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Efficacy of FRP hooping in masonry domes: a simple numerical approach†

Alessandro Gandolfi [1]*, Natalia Pingaro [1] and Gabriele Milani [1]

1 Department of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133, Milan (Italy)

* Correspondence: alessandro.gandolfi@polimi.it

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Abstract: A simple numerical approach to predict the efficacy of FRP hooping in historical masonry domes is presented. The dome is modelled with 8-noded elastic hexahedron elements connected by 1D trusses/springs on meridians and on parallels, where all the non-linearity takes place. The aim is to simulate the nonlinear behaviour of domes through a FEM commercial software equipped only with non-linear 1D elements, namely point contacts and cutoff bars. The constitutive behaviour of the trusses is assumed either perfectly brittle or perfectly ductile. A possible orthotropic behaviour and the no-tension material case can be reproduced. An external retrofitting is simulated using trusses with an elastic-perfectly ductile behaviour, assuming a perfect bond between substrate and reinforcement and imposing an ultimate strength for the trusses which takes into account in a conventional way the possible debonding/delamination from the substrate. The Italian code CNR DT200 and the existing specialized literature are used as reference. The models are benchmarked on a masonry dome reinforced with three hooping FRP strips and experimentally tested at the University Architecture Institute of Venice IUAV, Italy. The procedure is validated through extensive comparisons with available experimental data and numerical results obtained in the literature with a variety of different models. By the extensive comparisons carried out and discussed, the robustness and simplicity of the procedure are proven.

Keywords: FRP; hoop reinforcement; masonry domes; non-linearity; collapse load; FEM; point contact; plastic hinge; cutoff bars

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1. Introduction

The verification methods here presented aim at determining the collapse load of a hemispherical masonry dome subjected to axisymmetric load on the crown. The same method is considered for the reinforcement of historical or overloaded domical structures by means of FRP strips.

The case study is a physical model with the same characteristics (about 2.2m in diameter, 0.12m thick with 0.20m wide oculus on the top) and mechanical properties as detailed in [1]. It was built and tested under vertical load and in case of reinforcement until collapse in the laboratory of the IUAV in Venice [2]. From these few mechanical tests forward, a series of numerical [1], [3]–[8] and analytical [7], [9] computations have been performed to find the ultimate collapse load and the position of plastic hinges.

The method finds its significance and utility for both research and professional works. Indeed, all the methods found in the literature are in the abovementioned cases expensive in terms of time and computational capacity or neglect important features of masonry [10], while the proposal here exposed is fast, it can be implemented in a simple finite-elements (FE) software and considers masonry tensile resistance and orthotropy.

In the next paragraphs, the reader may find brief information about two ways of modelling for nonlinear analysis (involving brittle and ductile elements) and consequent results. Eventually, the second method of elastic perfectly ductile elements is applied to simulate FRP strips behaviour and demonstrate their efficacy.

2. Modelling Methods

The modelling and the nonlinear analysis have been done in Strand7 environment, a commercial software. A meridian slice of a hemispherical dome (the reader may find an example in Figure 2.a) has been taken to speed up computations and, eventually, the results have been made commensurate to the whole dome by simple math.

To replicate the axisymmetric load applied to the physical model, a point load on the upper crown, close to the oculus is set by imposing a unit displacement.

Considering that failures in such constructions often occurs at mortar joints level and not in the clay bricks, these have been modelled as elastic hexahedral elements (3D) and are never expected to fail. As masonry domes show material and geometrical nonlinearities, these have been lumped in mortar joints, modelled by 1D Finite Elements – already implemented in the software used –, namely Point Contacts (PC) and Cutoff Bars (CoB). The next two paragraphs detail and explain the choice.

2.1. *Elastic perfectly brittle joints: Point Contact*

In the first way of modelling, joints nonlinearity is set by elastic perfectly brittle PC, which are categorised as 1D "beam elements" [11] and used under Heyman's hypotheses of no-tension materials and small displacements hypothesis. They are set to work in compression only. No sliding is accounted.

2.1.1. Results

The results of such a modelling are shown in the sensitivity analysis in Figure 1. It compares the collapse loads resulting from small changes in the tensile resistance assigned to horizontal joints only ($f_{T,h}$), while vertical joints were set to be no-tension. The curves show some mutual difference in their behaviour in the first steps of the simulation (rectangle a in Figure 1), with progressive loss of structural stiffness and development of plastic hinges. While in the steps toward the end, they tend to the asymptote value given by the lower-bound Limit Analysis (LA) found in [9] (rectangle b in Figure 1).

