi(\tilde{\mathbf{r}}_{1},t)] \rangle exp\left[-i\frac{2\pi}{\lambda f}(\mathbf{r}_{2},\tilde{\mathbf{r}}_{2} - \mathbf{r}_{1},\tilde{\mathbf{r}}_{1})\right] d\tilde{\mathbf{r}}_{1}d\tilde{\mathbf{r}}_{2}$$

$$(3) \quad 31$$

where κ is a constant physical quantity having dimensions of length square. As the 32 LED source is spatially incoherent any two points of the source are mutually uncorrelated 33 and hence represented by a two-dimensional (2-D) Dirac-delta function 34

$$\langle exp[i(\phi(\tilde{r}_2,t)-\phi(\tilde{r}_1,t)]\rangle = \delta(\tilde{r}_2-\tilde{r}_1)$$
³⁵

The Eq. (3) is now reduced to

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2) = \frac{\kappa}{\lambda^2 f^2} \left\{ \int \eta(\tilde{\mathbf{r}}; \lambda) \exp\left[-i\frac{2\pi}{\lambda f} \tilde{\mathbf{r}}.(\mathbf{r}_2 - \mathbf{r}_1)\right] d\tilde{\mathbf{r}} \right\} d\lambda$$
(4) 37

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In the above equation an integration is also performed over the wavelength range λ 1 as the source is non-monochromatic and have a finite spectral bandwidth. The integral 2 inside the curly bracket represents van Cittert-Zernike theorem. 3



3. Experiment and results

Figure 2. Experimental set up square Sagnac radial shearing interferometer: P: Polarizer, L: Lens, BS: Beam Splitter, M: Mirror, QWP: Quarter Wave Plate

Fig. 2 shows the experimental set up to record the spatial coherence function. The 9 first part of the set up shows the recording of object information as explained in previous 10 section. A polarizer P1 is kept at 45° just after the LED source to make the unpolarized 11 light coming from the LED polarized. The field is Fourier transformed using a lens L1 kept 12 at its focal length $f_1 = 60$ mm from the target, number '5'. The black dotted line shows the 13 Fourier plane where the spatial coherence function represented by Eq. (4) is present. In 14the later part, a square Sagnac radial shearing interferometer with a telescopic lens system 15 L2 (f_2 = 120 mm) and L3 (f_3 = 125 mm) is designed to measure the coherence function. The 16 incoming field is divided by a polarizing beam splitter with orthogonal polarization states 17 in two parts and between the two oppositely counter propagating beams one gets magni-18 fied with magnification $\alpha = f_3/f_2$ and the other gets demagnified with magnification 19 $\alpha^{-1} = f_2/f_3$. Finally, at the output plane shown by blue dotted lines we get radially 20 sheared copies of the two fields. The output plane parameters are now scaled as $r_1 =$ 21 $\alpha^{-1}r$ and $r_2 = \alpha r$ and the coherence function $\Gamma(r_1, r_2)$ is now mapped as $\Gamma(\alpha^{-1}r, \alpha r)$. 22 The output plane doesn't fit the CMOS camera area (Thorlabs DCC3240M, imaging area 23 6.78 mm x 5.43 mm with pixel size 5.3 μ m) so, an imaging lens L4 with focal length f_4 = 24 150 mm is used to demagnify the image such that it fits the camera aperture. At the output, 25 just before the camera, a quarter wave plate (QWP) is kept at 45° from its fast axis to con-26 vert the two linear orthogonal polarization states in right circular and left circular polari-27 zation states, respectively. Later, a polarizer P2 is kept and rotated by angle θ such that 28 it introduces a phase shift 2θ between the two incoming beams. Five off-axis interfero-29 grams with phase shift 0, $\pi/2$, π , $3\pi/2$ and 2π are recorded by giving a tilt using mirror 30 M2 between the two counter propagating beams. The fringe contrast γ and correspond-31 ing phase ϕ' can be calculated as mentioned in references [11] and [12], respectively. 32 Thus, the complex spatial coherence function is built as $\Gamma(\alpha^{-1}\mathbf{r}, \alpha\mathbf{r}) = \gamma \exp(i\phi')$. 33

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Now, to reconstruct the object intensity distribution the spatial coherence function is 1 inverse Fourier transformed 2

$$\tilde{\eta}(\tilde{\boldsymbol{r}};\lambda) = (\alpha - \alpha^{-1})^2 \int \Gamma(\alpha^{-1}\boldsymbol{r},\alpha\boldsymbol{r}) exp\left[i\frac{2\pi}{\lambda f}(\alpha - \alpha^{-1})\boldsymbol{r}.\tilde{\boldsymbol{r}}\right] d\boldsymbol{r}$$
(5) 3



Figure 3. Digitally constructed coherence function (a)fringe visibility (b) corresponding phase and 6 (c) reconstructed object intensity distribution 7

Fig. 3 represents the digitally constructed (a) fringe visibility and (b) corresponding 9 phase using the five recorded interferograms and (c) the reconstructed object intensity 10 distribution showing the information of number '5'. 11

4. Conclusion

We have presented an experimental method using shearing for holography with the 13 incoherent source. The object information is recorded in the form of complex spatial co-14 herence function based on the principle of van Cittert-Zernike theorem and later it is an-15 alysed using a Sagnac radial shearing interferometer with the five-phase shifting algo-16 rithm. The object information is computationally acquired on inverse Fourier transform. 17

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