A New Software Reliability Growth Model with Testing Coverage and Uncertainty of Operating Environment

Presented by

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2. Software reliability growth model
   - SRGM with uncertainty of operating environment
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Introduction

- The probability of getting software without experiencing any fault for a while is known as software reliability.
- The quantitative software reliability measurement is expressed as the software reliability growth model (SRGM) during development.
- The software reliability growth model predicts software failure, software failure interval, release time, and failure rate.
- In 1979, Goel and Okumoto [1] proposed a software reliability growth model (SRGM) is called the exponential SRGM model and describes a software failure detection phenomenon.

[1] A. L. Goel and K. Okumoto, "Time-dependent error-detection rate model for software reliability and other performance measures," *IEEE Transactions on Reliability,* vol. 28, no. 3, pp. 206–211, 1979.

Introduction

- Time to time this model was modified by various researchers to develop a new SRGMs.
- Many software reliability growth models (SRGMs) [2],[3] have been proven to be successful in estimating the software reliability and the number of errors remaining in the software.
- Non-homogeneous Poisson process (NHPP) is one of the most effective mathematical tools to model software fault growth process.
- Many existing NHPP software reliability models have been used through the fault intensity rate function and the mean value functions $m(t)$ within a controlled testing environment to estimate reliability.

[2] M. Zhu and H. Pham, "A software reliability model with time-dependent fault detection and fault removal," *Vietnam Journal of Computer Science,* vol. 3, no. 2, pp. 71–79, 2016.

[3] X. Teng and H. Pham, "A new methodology for predicting software reliability in the random field environments," *IEEE Transactions on Reliability,* vol. 55, no. 3, pp. 458–468, 2006.

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Model Assumptions

For the proposed model, we consider the following assumptions:

1. The generation of fault in software follows the non-homogeneous Poisson process.
2. Fault detection rate is proportional to the remaining faults in the software.
3. After fault detection, the debugging process takes place immediately.
4. During the testing process, new faults are introduced into the software.
5. The testing coverage rate function is incorporated as the fault detection rate function.
6. Random testing environment affects the fault detection rate.

SRGM with uncertainty of operating environment

The assumption (6) is first suggested by Teng and Pham [4], and they have discussed that the models that capture the uncertainty of field environments are expressed as multiplying a random variable η with fault detection rate.

Considering above all the assumptions, the proposed model is

$$\frac{dm(t)}{dt} = \eta \frac{c'(t)}{1 - c(t)} \Big\{ N(t) - m(t) \Big\}, \; m(0) = 0. \tag{1}$$

where, $m(t)$ is total number of removed fault, $N(t)$ is total number of fault present at time t, η is random variable, $c(t)$ is testing coverage function.

[4] Teng X., Pham H., "A new methodology for predicting software reliability in the random field environments," *IEEE Transactions on Reliability*, 2006; 55(3): 458-468.

SRGM with uncertainty of operating environment

The fault content function is

$$N(t) = N + dm(t) \tag{2}$$

where d is the fault introduction rate.
The general solution for the MVF $m_\eta(t)$ is given by

$$m_\eta(t) = \frac{N}{1-d}\left(1 - e^{-\eta \int_0^t (1-d)\frac{c'(\tau)}{1-c(\tau)} d\tau}\right) \tag{3}$$

In order to find the mean value function $m(t)$, we have assume that, the random variable η follows Exponential distribution, i.e., $\eta \sim \exp(\alpha)$ and the probability density function of η is given by

$$f(\eta) = \alpha e^{-\alpha\eta}, \quad \alpha > 0 \text{ and } \eta \geq 0. \tag{4}$$

SRGM uncertainty of operating environment

An application of Laplace transformation of equation (3) using Exponential distribution for random variable η, the mean value function $m(t)$ is given by

$$m(t) = \frac{N}{1-d}\left\{1 - \left(\frac{\alpha}{\alpha + (1-d)\int_0^t \frac{c'(\tau)}{1-c(\tau)} d\tau}\right)\right\} \tag{5}$$

We have considered the following testing coverage rate function $c(t)$ as follows

$$c(t) = 1 - e^{1-c^{t^b}}, \; c > 1, \; b > 0 \tag{6}$$

After substituting $c(t)$ in equation (5), we obtained the following closed form of the solution of the mean value function $m(t)$ as

$$m(t) = \frac{N}{1-d}\left\{1 - \left(\frac{\alpha}{\alpha + (1-d)(c^{t^b} - 1)}\right)\right\}. \tag{7}$$

Different SRGM

Table 1 summarizes the MVF of the proposed model and other selected models, which are taken for comparison.