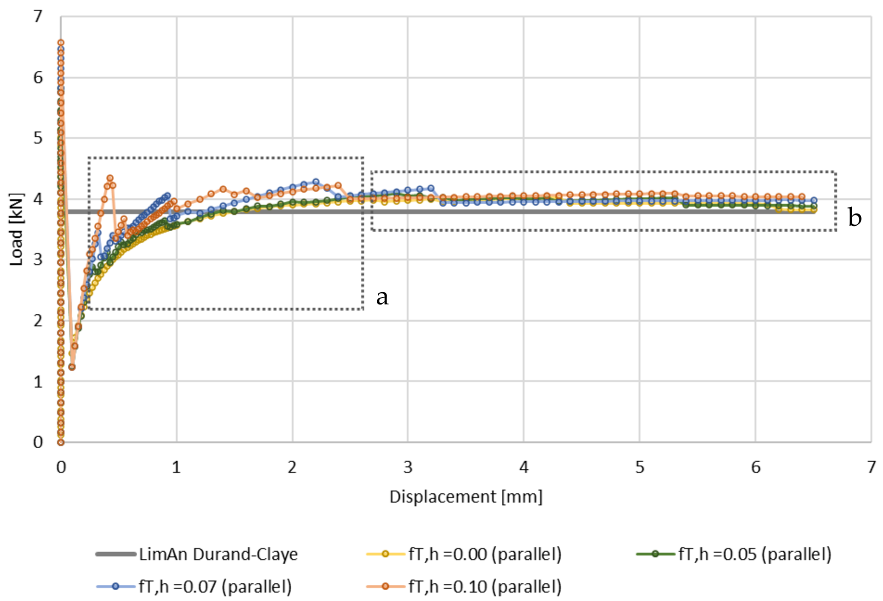
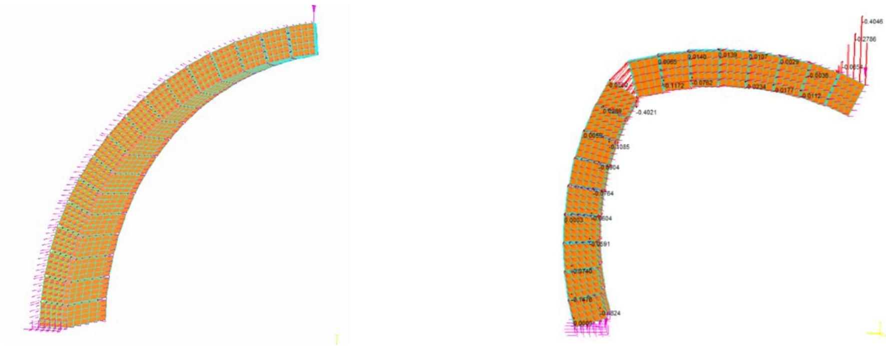


Figure 1: sensitivity analysis for no-tension material hypothesis model (brittle PC).

In spite of a largely conservative collapse load value, such an analysis gives an accurate position of plastic hinge (at 45° from the vertical axis), visible in the deformed shape in Figure 2.b.



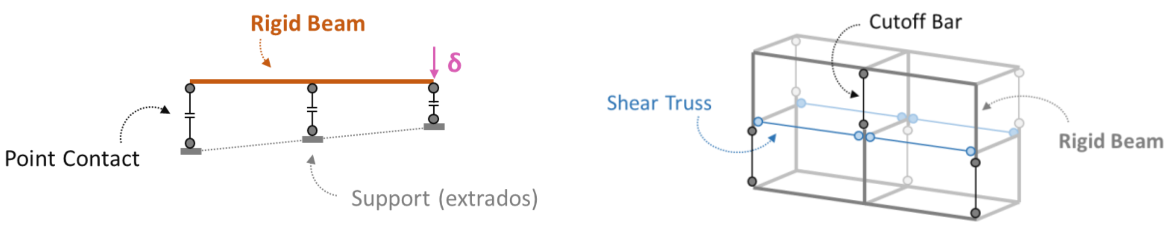
(a) (b)

Figure 2: (a) initial meridian slice of hemispherical dome reported as example. (b) Formation of the intermediate plastic hinge at about 45° from the vertical axis. Front view of the meridian slice.

