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Proceeding Paper

A stochastic bilevel DEA-based model for resource allocation †

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† **The 1st International Online Conference on Mathematics and Applications, 1-15 of May 2023**.

Abstract: Optimal allocation of limited resources along with output target setting are critical in pursuing sustainability and competitiveness of organizations. The process of resource distribution is usually implemented through a central unit that routes resources to the subordinate decision-making units (DMUs) along with DMUs lower bounds of desired efficiency. Moreover, the central unit has the authority to set the overall expected output targets so as to maximize organizational effectiveness. In this paper, we investigate evaluation efficiency issues using a type of bilevel network data envelopment analysis (DEA) approach in a stochastic framework. The proposed bilevel DEA model takes into account stochastic conditions and optimizes centralized resource allocation and target setting imposing lower bounds on the efficiencies of all DMUs affiliated to the organization. Consequently, the total input consumption is minimized and the total output production is maximized while considering additional bounds and availability constraints for inputs. In the proposed bilevel model, uncertainty is introduced through the upper level (leader) problem that attempts to maximize organizational effectiveness while in the lower level (follower) problem it evaluates the efficiency of the DMUs. A solution methodology for the bilevel network DEA-based model is presented and numerical results are obtained using data from the literature. The obtained results are compared with those published in other case studies for centralized resource allocation DEA models.

Keywords: Bilevel optimization; DEA; Stochastic environment; Resource allocation

Citation: Lastname, F.; Lastname, F.; Lastname, F. Title. *Proceedings* **2021**, 68, x. https://doi.org/10.3390/xxxxx

1. Introduction

To increase their competitiveness and enhance their sustainability, decision-making units (DMUs) seek to allocate their resources optimally, maximize their output towards some rational targeting and evaluate their efficiency systematically. At the same time, organizational resource allocation is of great importance not only because resources are limited, but due to the fact that resource allocation has a serious impact on effectiveness, target setting and production planning. The process of resource distribution and target setting in organizations is usually implemented through a central unit that decides upon the resources supplied to the subordinate decision-making units (DMUs) along with DMUs lower bounds of desired efficiency. Moreover, the central unit has the authority to set the overall expected output targets so as to maximize the organizational effectiveness.

Data Envelopment Analysis (DEA) is a mathematical programming technique that has been extensively used for measuring the relative efficiency of homogeneous DMUs with multiple inputs and outputs [1-2]. Traditional DEA models can assess the efficiency of individual DMUs. However, they are not suitable when one considers the scenario of multiple DMUs operating under the control of a higher-level, supervising entity. In such a scenario it is common that the higher-level entity seeks to maximize the efficiency of each DMU and at the same time minimize the total input consumption and/or maximize the total output production. This type of network structure is typical in organizations consisting of a central unit that makes centralized decisions affecting a number of subordinate decision-making units. The central unit decides for the allocation of limited resources to the DMUs and sets the output targets which are influenced by the number of resources distributed to the DMUs. Typical organizations that exhibit this structure might include banks, hospitals, universities, etc.

DEA has been widely used for centralized resource allocation and various DEA based resource allocation models have been published in the relevant literature. Beasley [3] presented DEA based models for allocating fixed costs and input resources to DMUs as well as for target setting. Moreover, he reinterpreted DEA as a holistic selection of weights for all DMUs so as to maximize the average efficiency while in traditional DEA weights are chosen separately for each DMU. Lozano and Villa [4] present two DEA models for centralized resource allocation which minimize total input consumption or maximize total output production while considering the efficiency of the individual DMUs. Wu [5] presented a bilevel DEA model that optimizes the firm performance in decentralized companies. The model allocates resources between the two stages of a DMU, the (first) stage of the leader and the (second) stage of the follower, in a cost-efficient way. Hakim [6] proposed a bilevel DEA model for centralized resource allocation, where the organizational efficiency is maximized satisfying at the same time a lower bound on the efficiency of each DMU. Ang et al. [7] propose two-stage DEA models with bilevel formulations where the upper-level maximizes the organizational effectiveness while the lower-level model constrains the efficiency of all DMUs simultaneously. Furthermore, they consider two-stage DMUs, where inputs of the first stage are converted into intermediate measures which in turn are converted into outputs in the second stage.

