computer sciences and mathematics forum

MDPI

Proceedings Paper

# Application of Soft Sets and Neutrosophic Sets for Introducing a Multi-Valued Logic in Ethics †

**Michael Gr. Voskoglou [1,*], Joachim Feuerstein [2] and Evangelos Athanassopoulos [3]**

[1] Department of Applied Mathematics, Graduate Technological Educational Institute of Western Greece, 22334 Patras, Greece
[2] Independent Researcher, Am Staden 11, 66121 Saarbrucken, Germany; augentrost2011@yahoo.de
[3] Independent Researcher, Giannakopoulou 39, 27300 Gastouni, Greece; evatha@gmail.com
* Correspondence: voskoglou@teiwest.gr or mvoskoglou@gmail.com
† Presented at the 1st International Online Conference on Mathematics and Applications; Available online: https://iocma2023.sciforum.net/.

**Abstract:** The use of a multi-valued logic in ethics is not a new idea, but, to the best of our knowledge, there is not any integrated proposal for its application reported in the literature until now. In this work soft and neutrosophic sets are used for a mathematical representation of the ethical rules, and in extension of the moral theories. Our target is not to add, in the already existing long catalogue, another theory about ethics, but instead to create a new basis enabling a modern approach to the subject.

**Keywords:** ethics; morality; moral dilemmas; fuzzy logic; soft sets

**Citation:** Voskoglou, M.G.; Feuerstein, J.; Athanassopoulos, E. Application of Soft Sets and Neutrosophic Sets for Introducing a Multi-Valued Logic in Ethics. *Comput. Sci. Math. Forum* **2023**, *6*, x. https://doi.org/10.3390/xxxxx

Published: 28 April 2023



## 1. Introduction

The terms *ethics* and *morality* are frequently used interchangeably, loosely meaning a framework of rules and behaviors about "good" and "bad" or "right" and "wrong" conduct. Many people, however, think of morality as something which is personal and of ethics as the standards of "good" and "bad" or "right" and "wrong" distinguished by a certain community or social setting. Assume, for example, that your local community considers that sex before marriage is immoral, but you have no strong feelings about it. In this case your personal morality contradicts the ethics of your community.

Humans are characterized by deep differences among each other, the result of which is the lack of unity and the absence of a universal will. The development, therefore, of a universal system of principles, rules and laws for the right human behavior, or in other words of a universally acceptable theory of ethics, seems to be impossible or, at least, very difficult. This becomes more difficult due to the fact that the various theories about an ideal system of ethics, developed during human history, are based on the principles of the *bivalent logic (BL)*, of Aristotle (384–322 BC). As a result, every human action or behavior is characterized only as "good" or "bad", without taking into account how good or bad it is, which frequently creates *moral dilemmas.*

A moral dilemma is a situation in which a person has moral reasons to do two (or more) actions, he/she can do each of them, but not both (or all) of them. No matter what he/she does, he/she will do something wrong, thus seems condemned to moral failure. The *problem of the trolley* [1] is probably the most well-known example of a moral dilemma. One of the many variations of this dilemma is designed in Figure 1.

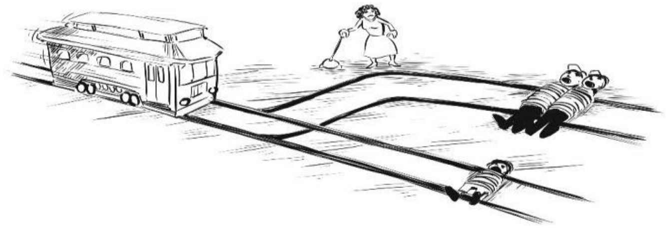

**Figure 1.** A variation of the trolley's problem.

The mother in Figure 1 is looking to the train, which cannot be stopped, approaching with high speed to her son, who is trapped on the rail. The only thing that she can do is to use the bar in order to direct the train to the other side of the rail, where unfortunately two other people are also trapped. What must she do?

At that time the poor mother had no time to think. Most probably, therefore, her motherly instinct will push her to turn the train to the other side for saving the life of her son, but killing the other two people. According to the ethical theory of utilitarianism [2], however, whose basic principle is to prescribe actions maximizing the well-being and happiness of all the individuals affected by them, this is an immoral action.

Such kind of dilemmas were the reason for starting to make evident to a number of ethical philosophers the need of introducing a multi-valued logic in ethics, where degrees of truth are used. As far as we know, however, apart from some general suggestions (e.g., [3]) or specific examples (e.g., [4]), nobody has attempted until now to propose a concrete and general way to materialize this idea in practice.

In this work *soft sets (SSs)* and *neutrosophic sets (NSs)* are proposed as tools for the introduction of a multi-valued logic in ethics.