2.2. *Elastic perfectly ductile joints: Cutoff Bars*

The second type of model sees nonlinear joints configured as elastic perfectly ductile truss elements (1D) [11] with predefined tensile and compressive strength, namely CoB. Cutoff values for tension and compression are suitably tuned to simulate masonry orthotropy.

Additionally, a rigid base larger than that of a point load (a scheme in Figure 3.a) is added to simulate the load distribution of a real architectural element on the crown of the dome. Then, to prevent out-of-plane sliding, shear resistance has been provided by a complex joint construction involving rigid beams, CoB and shear trusses (may the reader refer to Figure 3.b for major clarity). The failure of this structure enlightens the formation of plastic hinge. For seek of brevity, no more information will be reported here.



(a) (b)

Figure 3: (a) scheme of the load pattern for load distribution; (b) scheme of the shear resisting joint.

2.2.1. Results

The results of the second model are shown in the essential sensitivity analysis of Figure 4. Each curve results from tuning the tensile resistance values along horizontal ($f_{T,paralel}$) and vertical ($f_{T,meridian}$) directions. The values, collected in Table 1, are always multiplied by the influence area of the joint, which depend on the position along the meridian.

Table 1: data for an essential sensitivity analysis

$f_{T,parallel}$ [MPa]		$f_{T,meridian}$ [MPa]
Upper	Haunch	

1	0.00	0.00	0.00
2	0.07	0.07	0.05
3	0.05	0.05	0.08
4	0.05	0.07	0.12

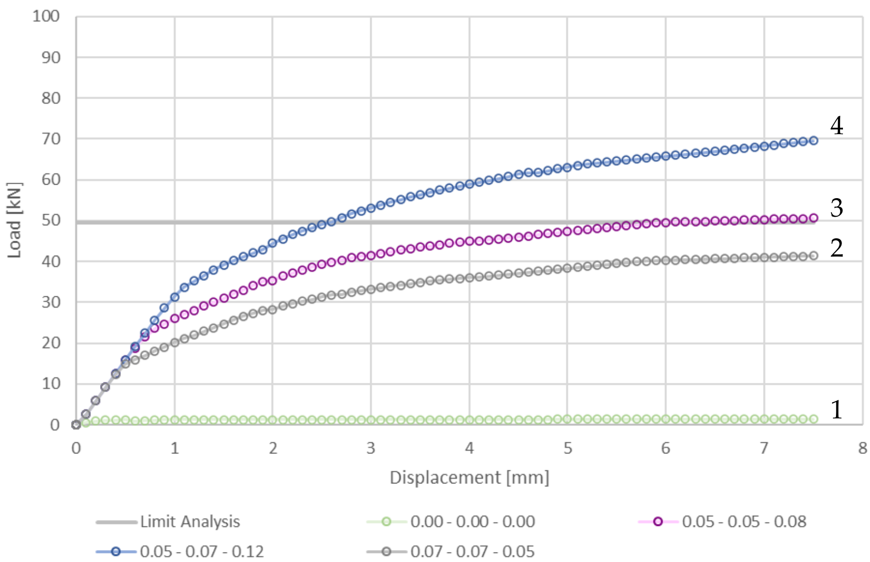


Figure 4: Essential sensitivity analysis accounting orthotropy (ductile CoB).

In Figure 4, it can be noted that a sensible increase in load bearing capacity can be achieved by acting on the $f_{T,meridian}$. The comparison with no-tension material case (curve 1) shows a far higher capacity when tensile resistances are accounted. The other curves represent the significant cases of the tuning of variables considering orthotropy and leading to the accordance with literature above lower-bound LA [7] (Curve 3).

In Figure 7.a, the deformed shape of the meridian slice is shown. As can be seen, there is no evident position of intermediate plastic hinge, hence it is said to be "smeared".

The chart in Figure 5 shows the validation of the best result from the sensitivity analysis against those coming from FE ([1], [7], [8]) and DE ([3], [4]) approaches from literature.