In this paper, we evaluate organizational efficiency using a bilevel network data envelopment analysis (DEA) approach in a framework that introduces an aspect of uncertainty. The proposed model is based on the bilevel DEA model presented in Hakim et al. [6] and includes stochastic conditions. It attempts to optimize centralized resource allocation and target setting by imposing scenarios of lower bounds on the efficiencies of all DMUs belonging to the organization. Consequently, the total input consumption is minimized and the total output production is maximized at the same time, while additional bounds and availability constraints (that is a stochasticity dimension) for inputs are considered. Concretely, in this bilevel DEA model, stochasticity takes the form of discrete scenarios associated with a user-defined occurrence probability. Each discrete scenario sets a value to the stochastic parameter of the model regarding input availability. Then the expected total benefit for an organization is determined through a weighted average of the obtained optimal solutions based on the scenarios and their realization probabilities.

2. A stochastic bilevel DEA model for centralized resource allocation

Roughly speaking, a *bilevel program* is a mathematical programming problem whose feasible space encapsulates the parametric solution of another mathematical program. Moreover, such a structure consists of an upper-level optimization problem (leader's problem) and a lower-level optimization problem (follower's problem). Objective function, decision variables and parameters are defined for the upper and the lower-level problems, respectively. Bilevel Programming has been used to model complex network structures of DMUs in a broad area of fields such as banking, engineering, supply-chain management, ecology, etc. [8].

Conventional DEA is a popular method for efficiency evaluation of DMUs where all input and output data used are assumed to be accurate and deterministic. However, in many real-world problems, input and output data may be erroneous and unavailable due to information loss, human errors and lack of historical data. For this reason, conventional DEA models have received a great deal of criticism leading to a variety of extensions that handle this drawback (such as fuzzy logic, stochastic approaches, models for imprecise data etc.) [9]. Apart from uncertainty in data, a solution may turn to infeasible or suboptimal when it comes to implementation. Ben-Tal et al. [10] showed that even a small perturbation in data can lead to a considerable change in feasibility of the optimal solution.

For DMU efficiency evaluation with uncertain data, two approaches are the most common, robust and stochastic optimization. Soyster [11] set the foundations for robust optimization, by assigning each uncertain parameter in convex programming problems to its worst-case value within a set. Ben-Tal and Nemirovski ([10,1212]) and El-Ghaoui and Lebret [13], who considered this approach as too conservative, allowed the uncertain parameters to uncertainty sets without the most unlikely values and derived tractable mathematical programs. Bertsimas et al. ([14,15]) proposed a robust optimization approach where the constructed problems remain in the same class. Mulvey et al. [16] presented an approach that uses goal programming with a description of problem data based on scenarios. The number of studies that deal with stochasticity in the DEA framework is continuously growing. Omrani et al. [17,18] developed a multi-objective DEA model to determine three types of efficiency, i.e., profitability, operational, and transactional for bank branches with uncertain data. The uncertainty in data is treated with discrete scenarios. The proposed models are tested in the case of 45 Iranian Agriculture bank branches under four different scenarios. Shakouri et al. [19] present a p-robust DEA model to evaluate the efficiency of DMUs under uncertainty in data where input parameters are given from different scenarios. Their model is tested for efficiency assessment of Iranian banking sector. Moreover, Shakouri et al. [20] presented Network DEA models based on Stackelberg and game theory under uncertainty. They too applied their models in an analysis of bank branch performance.

In this paper, we propose a stochastic bilevel DEA-based model which maximizes the overall efficiency considering at the same time a lower bound for the efficiency of the DMUs. In the proposed bilevel model there are two submodels, the upper-level model and the lower-level model. The upper-level model determines the inputs and the outputs which optimize the overall efficiency by maximizing the total benefits (total outputs minus total inputs). The lower-level model computes the weights associated with the inputs and the outputs that maximize the efficiency of each subordinate DMU. Our approach exploits the leader-follower relations in the bilevel framework that cannot be easily captured otherwise. To take into consideration the uncertainty aspect, we discretize the stochastic nature of resources upper bounds in the bilevel problem using the approach of different scenarios. Each discrete scenario sets a value to the stochastic parameter of the model regarding input availability along with a probability of realization that reflects the decision maker's confidence or aspiration of the specific scenario to finally be realized. Throughout the paper, we use the following notations and definitions:

Notation	**Definitions**
n	the number of DMUs
m	the number of input resources
s	the number of output targets
p_r	unit price for output r
q_i	unit price for input i
${e_k}^*$	the optimal efficiency score for DMU k
e_{kj}	cross-efficiency of DMU j with respect to DMU k
X_{ij}	the observed input i for DMU j

Y_{rj}	the observed output r for DMU j
Le_k	the lower bound for efficiency of DMU k
Lx_{ik}	the lower bound for input resource i of DMU k
Ux_{ik}	the upper bound for input resource i of DMU k
Ly_{rk}	the upper bound for input resource i of DMU k
Uy_{rk}	the upper bound for input resource i of DMU k
x_{ik}^t	the input resource i for DMU k
y_{rk}^t	the output target r for DMU k
v_{ik}^t	the weight attached to input resource i of DMU k
u_{rk}^t	the weight attached to output target r of DMU k
w^t	occurrence probability of scenario t

Suppose we have n DMUs which have a bilevel network structure and each DMU j $(j = 1, \dots, n)$ uses m inputs x_{ik} to produce s outputs y_{rk}. Let $T = \{1, \dots, N\}$ be a set of discrete scenarios where each of them has an occurrence probability w^t. For some scenario t∈T, the upper-level optimization model for DMU k $(k = 1,2, \dots, n)$ is the following:

$$\max_{x_{ik}^t, y_{rk}^t, \lambda_{jk}^t} \sum_{r=1}^{s} p_r \sum_{k=1}^{n} y_{rk}^t - \sum_{i=1}^{m} q_i \sum_{k=1}^{n} x_{ik}^t \tag{1}$$

s.t.

$$Le_k \le e_k{}^* \qquad (k = 1,2, \dots, n) \tag{2}$$

$$x_{ik}^t \ge \sum_{j=1}^{n} \lambda_{jk}^t X_{ij} \qquad (i = 1,2, \dots, m; k = 1,2, \dots, n) \tag{3}$$

$$y_{rk}^t \le \sum_{j=1}^{n} \lambda_{jk}^t Y_{rj} \qquad (r = 1,2, \dots, s; k = 1,2, \dots, n) \tag{4}$$

$$\sum_{j=1}^{n} \lambda_{jk}^t = 1 \qquad (k = 1,2, \dots, n) \tag{5}$$

$$\lambda_{jk}^t \ge 0 \qquad (j = 1,2, \dots, n; k = 1,2, \dots, n) \tag{6}$$

$$\sum_{k=1}^{n} x_{ik}^t \le b_i^t \qquad (i = 1,2, \dots, m) \tag{7}$$

$$Lx_{ik} \le x_{ik}^t \le Ux_{ik} \qquad (i = 1,2, \dots, m; k = 1,2, \dots, n) \tag{8}$$

$$Ly_{rk} \le y_{rk}^t \le Uy_{rk} \qquad (r = 1,2, \dots, s; k = 1,2, \dots, n) \tag{9}$$

In the objective function (1) the unit costs p_r of the inputs and the unit prices q_i of the outputs are determined by the central unit. The optimal value of the objective function is denoted with z^t. Constraint (2) ensures that the efficiency of each subordinate DMU satisfies a lower bound set by the central unit. Constraints (3)-(4) ensure that the new input resources and the output targets belong to the production possibility set constructed by the observed inputs-outputs of all DMUs. The upper-level model is based on the variable returns to scale (VRS) assumption due to constraint (5), however it can also assume constant returns-to-scale (CRS) ignoring the latter constraint. Constraint (7) restricts availabilities of resources with the stochastic parameter b_i^t. Constraints (8)-(9) set the lower and upper bounds for input resources and output targets, respectively. The lower-level optimization model for DMU k $(k = 1,2, \dots, n)$ under scenario t is the following:

$$e_k^* = \max_{v_{ik}^t, u_{rk}^t, l_k^t} \frac{\sum_{r=1}^{s} u_{rk}^t y_{rk}^t - l_k^t}{\sum_{i=1}^{m} v_{ik}^t x_{ik}^t} \tag{10}$$

s.t.