## 2. Mathematical Background

### 2.1. Neutrosophic Sets

Aristotle's BL, which is characterized by the "*Principle of the Excluded Middle*" (a proposition is either "true" or "not true"), used to be for many centuries in the center of human reasoning, playing a dominant role to the development of science. Opposite views, however, appeared early in human history, supporting the existence of a third area between "true" and "not true"; e.g., by Buddha in India around to 500 BC, by Plato (427–377 BC), much later by the Marxist philosophers, etc. Complete propositions for multi-valued logics appeared, at any case, only in the beginning of the 20th century by Lukasiewicz, Tarski and others [5] (Section 2). According to Lukasiewicz's "*Principle of Valence*", in reality there exist intermediate truth values between the two extremities "true" and "not true".

Zadeh, in order to tackle mathematically the existing in everyday life partial truths, extended in 1965 the notion of a crisp set to the notion of *fuzzy set* (*FS)* by generalizing the characteristic function of crisp sets to the *membership function* of FSs [6]. Based on the notion of FS, Zadeh also introduced the *fuzzy logic (FL)* [7], in which propositions may have infinitely many truth-values in the interval [0, 1]. FSs and FL were also used for handling the existing in real world uncertainty. FL extends the traditional BL, reducing to it when no uncertainty exists in the corresponding situation. For more details FSs and FL we refer to [8].

Various generalizations of the FS theory have been proposed during the last years for tackling better all the formsb of uncertainty. A brief description of the most important among these generalizations can be found in [9].

On the purpose of simulating the existing imprecision in human reasoning, K.T. Atanassov enriched in 1986 the Zadeh's membership degree with the degree of *non-membership* and introduced the notion of *intuitionistic fuzzy set (IFS)*, which is a generalization of FS [10].

F. Smarandache [11] extended in 1995 the notion of IFS to the concept of *neutrosophic set (NS)* by adding the degree of *indeterminacy* or *neutrality*. The term neuttrosophic is the result of the synthesis of the words "neutral" and "sophia", which means in Greek language "wisdom". In this work we need only the simplest version of the concept of NS, which is defined as follows:

**Definition 1.** *A single valued NS (SVNS) A in U is of the form*

$$A = \{(x, T(x), I(x), F(x)): x \in U, T(x), I(x), F(x) \in [0, 1], 0 \leq T(x) + I(x) + F(x) \leq 3\} \quad (1)$$

In (1) T(x), I(x), F(x) are the degrees of *truth* (or membership), indeterminacy and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we write A <T, I, F>.

For example, let U be the set of students of a class and let A be the SVNS of the good students of U. Then each student x of U is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For instance, x(0.7, 0.1, 0.4) ∈ A means that there is a 70% belief that x is a good student, a 10% doubt about it and a 40% belief that x is not a good student. In particular, x(0, 1 ,0) ∈ A means that we do not know absolutely nothing about x's affiliation with A.

When the sum T(x) + I(x) + F(x) in a SVNS A in U is <1, then x is characterized by *incomplete* information, when it is equal to 1 by *complete* information and when it is >1 by *inconsistent* (i.e., contradiction tolerant) information. A SVNS may contain simultaneously elements characterized by all the previous types of information. For general facts on SVNSs we refer to [12].

*2.2. Soft Sets*

A disadvantage of the notion of FS is that there is no exact rule for defining the membership function. The methods used for this are usually statistical or intuitive/empirical. Moreover, the definition of the membership function is not unique depending on the "signals" that the individual receives from the environment. For example, defining the FS of "tall men" one could consider all men having heights greater than 1.9 m as tall and another one all those having heights greater than 2 m. Consequently, the first individual would assign membership degree 1 to all men with heights between 1.9 and 2 m, whereas the second one would assign to them membership degrees less than 1. The only restriction for the membership function is that it must be compatible with common logic; otherwise the resulting FS does not give a reliable description of the corresponding real situation. This could happen, for instance, if in the previous example men with heights less than 1.6 m possessed membership degrees ≥0.5.

Several theories related to FSs have been proposed for overcoming this difficulty, like the *interval-valued FSs*, the *grey systems/numbers*, the *rough sets*, etc. [9].