No	Model Name	Mean Value Function (MVF)
1	Goel-Okumoto model [1]	$m(t) = a(1 - e^{-bt})$
2	Delayed S-shaped model [4]	$m(t) = a(1 - (1 + bt)e^{-bt})$
3	Yamada ID model-I [8]	$m(t) = \frac{ab}{\alpha+b}(e^{\alpha t} - e^{-bt})$
4	Yamada ID model-II [8]	$m(t) = a(1 - e^{-bt})(1 - \frac{\alpha}{b}) + \alpha at$
5	Yamada et al. (YExp) [7]	$m(t) = a\left\{1 - e^{-\gamma\alpha(1-e^{-\beta t})}\right\}$
6	Yamada et al. (YRay) [7]	$m(t) = a\left\{1 - e^{-\gamma\alpha(1-e^{-\beta t^2/2})}\right\}$
7	Pham-Zhang model [6]	$m(t) = \frac{1}{1+\beta e^{-bt}}\left((c + a)(1 - e^{-bt}) - \frac{ab}{b-\alpha}(e^{-\alpha t} - e^{-bt})\right)$
8	Pham-Zhang ID model [5]	$m(t) = a(1 - e^{-bt})(1 + (b + d)t + bdt^2)$
9	Proposed model	$m(t) = \frac{N}{1-d}\left\{1 - \left(\frac{\alpha}{\alpha+(1-d)(c^{t^b}-1)}\right)\right\}$

Table 1: Summary of software reliability growth model

Numerical and data analysis

The first data set (DS-I) discussed in this paper is collected from the online IBM entry software package[5] and the second data set is collected from testing system T at $AT\&T$[6].

The sum of squared residuals is used for parameter estimation by minimizing the squares of the difference between the observed data and the mean value function, which is given by

$$\sum_{i=1}^{k} \{m(t_i) - y_i\}^2$$

where $m(t_i)$ and y_i are estimated cumulative number of failures and the cumulative number of failures at time t_i, respectively, and k is the number of observations in the model.

[5] Ohba M., "Software reliability analysis models," *IBM Journal of Research and Development*, vol. 28, pp. 428-443, 1984.

[6] Ehrlich W., Prasanna B., Stampfel J., Wu J, "Determining the cost of a stop-test decision (software reliability)," *IEEE Software*, vol. 10, pp. 33-42, 1993.

Numerical and data analysis

The criteria used to compare our model with the selected existing models are mean-squared error (MSE) [2], predictive ratio risk (PRR) [2], bias [3], variance [3], and root mean square prediction error (RMSPE) [3].

The smaller value of all goodness-of-fit criteria gives a better software reliability model.

The estimated parameter value for DSI of the proposed model is $\hat{N} = 3.1569, \hat{d} = 0.9527, \hat{\alpha} = 6.5911, \hat{b} = 0.4806, \hat{c} = 3.8052$ and for DSII is $\hat{N} = 29.773, \hat{d} = 0.001, \hat{\alpha} = 0.0006, \hat{b} = 1.082, \hat{c} = 1.0001$.

Numerical and data analysis

The proposed model's performance in terms MSE, PRR, Bias, Variance, and RMSE criteria is shown in Table 2. The result shows that the proposed model fits the DSI better as compared to other selected models.

No	MSE	PRR	Bais	Variance	RMSE
1	8.973872	1.019703	0.890848	3.452888	3.565957
2	1.480686	26.32063	-0.23211	1.313173	1.333528
4	4.057239	0.371741	-0.65302	2.06001	2.164841
5	1.458347	2.676588	-0.05303	1.241017	1.242149
6	6.245228	0.822893	0.844226	2.56076	2.696332
7	1.959292	55.68604	-0.39852	1.434314	1.488648
8	1.278945	2.729673	-0.0696	1.165397	1.167473
9	1.533603	40.66148	-0.25021	1.344427	1.367511
10	**1.168283**	**0.381616**	**-0.04598**	**1.105182**	**1.106138**

Table 2: Comparison results for different models on DSI

Numerical and data analysis

Table 3 shows that the proposed model performs better regarding MSE, PRR, Bias, Variance, and RMSE criteria for DSII.