$$0 \leq \mathrm{e_{kj}} = \frac{\sum_{r=1}^{s} u_{rk}^t y_{rk}^t - l_k^t}{\sum_{i=1}^{m} v_{ik}^t x_{ik}^t} \leq 1 \qquad (j = 1,2,\dots,n) \tag{11}$$

$$u_{rk}^t \geq 0 \qquad (r = 1,2,\dots,s) \tag{12}$$

$$v_{ik}^t \geq 0 \qquad (i = 1,2,\dots,m) \tag{13}$$

The lower-level model is the standard DEA model as presented in Beasley [2]. The objective function (10) maximizes the efficiency of each DMU k. Constraint (11) restricts cross efficiency to take values between zero and one. The cross-efficiency e_{kj} is defined to be the efficiency of DMU k when it is evaluated using the weights that are used to compute the efficiency of DMU j. Constraints (12)-(13) impose the nonnegativity of the input and output weights. Due to the existence of the free variable l_k^t, the lower-level model computes the variable returns to scale efficiency of DMU k.

The stochastic bilevel DEA model is a non-linear programming problem which cannot be solved in its bilevel form. Thus, the proposed bilevel DEA model is converted to a single level optimization problem according to Theorem 1 in [5]. The single level problem for the proposed model is as follows:

$$\max_{x_{ik}, y_{rk}, \lambda_{jk},} \sum_{r=1}^{s} p_r \sum_{k=1}^{n} y_{rk}^t - \sum_{i=1}^{m} q_i \sum_{k=1}^{n} x_{ik}^t \tag{14}$$

s.t.

$$Le_k \leq e_{kk} = \frac{\sum_{r=1}^{s} u_{rk}^t y_{rk}^t - l_k^t}{\sum_{i=1}^{m} v_{ik}^t x_{ik}^t} \qquad (k = 1,2,\dots,n) \tag{15}$$

$$x_{ik}^t \geq \sum_{j=1}^{n} \lambda_{jk}^t X_{ij} \qquad (i = 1,2,\dots,m; k = 1,2,\dots,n) \tag{16}$$

$$y_{rk}^t \leq \sum_{j=1}^{n} \lambda_{jk}^t Y_{rj} \qquad (r = 1,2,\dots,s; k = 1,2,\dots,n) \tag{17}$$

$$\sum_{j=1}^{n} \lambda_{jk}^t = 1 \qquad (k = 1,2,\dots,n) \tag{18}$$

$$\lambda_{jk} \geq 0 \qquad (j = 1,2,\dots,n; k = 1,2,\dots,n) \tag{19}$$

$$\sum_{k=1}^{n} x_{ik}^t \leq b_i^t \qquad (i = 1,2,\dots,m) \tag{20}$$

$$Lx_{ik} \leq x_{ik}^t \leq Ux_{ik} \qquad (i = 1,2,\dots,m; k = 1,2,\dots,n) \tag{21}$$

$$Ly_{rk} \leq y_{rk}^t \leq Uy_{rk} \qquad (r = 1,2,\dots,s; k = 1,2,\dots,n) \tag{22}$$

$$0 \leq e_{kj} = \frac{\sum_{r=1}^{s} u_{rk}^t y_{rk}^t - l_k^t}{\sum_{i=1}^{m} v_{ik}^t x_{ik}^t} \leq 1 \qquad (k = 1,2,\dots,n; j = 1,2,\dots,n) \tag{23}$$

$$u_{rk}^t \geq 0 \qquad (k = 1,2,\dots,n; r = 1,2,\dots,s) \tag{24}$$

$$v_{ik}^{t} \geq 0 \qquad (k = 1,2,\dots,n; i = 1,2,\dots,m) \qquad (25)$$

The single-level problem is a non-linear/non-convex programming problem due to constraints (15) and (23). The superscript t in the variables of the single-level model is used mostly for notational convenience, since the model is separable over each scenario t. To estimate the optimal strategy for an organization concerning centralized resource allocation and target setting, we compute the expected total profit, that is a weighted average of the total optimized profits using as weights the occurrence probability for all scenarios.

3. Computational results

In this section, we present preliminary computational results that we obtained by solving the proposed bilevel model for an example using data that appeared first in [18]. The single-level DEA-based model is implemented in Python with the use of the Pyomo library and solved on a PC with 16GB RAM and CPU 2.6 GHz.

