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Imaging incoherent target using Hadamard basis patterns

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Abstract: In this paper, we present a correlation-based imaging technique in single pixel imaging scheme using Hadamard basis illumination. The Hadamard basis, which has the characteristics of a two-bit value (−1, +1) and sparsity in its transformed domain, has been used in the illumination patterns and successfully utilized for imaging the incoherent target. It gives image reconstruction even in low light condition. Such deterministic patterns also help to solve the problem of large number of measurements in single pixel imaging, and hence simplify the experimental implementation. Further to compare the quality of imaging with Hadamard basis patterns, we also compare imaging with Fourier basis patterns and simulation results of both methods, namely Hadamard and Fourier basis, are presented.

Keywords: Single pixel imaging, Hadamard basis patterns, Fourier basis patterns.

1. Introduction

Correlation based imaging technique is an unconventional approach where an object is reconstructed as the distribution of correlations of the light field to image the desired target [1]. Single-pixel imaging, part of the unconventional techniques, is a novel imaging scheme where an object can be imaged by a single pixel detector without having any spatial resolution. Pixelated detectors are mostly expensive and not capable to detect signal in the spectrum such as x-ray and infrared. Moreover, a low-cost imaging system can be designed with single pixel detector which can operate at other wavelength regions even in low light condition, and techniques such as ghost imaging, and non-line of sight (NLOS) imaging have greatly benefited from the single pixel detection [2].

Ghost imaging (GI) provides object information from correlation-based measurements between two light beams. One light beam which interacts with the object to be imaged is collected by a bucket detector having no spatial resolution. Another beam is collected with a spatially resolved detector such as a charged coupled device (CCD) which samples the light that has never interacted with the object by array of pixels. The GI was first demonstrated using the correlation of quantum entangled photons generated by spontaneous parametric downconversion. It was later demonstrated that GI can be performed with classical incoherent (pseudothermal) light source in the same way as quantum entangled photons. Computational ghost imaging is very similar to single pixel imaging where an object is illuminated with digitally structured light patterns, and corresponding to each light pattern single pixel detector records light signal [3]. The illumination light patterns are projected onto the object by spatial light modulator (SLM) or digital micro-mirror device (DMD). The light patterns can be random or can be deterministic illumination patterns which can reduce the measurement number and acquisition time in single pixel imaging scheme. One of such deterministic pattern is the Hadamard patterns which are made of orthogonal basis vectors. Hadamard matrix has the characteristics of a 2-bit binary value {−1, 1} and sparsity in its transformed domain. In Hadamard single pixel imaging (HSI), Hadamard matrices form illumination patterns to reconstruct the image of an object [4]. Fourier single-pixel imaging (FSI) is also another robust imaging technique based on deterministic model which uses Fourier basis patterns [5].

In this work, we present a single pixel correlation imaging technique using Hadamard illumination to image an incoherent target object. Such deterministic illuminations help in reducing the number of measurements and acquisition time. Object retrieval in the FSI with the 4 phase shifting requires 4×M×M number of measurements which can be reduced to the 2×M×M number of measurements for the imaging with the Hadamard pattern by differential measurement approach. MxM is the size of the image which has to be reconstructed. Hadamard basis patterns are binary (black-and-white), which makes HSI naturally suitable for single-pixel imaging systems. HSI is more noise-robust compared to FSI [2]. In order to qualitatively compare imaging with Hadamard and the Fourier basis patterns, we present simulation results for both cases. This reconstruction highlights that the Hadamard illumination patterns can be an alternate to the FSI depending on the nature of requirements.

2. Theory

In the HSI scheme, the Hadamard transform (H{ }) of an object A(x, y) is expressed as [6]

$$H\{A(x,y)\} = \sum_{x=0}^{L-1}\sum_{y=0}^{L-1} A(x,y)(-1)^{w(x,y,u,v)} \quad (1)$$

where x,y are coordinates in spatial domain and u,v are coordinates in Hadamard domain. L is order of Hadamard matrix and L is number of rows and columns of the image.

$$w(x,y,u,v) = \sum_{i=0}^{n-1} [g_i(u)x_i + g_i(v)y_i] \quad (2)$$

$$g_0(u) = u_{n-1}$$

$$g_1(u) = u_{n-1} + u_{n-2}$$

$$g_2(u) = u_{n-2} + u_{n-3}$$

.....

$$g_{n-1}(u) = u_1 + u_o \quad (3)$$

where n = $\log_2 N$ and u_i, v_i, x_i, y_i are the binary representations of u, v, x, and y respectively.

Hadamard basis pattern $P_H(x, y)$ is presented as follows

$$P_H(x,y) = \frac{1}{2}[1 + H^{-1}\{\delta_H(u,v)\}] \quad (4)$$

$$\delta_H(u,v) = \begin{cases} 1, & if\ u = u_0\ and\ v = v_0 \\ 0, & otherwise \end{cases}$$

where H^{-1} represents inverse Hadamard transform and $\delta_H(u,v)$ is the delta function.