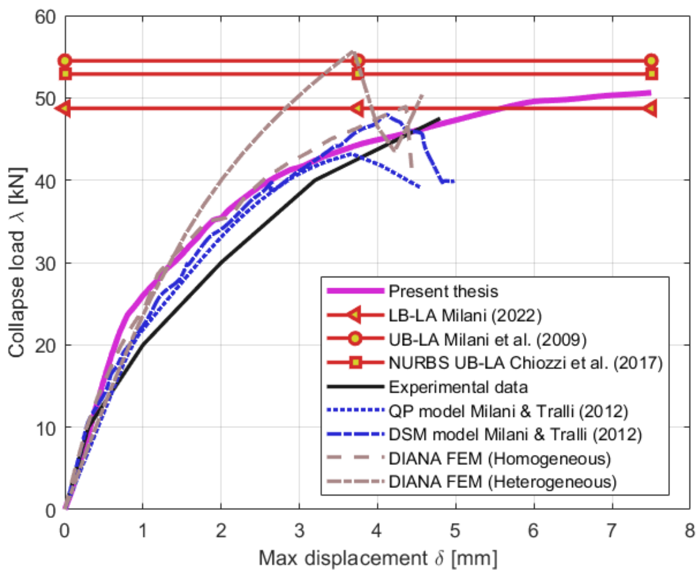


Figure 5: Validation of the results by comparison with results from the literature obtained by DE, FE methods and analytical calculations.

2.3. Elastic perfectly ductile Cutoff Bars applied to FRP hoop reinforcement

In order to prove the efficacy in applying hoop reinforcement on the extrados of domes, FRP strips have been simulated by addition of elastic perfectly ductile CoB with mechanical parameters and relative position derived from [1], [12].

The following chart Figure 6 contains collapse load computed by LA and DEM [1]. As the reader may note, the agreement between final results is satisfying enough. The area in between the DEM curve (DIANA Model, blue dotted line) and the FEM curve resulting from this method is due to a different way of modelling (more discretised nonlinearity in DIANA model). Compared to upper-bound LA, the FE model shows to be more conservative but proves that FRP strips increase the collapse load of a dome by preventing the formation of meridian cracks.

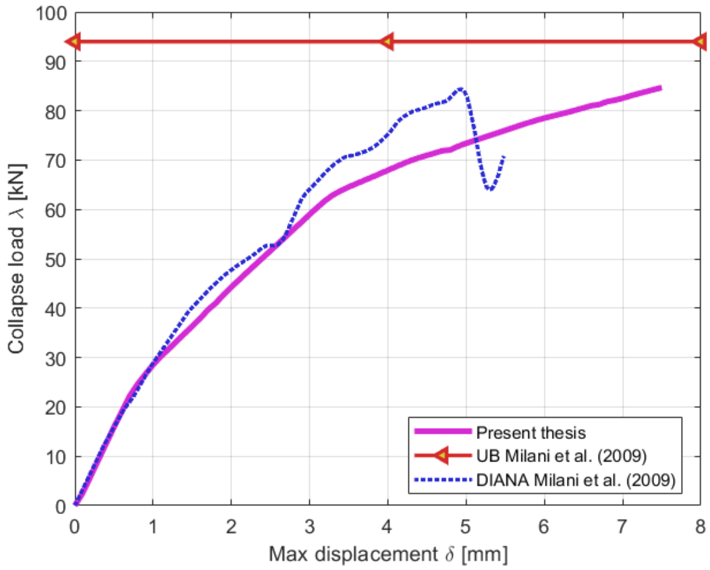


Figure 6: collapse load numerically computed after the addition of ductile CoB representing FRP strips application.

The following Figure 7 compares the deformed shapes of the unreinforced (a) and the reinforced (b) case, indicating where the FRP strips exert their confinement action against formation of meridian cracks and annular hinge.

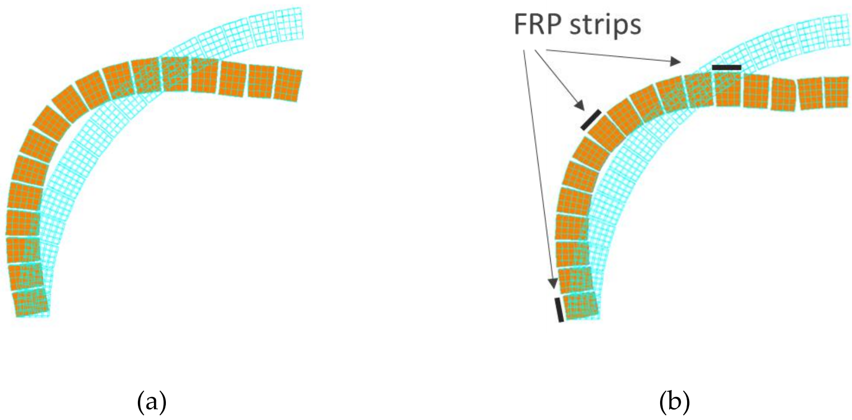


Figure 7: (a) unreinforced (note the smeared plastic hinge) vs (b) reinforced case deformed shapes.