The Russian mathematician D. Molodstov [13] introduced the notion of *soft set (SS)* for tackling the uncertainty in a parametric manner, as follows:

**Definition 2.** *Let E be a set of parameters, let A be a subset of E, and let f be a map from A into the power set P(U) of the universe U. Then the SS (f, A) in U has the form*

$$(f, A) = \{(e, f(e)): e \in A\} \quad (2)$$

For example, let U = {$T_1$, $T_2$, $T_3$} be a set of bicycles and let E = {$t_1$, $t_2$, $t_3$} be the set of the parameters $t_1$ = automatic, $t_2$ = smart and $t_3$ = expensive. Assume that $T_1$, $T_2$ are automatic bicycles, that $T_3$ is a smart bicycle and that $T_2$, $T_3$ are expensive bicycles. Then, a map f: E $\longrightarrow$ P(U) is defined by f($t_1$) = {$T_1$, $T_2$}, f($t_2$) = {$T_3$} and f($t_3$) = {$T_2$, $T_3$}. Therefore, the SS (f, E) in U is the set of the ordered pairs (f, E) = {($t_1$, {$T_1$, $T_2$}), ($t_2$, {$T_3$}), ($t_3$, {$T_2$, $T_3$})}.

The term "soft" is related to the fact that the form of (f, A) depends on the parameters of A. For general facts on SSs we refer to [14].

## 3. Representation of the Ethical Rules by Soft and Neutrosophic Sets

Here we show that, under the light of FL or any other multi-valued logic, the ethical rules of a human community can be represented by SSs or NSs. As a result, a moral theory, being a collection of ethical rules, could be considered as being a collection of SSs, or NSs. This consideration, which significantly reduces the ethical dilemmas, could be proved in future as being a new basis for a modern approach to ethics.

Let us consider, for example, the Moses' rule "Thou shalt not kill", which is more or less connected nowadays to the ethical beliefs of most people, regardless their religion. According to BL, therefore, assassination is an immoral action. The question, however, is: Have all assassinations the same degree of immorality? Is it the same, for instance, an assassination designed with every detail and performed under conditions of complete soberness (case $A_1$), with an assassination performed under a "boiling soul" condition (case $A_2$), or with the assassin being in defense trying to protect his/her life (case $A_3$), or even with the instinctive reaction of the mother (case $A_4$) in the moral dilemma with the train (Figure 1)? Consider also a neutral observer in place of the mother, who prefers to save the lives of the two people leaving the child to be killed (case $A_5$). The answer to this question is obviously negative.

In order to overcome this difficulty, let us consider the set U of all kinds of assassinations as the universal set of the discourse. We introduce the following parameters: $e_1$ = immoral, $e_2$ = immoral, but with lightning, $e_3$ = neutral and $e_4$ = moral. Set E = {$e_1$, $e_2$, $e_3$, $e_4$} and let f: E → P(U) be the function assigning to each parameter of E the subset of U consisting of all kinds of assassinations characterized by this parameter. Then case $A_1$ belongs to the set f($e_1$), $A_2$ to f($e_2$), $A_3$ and $A_4$ could belong to f($e_3$), $A_5$ could belong to f($e_4$), etc. Consequently, the SS (f, E) = {(e, f(e)): e ∈ E} represents mathematically the ethical rule "Thou shalt not kill".

Of course, this assignment still leaves some doubts and space for personal beliefs. One may consider, for example, that $A_3$ and/or $A_4$ belong to f($e_2$), or even that $A_5$ belongs to f($e_3$). The choice of the parameters is not unique too; one may add more parameters or change some of the previous ones (e.g., the parameter $e_3$). In any case, the previous manipulation is much better than the conclusions of the BL. Also, it can be improved further, if the choice of the parameters and the assassinations characterized by each one of them is decided not by a single person, but by a group (the wider possible) of specialists. In this way the previous choices will be statistically more objective. This scepticism could be also useful to the judges and the jurymen for reaching the right decisions in the courts.

An alternative option is to work with NSs instead of SSs. In case of the previous example, for instance, one could consider the NS of the "compatible" to the common ethics assassinations and characterize with neutrosophic triplets its elements as follows: $A_1$ (0, 0, 1), $A_2$ (0.2, 0.5, 0.7), $A_3$ (0.6, 0.8, 0.2), $A_4$ (0.7, 0.9, 0.2), $A_5$ (0.9, 0.3, 0), etc. This option is useful when one has doubts about the credibility of the parameters assigned to the elements of the universal set, but it still depends on the objectivity of the choice of the corresponding neutrosophic triplets. On the basis of this option, a moral theory could be represented by a collection of NSs.

The next dilemma of the *searching assassin* [15] (pp. 187–195), however, in which the moral principle of not killing conflicts to the obligation, according to Kant [16], of telling always the truth, marks out more emphatically the need of introducing degrees of truth to the moral principles. This happens, because assassination is much more serious than telling a lie, since its consequences, i.e., the loss of a human life, cannot be corrected.