No	MSE	PRR	Bais	Variance	RMSE
1	0.837889	0.126591	0.294310	1.087284	1.126412
2	1.409615	0.385109	-0.12266	1.251662	1.257659
3	1.394307	0.144932	-0.16151	1.225382	1.235980
4	0.718011	0.146291	0.168120	0.929816	0.944893
5	0.746532	0.135072	0.061377	0.896637	0.898735
6	2.027924	1.355159	-0.17989	1.477809	1.488717
7	1.526415	0.118110	0.879679	2.035660	2.217600
8	1.969308	2.921449	-0.17227	1.488848	1.498781
9	**0.708826**	**0.117530**	**0.051773**	**0.878641**	**0.880165**

Table 3: Comparison results for different models on DSII

Numerical and data analysis

For DSI, the comparison between the proposed and selected model's MVF is depicted in Figure 1. Figure 2 shows the relative errors for different models.

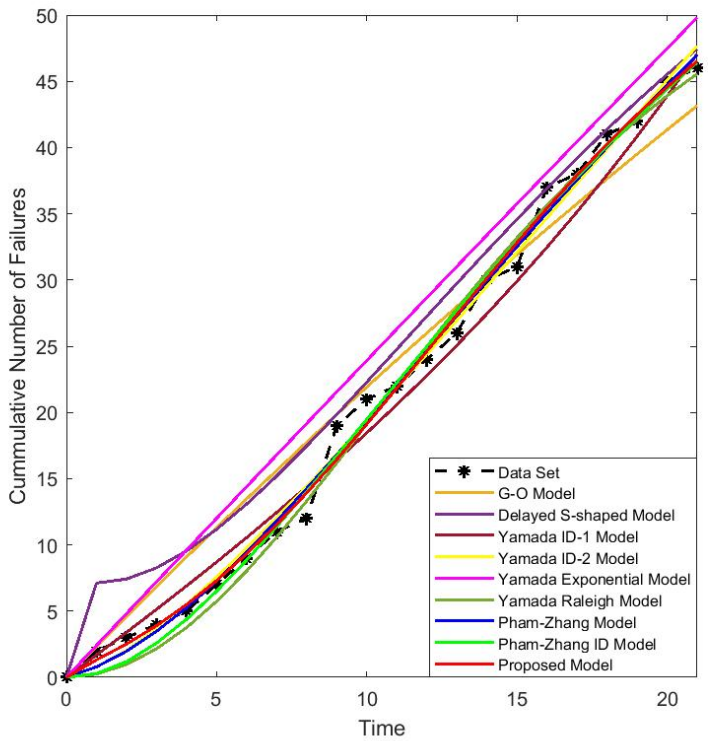

Figure 1: Comparison of actual cumulative failures and predicted failures (DSI)

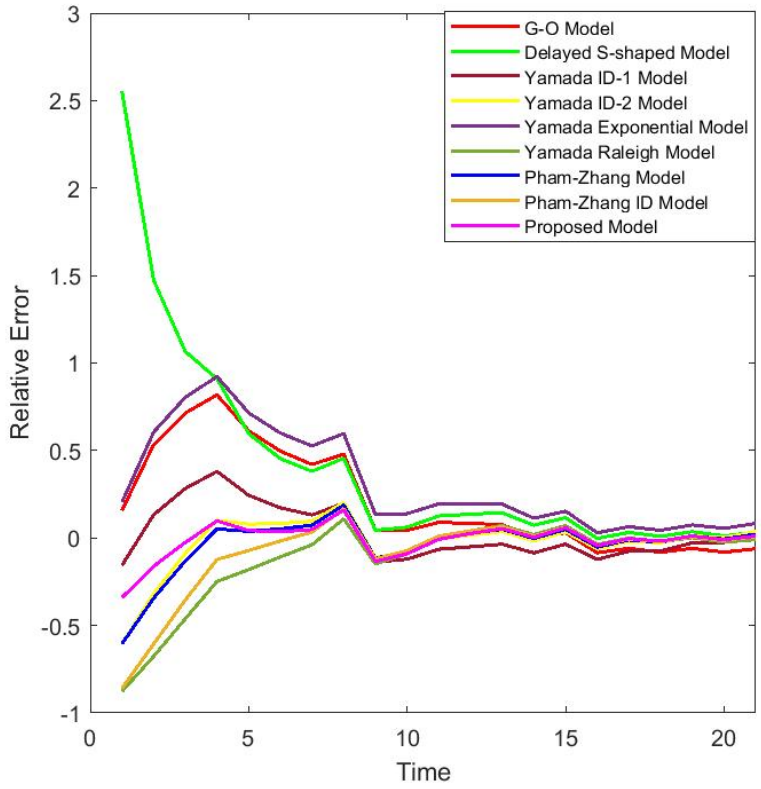

Figure 2: Relative errors vs Time (DSI)

Numerical and data analysis

Figure 3 shows the comparison between the proposed and selected model's MVF for DSII. The relative errors for different models is shown in Figure 4.