3. Conclusions

To recap the results achieved, the first method models masonry nonlinearity by elastic perfectly brittle PC, neglecting most of masonry tensile resistance (by Heyman's hypotheses). Whereas in the second method, masonry nonlinearity is expressed by elastic perfectly plastic CoB. The brittle behaviour of the first model ends in giving an accurate position of plastic hinge but a very conservative value in terms of collapse load. The contrary is achieved by the ductility imposed in the second model. This one has been used as a baseline for the application of FRP strips, again by addition of ductile CoB. Indeed, the collapse load computed for the reinforced case (Figure 6), is almost twice the one of the unreinforced case (Figure 5), demonstrating the efficacy of such interventions. The method itself proved to be robust, therefore a very useful and practical way to model such additions for retrofitting or safety purposes.

Despite the simplicity in modelling and the short computational time, the present method can be further implemented with automatic choices depending on some initial data to be input (from standards or surveys).

The methods can be applied to real masonry hemispherical domes both in unreinforced and reinforced cases. They both have proven to be numerically stable and robust; to be useful for the purpose of the verification (the first in finding intermediate plastic hinge position, and the second and third in finding the collapse load); and to be trustworthy (by comparison with analytical and numerical results). Hence, they may easily be used in professional life in parallel or in substitution of LA.

References

1. G. Milani, E. Milani, and A. Tralli, "Upper bound limit analysis model for FRP-reinforced masonry curved structures. Part II: Structural analyses," *Comput Struct*, vol. 87, no. 23–24, pp. 1534–1558, 2009, doi: 10.1016/j.compstruc.2009.07.010.
2. P. Foraboschi, "Strengthening of Masonry Arches with Fiber-Reinforced Polymer Strips," *Journal of Composites for Construction*, vol. 8, no. 3, pp. 191–202, 2004, doi: 10.1061/(asce)1090-0268(2004)8:3(191).
3. G. Creazza, R. Matteazzi, A. Saetta, and R. Vitaliani, "Analyses of Masonry Vaults: A Macro Approach based on Three-Dimensional Damage Model," *Journal of Structural Engineering*, vol. 128, no. 5, pp. 646–654, 2002, doi: 10.1061/(asce)0733-9445(2002)128:5(646).
4. G. Creazza, A. V. Saetta, R. Matteazzi, and R. V. Vitaliani, "Analyses of masonry vaulted structures by using 3-D damage model," *European Congress on Computational Methods in Applied Sciences and Engineering, ECCOMAS 2000*, no. September, pp. 11–14, 2000.
5. G. Milani, "Closed form solutions in Limit Analysis for masonry cloister vaults and domes subjected to concentrated vertical loads applied at the top crown," *International Journal of Masonry Research and Innovation*.
6. E. Milani, G. Milani, and A. Tralli, "Limit analysis of masonry vaults by means of curved shell finite elements and homogenization," *Int J Solids Struct*, vol. 45, no. 20, pp. 5258–5288, 2008, doi: 10.1016/j.ijsolstr.2008.05.019.
7. G. Milani, "Simple lower bound limit analysis model for masonry double curvature structures," *Comput Struct*, vol. 269, p. 106831, 2022, doi: 10.1016/j.compstruc.2022.106831.
8. A. Chiozzi, G. Milani, and A. Tralli, "A Genetic Algorithm NURBS-based new approach for fast kinematic limit analysis of masonry vaults," *Comput Struct*, vol. 182, pp. 187–204, 2017, doi: 10.1016/j.compstruc.2016.11.003.
9. D. Aita, G. Milani, and A. Taliercio, "Limit analysis of masonry domes with oculus and lantern: a comparison between different approaches," *Mathematics and Mechanics of Solids*. In Printing
10. M. Pavlovic, E. Reccia, and A. Cecchi, "A Procedure to Investigate the Collapse Behavior of Masonry Domes: Some Meaningful Cases," *International Journal of Architectural Heritage*, vol. 10, no. 1, pp. 67–83, 2016.
11. Strand7 Pty Ltd., *Theoretical Manual. Theoretical backgrund to the Straus7 finite element analysis system*. 2004.
12. CNR-DT 200 R1/2013, "Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Interventi di Consolidamento Statico mediante l'utilizzo di Compositi Fibrorinforzati Materiali, strutture di c.a. e di c.a.p., strutture murarie," 2013.