According to the dilemma of the searching assassin, stated by the contemporary to Kant French philosopher B. Constan, an individual who searches for his enemy on the purpose of killing him, comes to the house where his enemy is hidden, and asks the owner where he can find him. How moral could one characterize the possible answers of the owner of the house to the candidate assassin? Let us imagine some of these answers: $A_1$: I don't know him, $A_2$: I have not seen him during the last few days, $A_3$: He passed from here for a few minutes and then he left, $A_4$: He used to be here some time ago, but now he has left, $A_5$: He is hiding in the basement, etc. Obviously only the last of the previous answers is true. Consequently, with respect to the Kantian ethics and the principles of BL, this is the unique moral answer. Common logic, however, suggests that it is better to tell a lie to save a life. Considering, therefore, the SS (f, E) = {(e, f(e)): e ∈ E} in the universal set of all possible answers and with respect to the same with the previous example set E of parameters, one could put the previous answers $A_1$-$A_4$ to the set $f(e_2)$ and the answer $A_5$ to the set $f(e_1)$! Alternatively one could introduce the following neutrosophic triplets in the SVNS of the moral answers: $A_1$(0.3, 0.5, 0.6), $A_2$(0.4, 0.6, 0.4), $A_3$(0.5, 0.7, 0.3), $A_4$(0.8, 0.7, 0.2), $A_5$(0.2, 0, 0.8).

## 4. Conclusions

In this work the use of SSs and NSs was proposed for a mathematical representation of the ethical rules. To the best of our knowledge, this is the first integrated proposal for implementing FL (or another multi-valued logic) in ethics. This proposal, which reduces the moral dilemmas significantly, although it still leaves some doubts and space to personal beliefs, is much better than the outcomes of BL. Further research is needed, however, for obtaining safer results and conclusions.

**Author Contributions:** Conceptualization, M.G.V. and J.F.; methodology, M.G.V. and E.A.; validation, M.G.V.; formal analysis, M.G.V.; investigation, M.G.V., J.F. and E.A.; resources, M.G.V., J.F. and E.A.; data curation, M.G.V., J.F.; writing—original draft preparation, M.G.V.; writing—review and editing, M.G.V.; visualization, E.A.; supervision, M.G.V.; project administration, M.G.V. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

1. Thomson, J.J. Killing, Letting Die, and the Trolley Problem. *Monist* **1976**, *59*, 204–217.
2. Bentham, J. *An Introduction to the Principles of Morals and Legislation [1789]*; Dover Publications Inc.: Mineola, NY, USA, 2009.
3. Kosko, B. *Fuzzy Thinking: The New Science of Fuzzy Logic*; Hyperion: New York, NY, USA, 1993.
4. Papaioannou, Y. Fuzzy Logic in Science, Law and Ethics, 2013. Available online: https://blogs.elpais.com/atomium-culture/2013/01/fuzzy-logic-in-science-law-and-ethics.html (accessed on).
5. Athanassopoulos, E.; Voskoglou, M.G. A Philosophical Treatise on the Connection of Scientific Reasoning with Fuzzy Logic. *Mathematics* **2020**, *8*, 875.
6. Zadeh, L.A. Fuzzy Sets. *Inf. Control* **1965**, *8*, 338–353.
7. Zadeh, L.A. Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Syst. Man Cybern.* **1973**, *3*, 28–44.
8. Klir, G.J.; Folger, T.A. *Fuzzy Sets, Uncertainty and Information*; Prentice-Hall: London, UK, 1988.
9. Voskoglou, M.G. Generalizations of Fuzzy Sets and Related Theories. In *An Essential Guide to Fuzzy Systems*; Voskoglou, M., Ed.; Nova Science Publishers: New York, NY, USA, 2019; pp. 345–352.
10. Atanassov, K.T. Intuitionistic Fuzzy Sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96.
11. Smarandache, F. *Neutrosophy/Neutrosophic Probability, Set, and Logic*; Proquest: Ann Arbor, MI, USA, 1998.
12. Wang, H.; Smarandanche, F.; Zhang, Y.; Sunderraman, R. Single Valued Neutrosophic Sets. *Rev. Air Force Acad. (Brasov)* **2010**, *1*, 10–14.
13. Molodtsov, D. Soft set theory—First results. *Comput. Math. Appl.* **1999**, *37*, 19–31.
14. Maji, P.K.; Biswas, R.; Roy, A.R. Soft Set Theory. *Comput. Math. Appl.* **2003**, *45*, 555–562.
15. Sandel, M.J. *Justice. What Is the Right Thing to Do? [2009]*; Translated in Greek by Kioupkiolis, A.; Polis Editions: Athens, Greece, 2010.
16. Sullivan, R.J. *An Introduction to Kant's Ethics*; Cambridge University Press: Cambridge, NY, USA, 1994.