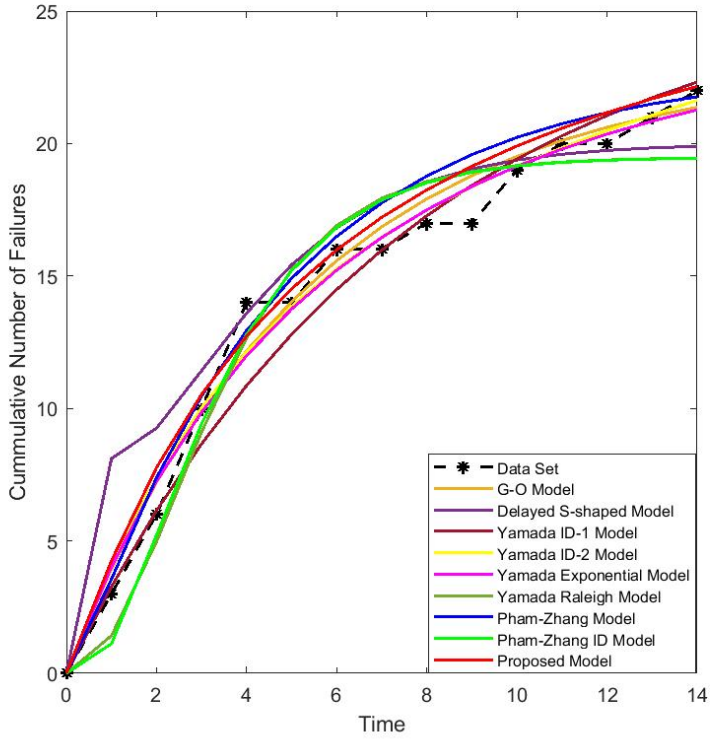

Figure 3: Comparison of actual cumulative failures and predicted failures (DSII)

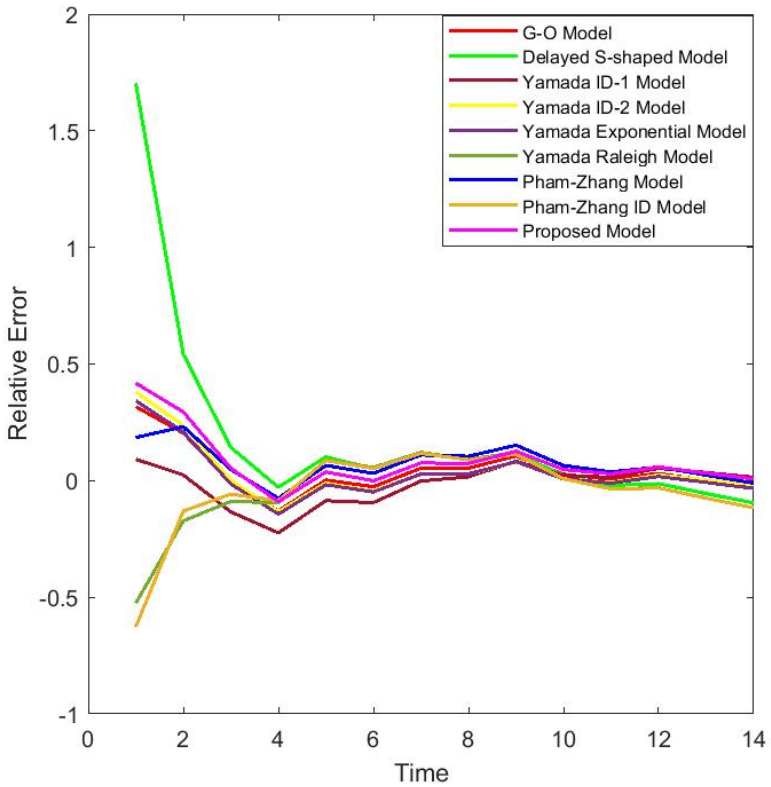

Figure 4: Relative errors vs Time (DSII)

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Conclusion

- We have presented a new SRGM that incorporates both testing coverage and the impact of the testing environment on the model.
- We observed from the numerical experiments that the proposed model gave better goodness-of-fit than the existing selected models.
- We concluded that the proposed model is more practical and better than other existing selected models.

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Thank you!