In order to consider imaging of an incoherent target, we consider an object obscured by a rotating diffuser as shown in Figure 1. The incoherent target is realized by a sequence of random scatterer which corresponds to each step of the rotating diffuser. A step of the diffuser is considered to introduce an independent and a uniformly distributed phase on the interval (-π,π). An object is illuminated by the Hadamard patterns $P_H(x, y)$ and single pixel detector(SPD) measures the responses corresponding to each pattern. Similarly, inverse Hadamard patterns i.e. $1 - P_H(x, y)$ are also projected in sequence onto the object and the same process is followed to record the responses at the SPD. D_+ and D_- are the SPD responses corresponding to the patterns $P_H(x, y)$ and $1 - P_H(x, y)$. Then Hadamard coefficient is derived from the two responses i.e D_+ and D_- by this differential measurement formula.

$$H(u, v) = D_+ - D_- \quad (5)$$

The object is reconstructed by taking inverse Hadamard transformation of $H(u, v)$. A schematic diagram of Hadamard illumination with the SPD is shown in Figure 1 where an object is placed behind a rotating diffuser. Hadamard patterns and inverse Hadamard patterns illuminate object as shown in Figure 1. SPD collects signal corresponding to each pattern. After that correlating these patterns object is reconstructed.

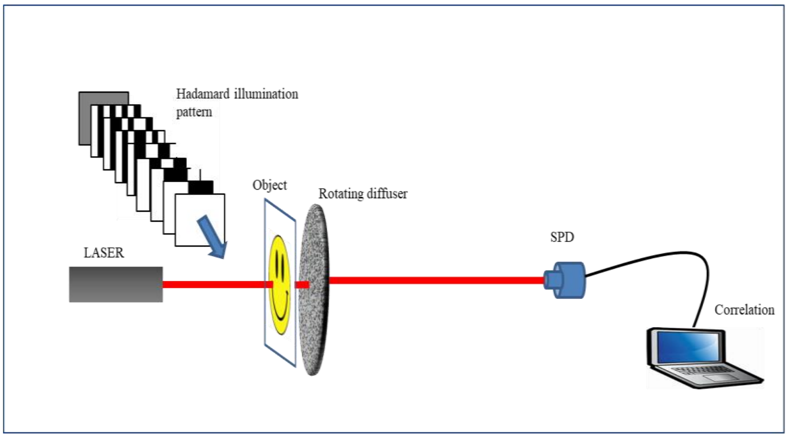
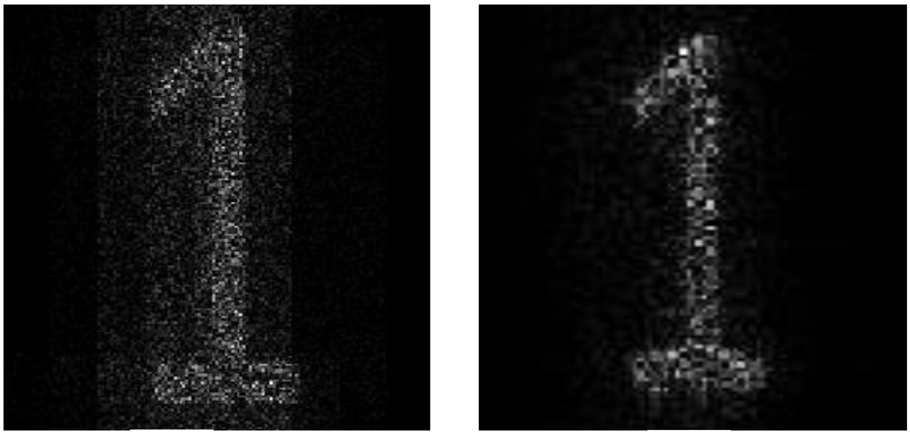


Figure 1. Schematic diagram of Hadamard single pixel imaging of incoherent target object

3. Results and discussion

We have shown here image reconstruction for an incoherent object "1" using both HSI and FSI. Simulation results using MATLAB are presented below. To image the incoherent target of size 256×256, 2×256×256 number of Hadamard patterns including Hadamard patterns ($P_H(x, y)$) and inverse Hadamard patterns ($1 - P_H(x, y)$) are projected onto the object. Hadamard patterns are formed according to equation number 4. Similarly, Fourier structured light is projected onto the object of size 256×256. To reconstruct the object total 4×256×256 number of Fourier patterns are needed to illuminate the object for applying four phase shift in FSI scheme.



(a) (b)

Figure 2. Reconstruction of an incoherent object using (a) HSI (b) FSI

4. Conclusion

In summary, a correlation-based single pixel imaging technique using Hadamard illumination is presented. The Hadamard patterns are formed with orthogonal basis vector consisting two binary values +1 and -1. Such deterministic pattern helps in single pixel imaging scheme by solving the problem of a huge number of measurements. We have successfully used Hadamard basis illumination to image an incoherent target hidden behind the diffuser. Differential measurement of Hadamard coefficient reduces noise and is easier than phase shifting in case of Fourier basis pattern. Further to compare reconstruction quality we have also shown simulation result using Fourier basis pattern.

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