In this example, 10 DMUs are considered which consume two inputs and produce two outputs. We consider three scenarios under uncertainty conditions, with realization probabilities $w^1 = 0.3,\ w^2 = 0.2,\ w^3 = 0.5$. For each scenario we have set the following upper bounds for the availability of the two inputs $b_1^1 = 98,\ b_2^1 = 90,\ b_1^2 = 100,\ b_2^2 = 110,\ b_1^3 = 90$ and $b_2^3 = 100$. The input and output costs and prices for the two inputs and the two outputs are $q_1 = 2,\ q_2 = 2,\ p_1 = 10,\ p_2 = 5$. The upper and lower bounds for inputs and similarly for outputs are determined with the following rule: the upper input bound Ux_{ik} (upper output bound Uy_{rk}) is 110% of the input x_{ik}^t (output y_{rk}^t) and the lower input bound Lx_{ik} (lower output bound Ly_{rk}) is the 90% of the input x_{ik}^t (output y_{rk}^t). Furthermore, the lower-bound efficiencies for the ten DMUs are from Table 5 in [5]. The data we use for inputs and outputs are discretionary and non-categorical and are presented in Table 2 in [5]. The results obtained for each of the three scenarios are shown in **错误!未找到引用源。**. These results describe the optimal allocation of resources and output targets achievement for an organization with 10 subordinate DMUs under each scenario. The profits are 198.533 for scenario 1 and 2 and 188.046 for scenario 3. The expected total profit is 193.29.

4. Conclusions

The stochastic bilevel DEA model that we presented considers leader-follower relations, the desirable efficiency for DMUs and data uncertainty at the same time. One of the main advantages of this model is that it enables decision-makers to obtain an optimal strategy for resource allocation and output targeting in a holistic manner taking into consideration the efficiency objectives of DMUs. To overcome the difficulties in estimating the future optimal organizational benefits when data uncertainty is taken into account, we rely on an approach that considers discrete scenarios for each of the stochastic model parameters in order to determine the total expected profit. The expected profit of each scenario is realized using some discrete probability distribution determining the occurrence event of each scenario. Of course, there exist limitations and several issues need to be further investigated. The number of scenarios that must be designed ex ante and the computational needs of the problem for a large number of scenarios are under investigation. Another issue is the development of a 2-stage stochastic model with recourse action and emphasis on observed inputs/outputs when it comes to uncertainty. Finally, additional work is needed in the process of actually specifying the scenarios and in improving the existing solution methodologies for the stochastic bilevel DEA model.

Table 2. Input and output results for the three scenarios

DMUs	x_{1k}^1	x_{2k}^1	y_{1k}^1	y_{2k}^1	x_{1k}^2	x_{2k}^2	y_{1k}^2	y_{2k}^2	x_{1k}^3	x_{2k}^3	y_{1k}^3	y_{2k}^3
1	8.1	8.1	2.2	1.1	8.1	8.1	2.2	1.1	8.1	8.1	2.2	1.1
2	10.8	7.2	3.3	1.1	10.8	7.2	3.3	1.1	10.8	7.2	3.3	1.1

3	6.3	10.8	2.2	2.2	6.3	10.8	2.2	2.2	6.3	10.8	2.2	2.2
4	6.6	10.0	5.1	3.3	6.6	10.0	5.1	3.3	6.0	10.0	5.0	3.0
5	10.4	5.5	4.4	4.1	10.4	5.5	4.4	4.1	9.6	5.5	4.11	3.89
6	7.2	9.0	3.3	3.3	7.2	9.0	3.3	3.3	7.2	9.0	3.3	3.29
7	12.3	9.4	6.3	5.4	12.3	9.4	6.3	5.4	10.8	10.0	5.8	5.4
8	14.0	6.0	8.0	2.0	14.0	6.0	8.0	2.0	12.7	6.6	7.45	2.19
9	10.8	10.8	1.1	5.7	10.8	10.8	1.1	5.7	10.8	10.8	1.1	5.7
10	8.4	8.2	3.3	5.1	8.4	8.2	3.3	5.1	7.7	8.3	3.3	4.7

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